An Introduction to Engineering Economics

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CHAPTER 2

Variable and Fixed Costs

Topics Covered

- Delineate Direct and Indirect Costs
- Familiarity with Cash Cost, Book Cost, Sunk Cost, and Opportunity Cost
- Fixed, Variable, and Incremental Costs
- Life Cycle of Enterprise
- Total Cost, Total Revenue, and Profit
- Break-Even Analysis
- Second Break-Even Point—What Does This Mean?

2.1 One-Time and Recurring Costs

An eminent expert in the field of business administration defines the ABCs of the MBA program as: (i) Accounting; (ii) Finance; and (iii) Economics. Executives tending to do well with limited resources usually thank accountants for where they derived their critical information. Sound accounting practices need a good understanding of costing principles. The types of costs can be delineated based on the frequency of their occurrence. For instance, *fixed costs* are usually one-time occurrences. *Variable costs* occur more frequently, usually once every billing period. *Incremental costs* are those that are incurred proportionally to the number of goods produced. Instances of fixed costs, variable costs, and incremental costs can be seen in Example 2.1.

In the Garlic Bread Nibbler snack factory as discussed below in Example 2.1, the following are examples of fixed costs:

- Property Tax
- Equipment Cost of Kneader
- Equipment Cost of Mixer
- Equipment Cost of Chopper
- Equipment Cost of Roaster
- Equipment Cost of Coloring Bowl
- Insurance Fees
- Interest on Borrowed Capital
- General Management & Adminis

Examples of variable costs are:

- Utility Costs: Gas Heating, Teleph
  Utilities, Air Conditioning
- Labor Benefits.

Examples of incremental costs:

- Wheat Flour
- Maize Flour
- Butter
- Salt.

Direct + Fixed = traceable to product and does not change = salary of project manager for a specific product

Direct + Variable = traceable to product but changes = tons of sand used in preparation of tiles

Indirect + Fixed = not traceable to product and does not change = rent of office building

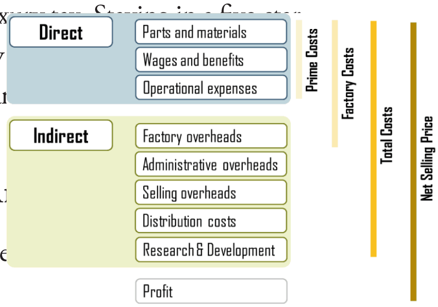
Indirect + Variable = not traceable to product and changes = HVAC and electricity bill

The planned costs per unit of output, anticipated labor hours, materials, and overhead categories are called *standard costs*. *Direct costs* can be measured and attributed to a specific activity or output. *Indirect costs* cannot be attributed to a single activity. Secretarial staff in an office building is an example of overhead costs. In addition to the above discussion, the following costs are also explained:

Cash Cost:	Transactions that involve payment of cash.
Book Cost:	This is a noncash cost such as depreciation on machinery, etc.

Sunk Cost:	Cost that has occurred in the past and has no relevance to estimates of future costs and revenues. It is a nonrefundable cash outlay. Examples of sunk cost are money spent on a passport or an earnest money deposit on a new home purchase.
Opportunity Cost:	Incurred because of the use of limited resources. For example, consider a recent graduate from a BS program at an Ivy League institution. When he signs up for graduate school, the salary he may have made from joining the work force is foregone. The difference in the salary in work force and the stipend received during graduate studies, say for five years, is called the opportunity cost.
Investment Cost:	The capital required in the acquisition phase is called investment cost.
O & M Cost:	Operation and Maintenance cost. This can be incurred by hiring people, purchasing machines, materials, information, and energy.
Working Capital:	These are funds required for the day-to-day operation of the enterprise.
Salvage Value:	The trade-in value of equipment after its full use is called salvage value.

A state of perfect competition is said to exist when there is more than one supplier of a product in demand. Some goods are *necessities* and some are *luxuries*. A necessity is a good that is vital to the function of the enterprise. For example, cash supply for the snack factory to pay its employees their wages is a necessity. Luxury is an item that is in surplus of need. For example, any guest in a five-star hotel is asked to pay a lux hotel is taxable as it is a surplus of need. Luxury for the survival of the enterprise. A fur coat is a



2.2 The Life Cycle of A

A life cycle exists for every business enterprise prise consists of the following stages:

(i) Identification of Need
(ii) Conceptual Preliminary Design
(iii) Bench Unit Trial or Prototype Testing
(iv) Detailed Design
(v) Production or Construction

(vi) Operation
(vii) Maintenance
(viii) Customer Use
(ix) Retirement and Disposal
(x) Recycling.

The cost associated with each step during the life cycle of an enterprise can be categorized according to the step in the cycle.

2.3 Total Revenue, Total Cost, and Profitable Region

The total cost incurred during the operation of an enterprise can be written as follows:

$$C_T = C_F + DC_V \tag{2.1}$$

where C_V is the variable cost, D is the demand in number of units, and C_F is the fixed cost. The price of the product usually falls with increase in demand in a linear manner.

$$p = a - bD \tag{2.2}$$

where p is the price and D is the demand in number of units; a is the price and zero demand and b is the rate of fall of price with increase in demand. It has been found that price is determined by supply and demand of goods in a market with perfect competition. The price changes by either falling or rising to equalize the goods produced and goods consumed. The price moves in such a fashion that some *equilibrium* is maintained. Other factors that may affect demand and supply—such as income of customers, price of raw materials that went into the manufacture of the product—do not figure into the price determination directly. Supply is usually driven by marginal cost. Additional output is determined by marginal cost.

The *law of demand* (*ceteris paribus*) states that the customers will be ready to buy more goods as the price decreases [1]. Eq. (2.2) is congruent with the law of demand. It is also consistent with the *elasticity* of supply and demand. Elasticity of demand is measured by the change in demand in response to a change in price. Elasticity of supply is measured by the change in supply in response to a change in price. If the quantity of demand changes

considerably with changes in unit price, the price-demand curve is said to be *elastic*. If the change in quantity was small, then the price-demand curve is said to be *inelastic*. Income elasticity of demand is not considered here. These are the changes that come about in supply and demand in response to change in income levels of the customers. There can be other factors that are not considered, such as cross elasticity of demand.

The total revenue generated by the business—TR—can be calculated as

$$\mathrm{TR} = \mathrm{pD} \tag{2.3}$$

$$\mathrm{TR} = \mathrm{D(a - bD)} \tag{2.4}$$

The total revenue reaches a maximum at a certain demand, D″. This can be calculated as follows:

$$\frac{dTR}{dD} = 0$$

$$a = 2bD''$$

$$D'' = \frac{2b}{a}$$

The break-even point is one where the total revenue, TR, exceeds total cost, C_T. This happens when

$$\mathrm{TR} = \mathrm{C_T} \tag{2.5}$$

Combining Eq. (2.1), Eq. (2.4), and Eq. (2.5):

$$0 = \mathrm{bD^2 + D(C_V - a) + C_F} \tag{2.6}$$

From Eq. (2.6), it can be realized that there can be *more than one* break-even point. The number of break-even points that will occur during a given enterprise depends on the order of the algebraic equation given by Eq. (2.6). For the price-demand relation given by Eq. (2.2), it can be seen from Eq. (2.6) that two break-even points can be expected. For enterprises where the price falls with demand in a nonlinear manner, more than two break-even points can be expected. The break-even points occur at D’ which can be calculated as

$$D' = \frac{(a - C_V) \pm \sqrt{(a - C_V)^2 - 4bC_F}}{2b} \tag{2.7}$$

The first break-even point occurs when the revenue from the sale of goods produced exceeds the costs incurred. The second break-even point occurs when there is overproduction. From the second break-even point forward, it is *counterproductive* to increase further the demand, D. Here the costs rise at a steady rate and the revenue reaches maxima and begins to fall with further increases in demand. At some point a *loss* is incurred due to excess production. The region in the revenue-demand graph where total revenue exceeds total cost is called a *profitable* region.

Example 2.1 Garlic Bread Nibbler Snack Factory

A team of professors and alumni got together to form a venture to make the Garlic Bread Nibbler Snack Factory. The raw materials procured were wheat, maize and corn flour, butter, and salt. The unit operations used in the preparation were kneader, mixer, baking oven, chopper, roaster, coloring, and packaging. The fixed costs that include leasing the building and land, insurance, taxes, and franchise fees to Snyder's of Hanover Pretzel Makers can be estimated at 1 million US$. The variable costs, including electric utilities, air conditioning, gas heating, telephone wires, etc. run the enterprise roughly 2 US$ per bag of snacks. The price-demand relation can be given by a *linear* relation such as

$$p = a - bD \tag{2.8}$$

When p is the price per bag and D is the number of bags of Nibblers produced, a and b can be given as \$10 and $5^* \, 10^{-6}$ (\$/bag). Find the two break-even points. Sketch the total revenue and total cost as a function of demand.

$$\text{Total Revenue, TR} = pD = aD - bD^2 \tag{2.9}$$

$$\text{Total Cost, } C_T = C_F + C_v D \tag{2.10}$$

$$C_F = 1 \text{ million US\$} \tag{2.11}$$

$$C_v = 2 \text{ US\$ per bag} \tag{2.12}$$

The total revenue and total cost as a function of bags of Nibbler produced is shown in Figure 2.1. At optimal production, the rate is about 114 bags per hour, assuming that the factory operates 24 hours, 7 days a

Table 2.1 Total Revenue, Total Cost for Garlic Bread Nibbler Snack Factory

# Bags	Price, p (per bag $)	TR, Total Revenue (Thousands of US$)	C_T, Total Cost (Thousands of US$)
0	10	0	1000
100,000	9.5	950	1200
200,000	9	1800	1400
300,000	8.5	2550	1600
400,000	8	3200	1800
500,000	7.5	3750	2000
600,000	7	4200	2200
700,000	6.5	4550	2400
800,000	6	4800	2600
900,000	5.5	4950	2800
1,000,000	5	5000	3000
1,100,000	4.5	4950	3200
1,200,000	4	4800	3400
1,300,000	3.5	4550	3600
1,400,000	3	4200	3800
1,500,000	2.5	3750	4000
1,600,000	2	3200	4200
1,700,000	1.5	2550	4400
1,800,000	1	1800	4600
1,900,000	0.5	950	4800
2,000,000	0	0	5000

week, and 52 weeks in a given calendar year. At this point, the revenue is a maximum. This can be seen in Figure 2.1 at 1 million bags produced in a year. The same can be obtained by the use of calculus as follows:

$$\frac{d(TR)}{dD} = 0 = a - 2bD \tag{2.13}$$

At maximum revenue,

$$D_{TR} = \frac{a}{2b} = \frac{10}{10 * 10^{-6}} = 10^6 \; bags \tag{2.14}$$

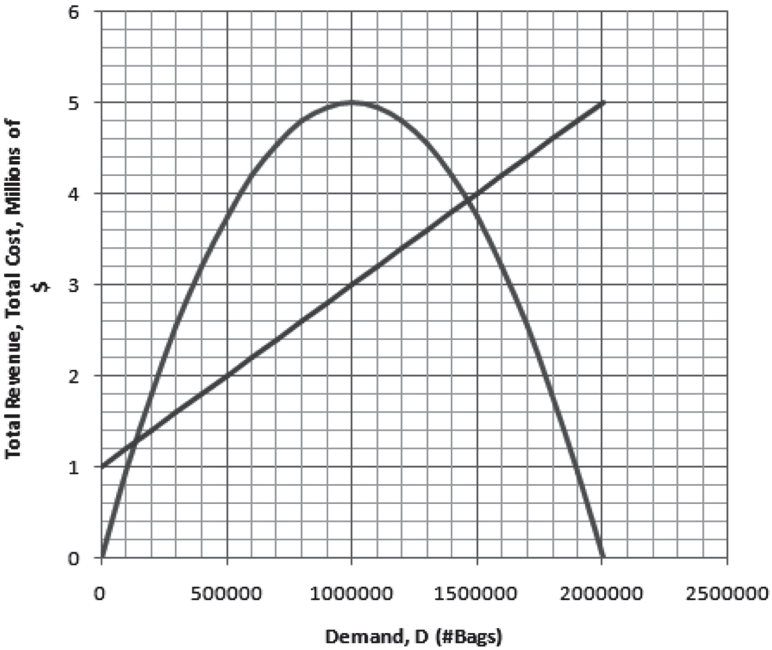


Figure 2.1 Total Revenue and Total Cost vs. Demand for Garlic Bread Nibbler Snack Factory

The profit generated can be calculated as

$$\begin{aligned}\text{Profit} = \text{P} = \text{TR} - \text{C}_\text{T} &= \text{aD} - \text{bD}^2 - \text{C}_\text{F} - \text{DC}_\text{v} \\ &= -\text{bD}^2 + \text{D}(\text{a-C}_\text{v}) - \text{C}_\text{F}\end{aligned} \tag{2.15}$$

For maximum profit,

$$\frac{dP}{dD} = 0 = -2bD + (a - C_v) \tag{2.16}$$

At maximum profit, the demand would be calculated as:

$$D_P = \frac{(a - C_V)}{2b} = \frac{(10 - 2)}{2 * 5 * 10^{-6}} = 800{,}000\ bags \tag{2.17}$$

800,000 bags checks out in the graphical solution or by generation of columns using a Microsoft Excel spreadsheet on a desktop computer.

The break-even points are when the total revenue equals the total cost. The first break-even point is when, for additional units produced, the

total revenue exceeds the total cost incurred. The second break-even point indicates a point of overproduction to a point of being counterproductive. Here, for additional units produced, the total revenue falls below the total cost.

$$TR = C_T \tag{2.18}$$

$$aD - bD^2 = C_F + DC_v \tag{2.19}$$

The break-even points can be obtained by the solution of the following quadratic equation:

$$bD^2 - D(a - C_v) + C_F = 0 \tag{2.20}$$

The first BEV (break-even) point occurs at a demand of 136,000–137,000 bags of Nibbler bags per year. The second BEV point occurs between 1,460,000 and 1,470,000 Nibbler bags per year. Further production of Nibbler bags would be counterproductive for revenue. The reason for the occurrence of the second break-even point is because the price falls inversely with the demand. When prices are low, the total revenue generated is not sufficient to exceed total cost. Total cost increases with demand in a linear manner.

2.4 Giffen and Veblen Goods

Not all goods obey the law of demand. Examples of *Veblen goods* are diamonds, luxury cars, fur coats, etc. These are status symbols. Families with higher incomes tend to buy them to show their higher status in society. The price of these goods rises with increase in demand. *Giffen goods* experience an increase in demand as the price rises. This does not obey the law of demand. During a potato famine, the price rises. Rather than less demand, poor families buy more potatoes to show their status.

An *elasticity of supply* can be defined as follows:

$$E_{sup} = \frac{\Delta S}{\Delta p} \tag{2.21}$$

Elasticity of supply as defined by Eq. (2.21) denotes the change in supply that can be expected for a small change in price. E_{sup} in most cases is positive, as stated by the law of supply. The supply curve is *concave*

curvature facing upward. The elasticity of supply for a considered good depends on factors such as availability of raw materials, length and complexity of production, time to respond, excess capacity, inventories, etc. The elasticity for gasoline is reported as 1.61 and that of cotton as 0.3. One of the assumptions of the law of demand is that the supply and demand are independent of each other.

An *elasticity of demand* can be defined in a similar manner to Eq. (2.21) as

$$E_{dem} = \frac{\Delta D}{\Delta p} \tag{2.22}$$

Elasticity of demand as defined by Eq. (2.22) denotes the change in demand that can be expected for a small change in price. E_{dem} in most cases is *negative* as stated by the law of demand. Price changes come about to equalize the product of price and demand. As demand increases, price decreases and vice versa. The demand y curve is *concave curvature* facing upward.

Example 2.2 Japanese Robots

The price-demand relation for a certain Japanese robot is given by

$$p = a - bD + cD^2 \tag{2.23}$$

How many break-even points do you expect? What is the demand at

(i) Maximum revenue?
(ii) Maximum profit?
(iii) Break-even point(s)?

$$\text{Total Revenue, TR} = pD = aD - bD^2 + cD^3 \tag{2.24}$$

The maximum revenue occurs at a demand that can be calculated as follows:

$$\frac{d(TR)}{dD} = 0 = a - 2bD + 3cD^2 \tag{2.25}$$

The extremas occur at

$$D' = \frac{2b \pm \sqrt{4b^2 - 12ac}}{6c} \tag{2.26}$$

For maxima, the second derivative of the TR (total revenue function) has to be negative, and evaluated at the extremum values of demand. The second derivative of TR is seen to be

$$\frac{d^2(TR)}{dD^2} = -2b + 6cD \tag{2.27}$$

This depends on the actual values of a, b, and c.

The cost function remains the same as

$$C_T = C_F + DC_V \tag{2.28}$$

The total profit can be calculated as:

$$\text{Total Profit} = (a\text{-}C_V)D - bD^2 + cD^3 - C_F$$

The demand at maximum profit can be estimated as follows:

$$\frac{dP}{dD} = 0 = 3cD^2 - 2bD + (a - C_V) \tag{2.29}$$

The maxima have to be confirmed by ensuring the second derivative of the profit function is positive evaluated at one of the extremum demand values. The second derivative of the profit function is given by

$$\frac{d^2P}{dD^2} = 6cD - 2b \tag{2.30}$$

The break-even points can be estimated when the total revenue and total cost are equal to each other.

$$(a - C_V)D - bD^2 + cD^3 - C_F = 0 \tag{2.31}$$

There are three break-even points that can be expected. These are obtained by the solution to the above cubic equation. The solution to cubic equations can be solved by two methods:

(i) Vieta's Substitution to Depressed Cubic Equation
(ii) and Numerical Solution.

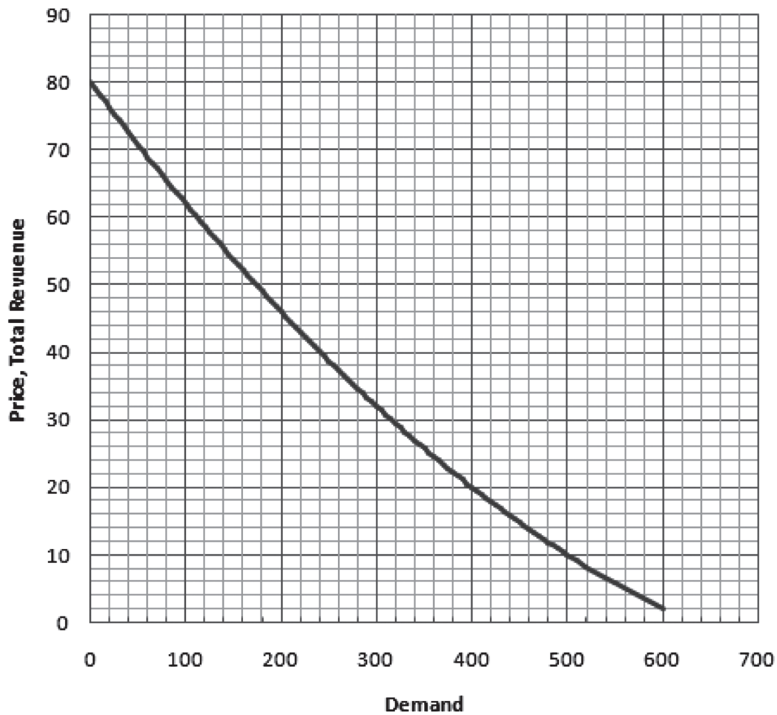


Figure 2.2 Price-Demand Curve for Chinese Lawn Mowers in Example 2.3

Example 2.3 Lawn Mowers from China

The price-demand relation for lawn mowers made by a Chinese manufacturer can be given by the following relation:

$$p = 80 - 0.19D + 10^{-4}(D^2) \tag{2.32}$$

This is shown in Figure 2.2.

The fixed cost to produce the lawn mowers, including taxes, license fees, and insurance is about \$2000. The variable cost that includes electric, gas, and water utilities can be taken as \$10 per lawn mower.

$$\begin{aligned} \text{Total Revenue, TR} &= pD = aD - bD^2 + cD^3 \\ &= 80D - 0.19D^2 + 1E\text{-}4D^2 \end{aligned} \tag{2.33}$$

The maximum revenue occurs at a demand that can be calculated as follows:

$$\frac{d(TR)}{dD} = 0 = a - 2bD + 3cD^2 \tag{2.34}$$

The extremas occur at

$$D' = \frac{2b \pm \sqrt{4b^2 - 12ac}}{6c} = \frac{+0.38 \pm 0.22}{0.0006} = 266.67 \;\&\; 1000 \tag{2.35}$$

For maxima, the second derivative of the TR, total revenue function, has to be negative, evaluated at the extremum values of demand. The second derivative of TR would be seen to be

$$\frac{d^2(TR)}{dD^2} = -2b + 6cD \tag{2.36}$$

Eq. (2.36) evaluated at the two roots for demand that causes an extremum value in the TR, total revenue function, are as follows:

$$\text{D}' = 266.67, \text{Eq. } (2.36) = -0.160002 \tag{2.37}$$

$$\text{D}' \text{ at } 1000, \text{Eq. } (2.36) = 0.22 \tag{2.38}$$

Thus the extremum of TR function at $D' = 1000$ corresponds to a minima. The other demand, $D' = 266.7$, corresponds to a maxima. The TR, total revenue function, is plotted using Microsoft Excel spreadsheets and shown in Figure 2.3. The maxima can be confirmed from the figure corresponding to a demand, $D' = 266.7$.

The cost function remains the same as

$$C_T = C_F + DC_V \tag{2.39}$$

The total profit can be calculated as

$$\text{Total Profit} = (a\text{-}C_V)D - bD^2 + cD^3 - C_F$$

The demand at maximum profit can be estimated as follows:

$$\frac{dP}{dD} = 0 = 3cD^2 - 2bD + (a - C_V) \tag{2.40}$$

The maxima have to be confirmed by ensuring the second derivative of the profit function is positive evaluated at one of the extremum demand values. The second derivative of the profit function is given by

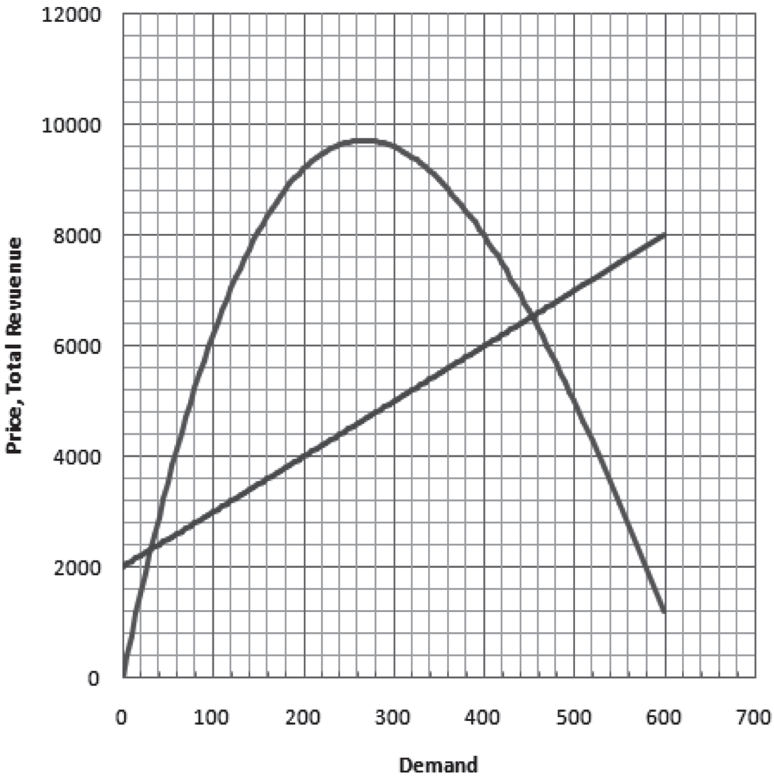


Figure 2-3 Maximum Total Revenue, Break-Even Points for Chinese Lawn Mowers in Example 2.3

$$\frac{d^2P}{dD^2} = 6cD - 2b \tag{2.41}$$

The break-even points can be estimated when the total revenue and total cost are equal to each other.

$$(a\text{-}C_V)D - bD^2 + cD^3 - C_F = 0 \tag{2.42}$$

The demand corresponding to maximum profit can be obtained by inspecting the table generated by an MS Excel spreadsheet. This can be seen to come about at a demand of about 225 lawn mowers. The demand, price, total cost, total revenue, and profit for 0–600 lawn mowers is shown in Table 2.2. The two break-even points can be seen from Table 2.2 to occur at 35 and 450 units.

A third break-even point can be expected, according to Eq. (2.42). In order to search for the third break-even point, the TR function was

Table 2.2 Total Revenue and Profit for Chinese Lawn Mowers

Law of Demand						
		Demand	Price	Total Revenue	Total Cost	Profit
a	80	D	p	TR	CT	P
b	0.19	0	80	0	2000	−2000
c	0.0001	20	76.24	1524.8	2200	−675.2
		40	72.56	2902.4	2400	502.4
C_v	10	60	68.96	4137.6	2600	1537.6
C_F	2000	80	65.44	5235.2	2800	2435.2
		100	62	6200	3000	3200
		120	58.64	7036.8	3200	3836.8
		140	55.36	7750.4	3400	4350.4
		160	52.16	8345.6	3600	4745.6
		180	49.04	8827.2	3800	5027.2
		200	46	9200	4000	5200
		220	43.04	9468.8	4200	5268.8
		240	40.16	9638.4	4400	5238.4
		260	37.36	9713.6	4600	5113.6
		280	34.64	9699.2	4800	4899.2
		300	32	9600	5000	4600
		320	29.44	9420.8	5200	4220.8
		340	26.96	9166.4	5400	3766.4
		360	24.56	8841.6	5600	3241.6
		380	22.24	8451.2	5800	2651.2
		400	20	8000	6000	2000
		420	17.84	7492.8	6200	1292.8
		440	15.76	6934.4	6400	534.4
		460	13.76	6329.6	6600	−270.4
		480	11.84	5683.2	6800	−1116.8
		500	10	5000	7000	−2000
		520	8.24	4284.8	7200	−2915.2
		540	6.56	3542.4	7400	−3857.6
		560	4.96	2777.6	7600	−4822.4
		580	3.44	1995.2	7800	−5804.8
		600	2	1200	8000	−6800

calculated for demands greater than 600 and plotted using an MS Excel spreadsheet and shown in Figure 2.4. A *third break-even point* can be seen at a demand corresponding to about 1420 units, and can be identified from Figure 2.4. This is subject to the applicability of the law of demand given by Eq. (2.32). For demand units greater than 629 lawn mowers, it can be seen from Eq. (2.32) that a "negative" price is associated with the lawn mowers. What is going on in the market is not clear in this regime. The Chinese manufacturer has to pay money to have customers haul away their product. This is also not that uncommon given the cash-down and free promotional offers, etc. advertised time and again on TV commercials. The price comes back again to positive territory for demand greater than 1270 lawn mowers. Maybe at this point the advertisement program and customer reports have worked in favor of the Chinese manufacturer. From this point onward the price increases (!), with increase in further demand. Thus the profits increase with further increase in demand after the third break-even point. It does not decline into a nonprofitable region like in the Garlic Nibbler Snack Factory in Example 2.1. Then again, in

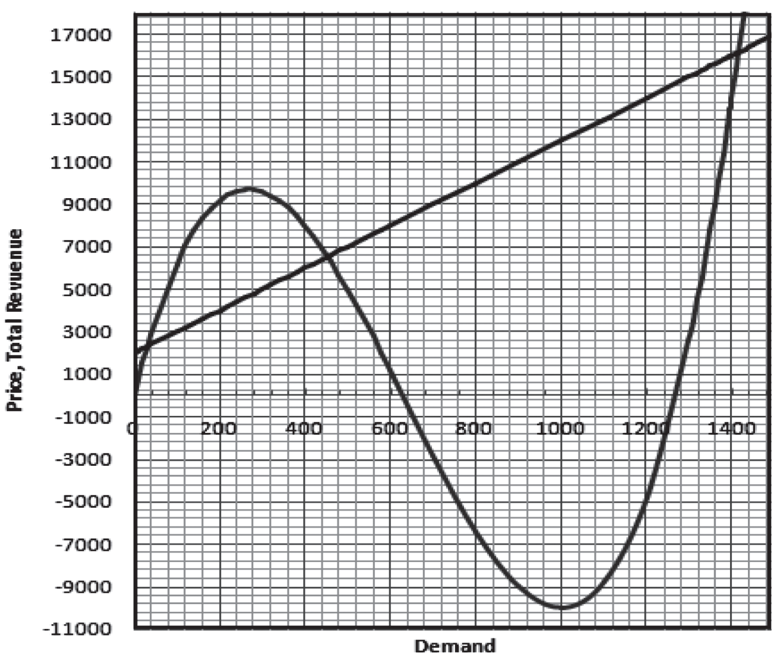


***Figure 2.4** Third Break-Even Point for Manufacture of Chinese Lawn Mowers in Example 2.3*

that example, the price-demand relation was linear. It obeys the law of demand and price falls with increase in demand.

The break-even points, the demand at which the TR (total revenue) reaches a maximum value, the demand at maximum profit needs to be calculated for every enterprise. The methodology is explained in the above sections. Much depends on the price-demand relation. If it is linear, then enterprise fortunes will conform to behavior as explained in Example 2.1. If the price-demand relation follows a quadratic or quadrilinear relation, the enterprise fortunes will behave in a fashion as explained in Example 2.3.

~~2.5 Dualistic Relations~~

Before the dawn of the industrial revolution and scientific progress, nations were affected by periodic famines. The marriage rate was inversely proportional to the price of bread. Floods, droughts, hurricanes, tsunamis, earthquakes, volcano eruptions, and other natural disasters resulted in the deaths of millions of the populace. Natural disasters were known; not much was done about them. The 21st millennium is a period of abundant supply of goods and services. The boom-and-bust cycles of economies sometimes leads to periods of depression. Present-day marriage and divorce rates rise and fall as a function of job opportunities rather than the cost of bread. Famine due to crop failure is averted by trade and the exchange of food supplies from other parts of the globe. The economic policy of the government is to foster economic progress, minimize damage from the bust cycles of the economy, and improve and maintain high standards of living.

Paul Samuelson [2] presented the first lesson in economics: Things are often not what they seem. For instance, if all farmers work hard and a bumper crop is realized, the total farm income falls! In his Nobel lecture in 1970, Samuelson drew an analogy from optics and thermodynamics to economics. He considered a profit-maximizing firm that sells its output a long a demand curve, in which price received is a nonincreasing function of the amount sold. Furthermore, suppose that the output can be generated from 2, 3, or 99 different inputs. He assumed that the production function relating outputs to inputs is smooth and concave. These time and age neural networks can be designed based on this example. Storing the 99 inputs and outputs on the computer is an interesting problem in

CHAPTER 3

Time Value of Money

Topics Discussed

- Simple and Compound Interest
- Future Worth of Series of Uniform Payments, A
- Present Worth of Series of Uniform Payments, A
- Inverse Problem of Finding i
- Inverse Problem of Finding N
- Gradient Series
- Variable Interest Rate
- Deferred Payments

3.1 Simple and Compound Interest

Capital is often needed for starting new business enterprises. Capital can be dollar currency or other assets such as stocks, bonds, and money market accounts owned in the form of a diversified portfolio. The worth of money can be calculated to the nearest dollar and cent. This chapter is devoted to calculations of the cost of money. It is built on the fundamental notion of interest charge on capital.

The simple interest, I on a principal P, can be calculated as follows at a given interest rate i and pay period N:

$$I = P\,i\,N \tag{3.1}$$

where I is the simple interest amount, P the principal amount, i the interest rate expressed as a fraction, and N is the pay period. Over the same pay period, the compound interest and amount owed can be calculated

as follows. At the end of the first pay period, the future amount, F, or amount owed is given by

$$F = P + iP = P(1 + i) \tag{3.2}$$

At the end of the second pay period, the future amount is given by

$$F = P(1 + i) + iP(1 + i) = (1 + i)(P + iP) = P(1 + i)^2 \tag{3.3}$$

In a similar manner, at the end of pay period N the future amount is given by

$$F = P(1 + 1)^N \tag{3.4}$$

The principal, P, is multiplied with the algebraic expression given by Eq. (3.4), in order to obtain the future amount F. This algebraic expression is called a *single payment compound factor*. Eq. (3.4) can be read from Figure 3.1 for interest rates, i ranging from 0 to 30% and pay period, N, ranging from 2 through 12. The single payment compound factor, P/F, given by Eq. (3.4) can be read from Figure 3.2 for interest rates i ranging from 0 to 30% and pay period, N, ranging from 15 through 100. The P/F axis is in logarithmic scale to accommodate the larger P/F values associated with larger pay period N at larger interest rates, i.

Eq. (3.4) may be rearranged to given P given F, i and N as

$$P = \frac{F}{(1 + i)^N} \tag{3.5}$$

The future amount, F, is multiplied with the algebraic expression given by Eq. (3.5), in order to obtain the present amount P. This algebraic expression is called a *single payment present worth factor*.

The pay period N can be calculated given the F, P, and i as follows:

$$N = \frac{\log\left(\frac{F}{P}\right)}{\log(1 + i)} \tag{3.6}$$

The interest rate i can be calculated given F, P, and N as follows:

$$i = \left(\frac{F}{P}\right)^{\frac{1}{N}} - 1 \tag{3.7}$$

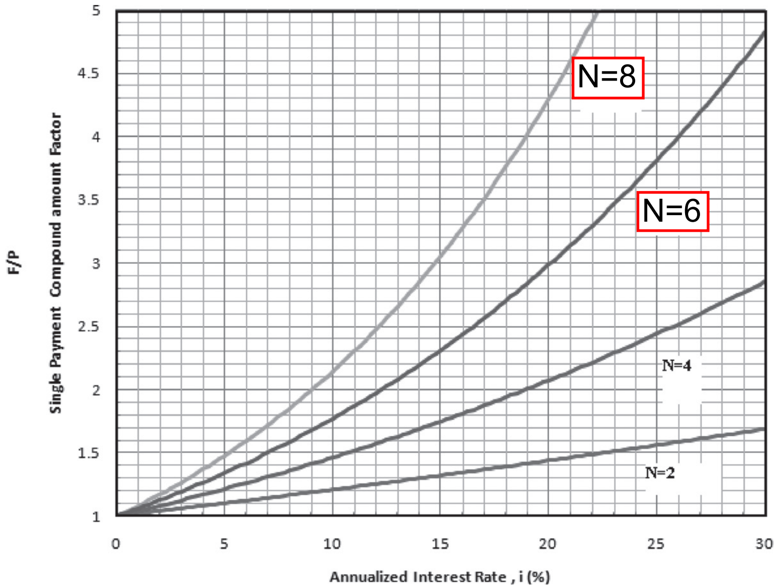


Figure 3.1 Single Payment Compound Amount Factor, F/P for $0 \leq i \leq 30\%$ and $2 \leq N \leq 12$

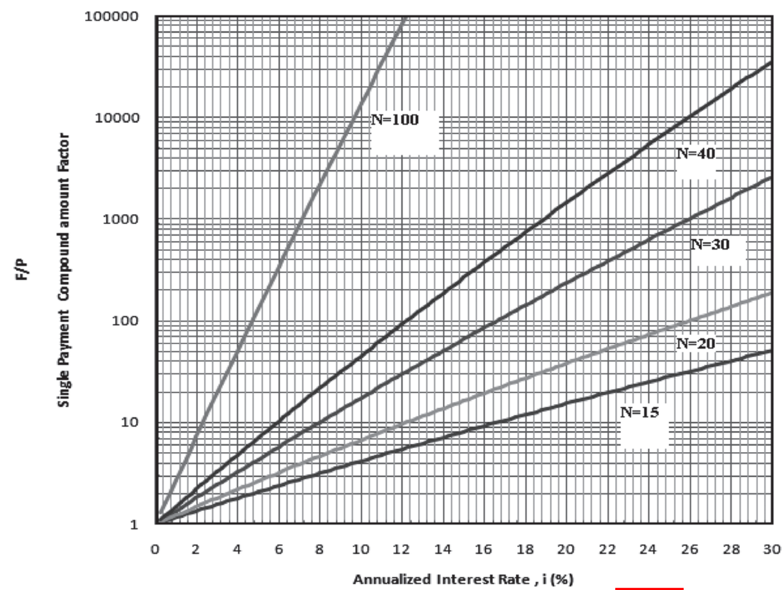


Figure 3.2 Single Payment Compound Amount Factor F/P for Interest Rate $0 \leq i \leq 30\%$ and Pay Period $15 \leq N \leq 100$

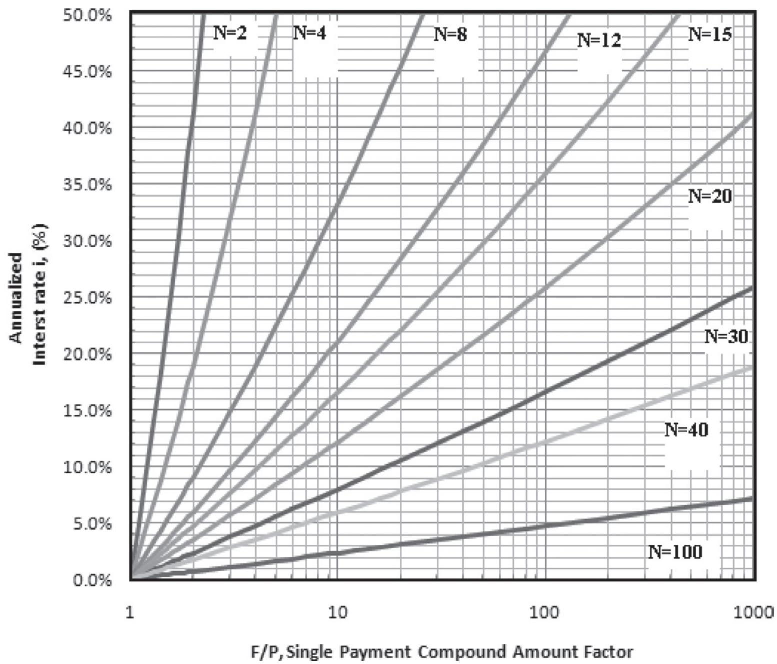


Figure 3.3 Interest Rate, i (%) vs. Single Payment Compound Amount Factor for Different Pay Periods, N

Eq. (3.7) is shown in an easily readable chart in Figure 3.3. The interest rates can be read from Figure 3.3 when F/P, the single payment compound factor and pay period N, are known. Different lines in the figure are for different pay periods N ranging from 2 through 100 years.

The compound interest can be calculated by subtracting P from the future amount given by Eq. (3.4).

$$\cancel{1} = P\left(\left(1 + \cancel{1}i\right)^{N} - 1\right) \tag{3.8}$$

Example 3.1 Compound Interest Calculated on an Everyday Basis

Your local bank branch recently announced a new savings plan with an interest rate of 0.0274% compounded daily. What is the interest payment on $52,000 at the end of one year?

$$F = P(1 + i')^{N} = 50{,}000{*}(1.000274)^{365} = \$55258.34 \tag{3.9}$$

From Eq. (3.8), the interest payment, I = $55,258.34-$50,000 = $5,258.34.

The history of mankind has seen the *uses* and *abuses* of compound interest. The farmers in India were persecuted by the feudal lords, or *Zamindars*, for late payment or nonpayment of loans. This was called bonded labor. Here the sons inherited the debts of their fathers and had to work them off. Over time, the compound interest ballooned in payment until the laborers could not repay the debt. Upon India's declaration of independence from the British Empire, the first prime minister, Jawaharlal Nehru, along with another prime minister who smiled on farmers, Mrs. Indira Gandhi, used to forgive the debts of farmers. The two political heads abolished bonded labor.

Bonded labor is an arrangement by which a person pays off his loan with direct labor in lieu of currency over a long period, sometimes even lifetimes. The term *debt slavery* is reserved for arrangements where the value of the work done is significantly greater than the original sum of money borrowed.

Peons are laborers who are bound in servitude until their debts are paid in full. Employers are allowed to extend credit to laborers to buy goods and services from employer-owned stores at exorbitant prices. This was used in the South after the Civil War. Poor African American and Caucasian farmers were called *sharecroppers*. They were given credit to purchase seed, fertilizer, and oxen from the landlord and made payments to him via a share of the crops.

3.1.1 *Effective Interest Rate*

Effective interest rate is the interest rate that gets compounded on an annual basis. Not all applications of interest rate are charged on an annual basis. Other applications need interest rate compounded on different bases, such as

(i) Daily
(ii) Weekly
(iii) Monthly
(iv) Quarterly
(v) Semiannual.

The interest rate compounded on any pay period is called the nominal interest rate. Federal laws such as the *truth-in-lending* law passed in Congress in 1969 require a statement regarding the APR (annual

percentage rate) being charged on promissory notes and contracts involving monetary transactions. Compounding is not included.

An *effective interest rate*, i can be defined as a function of the nominal interest rate as follows [1]:

$$i = \left(1 + \frac{i'}{P}\right)^P - 1 \tag{3.10}$$

where i′ is the *nominal interest rate* and P is the number of compounding periods per year. This is the interest earned on the principal on one year. Effective interest rates are expressed on an annual basis. For example, the effective interest rate for a nominal rate of 12% compounded on a monthly basis would be

$$i = \left(1 + \frac{0.12}{12}\right)^{12} - 1 = 0.12683 \tag{3.11}$$

A number of banks post their interest on savings accounts on a monthly basis and credit card companies calculate the interest charges due on the unpaid balance on a daily basis. A summary of effective interest rate i and nominal rate i’ for different compounding periods is calculated using Eq. (3.10) and summarized below in Table 3.1.

3.2 Uniform Series of Payments

Cash Flow Diagrams are illustrations that show all the monetary transactions during the time of an enterprise. They can be used to calculate the equivalence of a series of uniform payments in terms of its present worth, P, or in terms of its future worth, F. For example, consider the uniform payments A made as shown in Figure 3.4:

3.2.1 Given A, I, and N Find F

The equivalence of the uniform series of payments A for a said period N at a fixed interest rate i as shown in Figure 3.1 can be established. The future equivalent worth of the uniform series of payments can be estimated as follows: The payment A at time period 1 will grow after N years on account of compound interest using Eq. (3.4) to $A(1 + i)^{\cancel{N}}$. In a similar manner, the payment A at time period 2 will grow on account

Table 3.1 Effective Interest Rate i, as a Function of Nominal Interest Rate i'

	Nominal Rates								
	P	28%	24%	21%	18%	15%	12%	9%	6%
Daily	365	32.30%	27.11%	23.36%	19.72%	16.18%	12.75%	9.42%	6.18%
Weekly	52	32.21%	27.05%	23.32%	19.68%	16.16%	12.73%	9.41%	6.18%
Monthly	12	31.89%	26.82%	23.14%	19.56%	16.08%	12.68%	9.38%	6.17%
Quarterly	4	31.08%	26.25%	22.71%	19.25%	15.87%	12.55%	9.31%	6.14%
Semiannual	2	29.96%	25.44%	22.10%	18.81%	15.56%	12.36%	9.20%	6.09%
Annual	1	28.00%	24.00%	21.00%	18.00%	15.00%	12.00%	9.00%	6.00%

of compound interest accrued to $A(1+i)^{\cancel{N-1}}$ at the end of the pay period. Thus, the future equivalent worth of the uniform series of payments A can be given by

$$F = \cancel{A(1+i)^N +} A(1+i)^{N-1} + A(1+i)^{N-2} + \cdots + \cdots\cdot A(1+i) + A \tag{3.12}$$

Eq. (3.12) can be seen to be a geometric series. This can be summed up by multiplying Eq. (3.12) by the ratio of the series, (1 + i) as follows:

$$F(1+i) = \cancel{A(1+i)^N +} A(1+i)^N + A(1+i)^{N-1} + \cdots + A(1+i)^2 + A(1+i) \tag{3.13}$$

Subtracting Eq. (3.12) from Eq. (3.13), it can be seen that all but the term A in Eq. (3.10) and $A(1+i)^N$ in Eq. (3.13) gets canceled, and

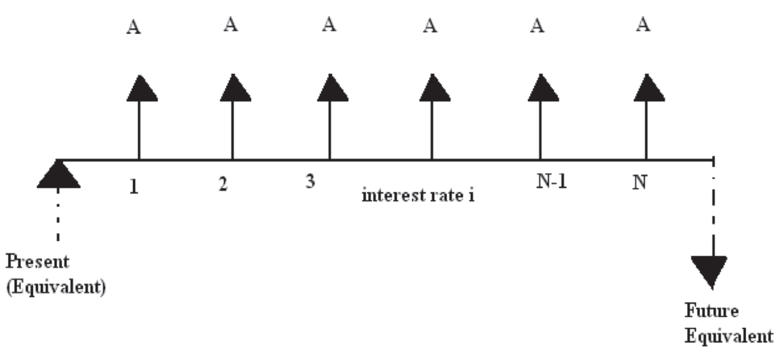


Figure 3.4 Cash Flow Diagram for Uniform Series of Payments A

$$F(1 + i - 1) = A(1 + i)^N - A \tag{3.14}$$

Or

$$F = \frac{A\left((1 + i)^N - 1\right)}{i} \tag{3.15}$$

The uniform series payment A needs to be multiplied with the algebraic expression given by Eq. (3.13) in order to obtain the future worth, F. This algebraic expression is called the *uniform series compound amount factor*. Given the future amount F, interest rate I, and pay period N, the uniform series payment A can be calculated by rearranging Eq. (3.15) as

$$A = \frac{iF}{(1 + i)^N - 1} \tag{3.16}$$

The future equivalent worth, F, needs to be multiplied with the algebraic expression given by Eq. (3.16) in order to obtain the uniform series payment A. This algebraic expression is called a *sinking fund factor*. The sinking fund factor given by Eq. (3.16) is shown in Figure 3.5 for values of interest rates, i (%) ranging from 0 to 30% and pay period N, ranging from 12 to 100. The sinking fund factor, A/F, given by Eq. (3.16) is shown as charts in Figure 3.6 for values of interest rates i(%) ranging from 0 to 30% and pay period N ranging from 2 to 8. It can be seen from Figures 3.5 and 3.6 that the nature of the function A/F vs. interest rate changes about N = 10. The power ponentiated expression $(1 + i)^N$ after N = 10 changes in a steeper manner. The curvature of the sinking fund factor vs. interest rate changes more rapidly at higher pay period values of N.

~~Example 3.2 Doubling Time of Investment~~

An investor deposits $1000 every month into his mutual fund with a savvy investment banker. The money was invested into stocks of companies that were marked as aggressively growing in the coming years. In how many pay periods would the future worth of the investments double in value?

In order for the Future worth, F, to double in value after pay period N, F = 2 NA (NA would be the value of the fund if the interest rate was 0%)

Using Eq. (3.13)

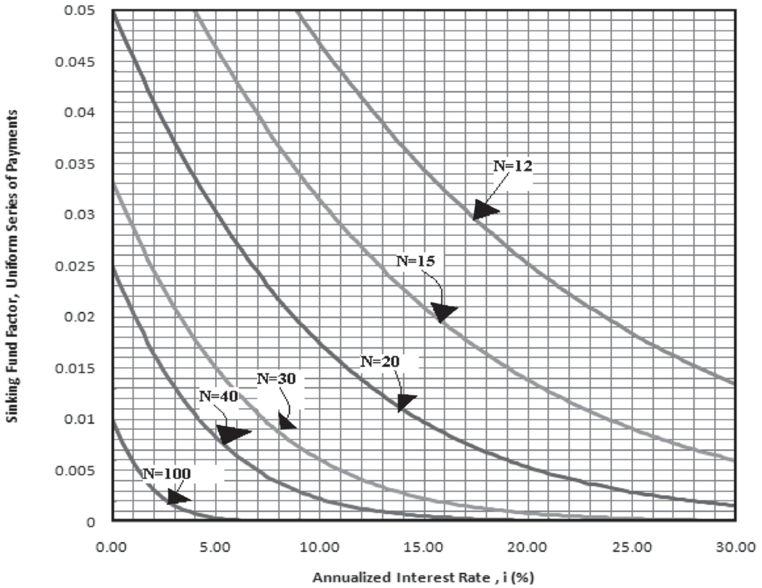


Figure 3.5 Sinking Fund Factor, A/F for Interest Rates, $0 \leq i \leq 30\%$ and Pay Period $12 \leq N \leq 100$

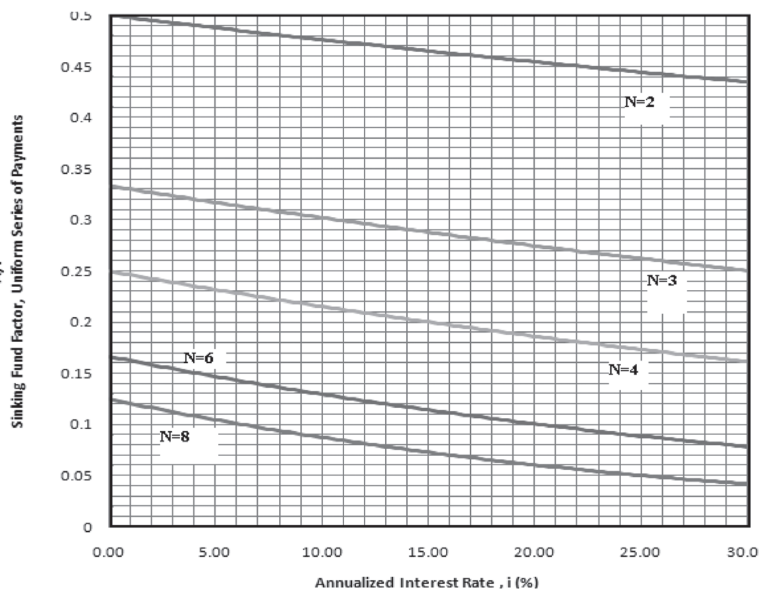


Figure 3.6 Sinking Fund Factor, A/F for Interest Rates $0 \leq i \leq 30\%$ and Pay Periods, $2 \leq N \leq 8$

$$F/A = 2N = \frac{F}{A} = 2N = \frac{\left((1 + i)^N - 1\right)}{i} \tag{3.17}$$

Expressing $(1+i)^N$ as a binomial series such as $1 + Ni + N(N - 1)i^2/2!$ at a truncation error of the order of magnitude of $O(i^3/3!)$, the above expression can be seen to be:

$$i = \frac{2}{(N - 1)} \tag{3.18}$$

The annualized interest rate i needed for the series of uniform payments A to double in value on account of compound interest paid—i.e., F = 2AN is shown in Table 3.2.

3.2.2 Given A, I, and N, Find P

The present worth, P, of the uniform series of payments, A, for the cash flow diagram shown in Figure 3.1 can be calculated by combining Eq.(3.15) and Eq. (3.4) to yield

Table 3.2 Interest Rate vs. Doubling Time of Investment(s)

End of Year #	Interest Rate Annualized, i
2	24%
3	12%
4	8%
5	6%
6	4.8%
7	4%
8	3.4%
9	3%
10	2.67%
11	2.4%
12	2.18%
13	2%
14	1.85%
15	1.71%

$$P = \frac{A\left((1+i)^N - 1\right)}{i(1+i)^N} \tag{3.19}$$

The algebraic expression given by Eq. (3.10) that the uniform series payment, A, needs to be multiplied with in order to obtain present worth, P, is called the *uniform series present worth factor*. Given the present amount P, interest rate I, and pay period N, the uniform series payment A can be calculated by rearranging Eq. (3.19) as

$$A = \frac{i(1+i)^N P}{(1+i)^N - 1} \tag{3.20}$$

The present worth P needs to be multiplied with the algebraic expression given by Eq. (3.20) in order to obtain the uniform series payment A. This algebraic expression is called a *capital recovery factor*. The capital recovery factor A/P for interest rates 0 ≤ i ≤ 30% for pay periods 0 ≤ N ≤ ∞ is shown as easily usable charts in Figure 3.8. It can be seen from the figure even for N = 12 the capital recovery factor is only 4% from the values at perpetual payment for N is ∞, infinity for higher values of interest rate.

Perpetual payment plan is one when the pay period N becomes infinity. The limit when N tends to infinity in Eq. (3.20) can be calculated as

$$A = \underset{N \to \infty}{Lt} \frac{iP}{1 - \frac{1}{(1+i)^N}} = iP \tag{3.21}$$

In Appendix A annuity tables are provided to read F/P, P/F, F/A, A/F, P/A, and A/P values for various values of interest rates ranging from 1/8% to 30% and pay periods N ranging from 1 to 100. In addition to the annuity tables for the uniform series of payments A, annuity tables for uniform gradient series of payments G are also provided in Appendix A.

Example 3.3 Perpetual Payment and Father's Day Prize

The general manager of a factory that makes oil-fired steam boilers wants to establish a prize in a high school that caters to the children of the employees of the boiler plant factory. The prize is forever and for the best all-around student in the penultimate year of schooling. It is awarded as a once-in-a-school-time prize, from nursery school to graduating high

school. Calculate the present amount that needs to be paid at an annualized interest rate of 10% for the prize amount to be $150 during Father's Day celebrations. Father's Day occurs once a year on June 21st, when daytime hours and nighttime hours are equal to each other.

$$\text{From Eq. (3.17), } P = \frac{A}{i} = \frac{150}{0.1} = 1500 \tag{3.22}$$

Example 3.4 0% Interest Rate

Calculate the present worth of a series of uniform payments of $300 per month for 72 months. This is for leasing a car from the car dealer near you. The interest rate has dropped to near zero, such that the effective interest rate i can be considered with little error to be zero.

Use of Eq. (3.19) for this example may seem like P would be infinity! This is because i is zero. On careful examination of Eq. (3.19), it can be seen that when i becomes zero, Eq. (3.19) takes on the $\frac{0}{0}$ indeterminate form. The present worth can be re-derived from Figure 3.1 when interest rate is 0. When interest rate is 0, there is no interest charged or compound interest levied. Therefore,

$$P = A + A + A + \cdots\cdots + A = AN \tag{3.23}$$

Thus, when i=0, Eq. (3.19) is not applicable to calculate P. But it can be seen to be equal to AN, where N is the pay period and A is the uniform series payment. This is the *asymptotic limit* of the principal amount when the interest rate i tends to 0%. The federal fund rates are 0.0% recently in the year 2010. During years when there is *deflation* instead of inflation, this may be more applicable.

Example 3.5 Nest Egg of Alice Smith

Alice Smith signed up with her company's 401(k) retirement savings plan. She pays $330 per month. Her employer matches dollar for dollar and pays as much per month into her retirement savings account. She is 26 years old. When she retires at 65, she hopes to obtain two lump-sum distributions. She can expect a return of 10% compounded annually as interest for her plan. Calculate the future equivalent amount of Alice Smith's retirement savings plan.

Eq. (3.15) can be used to calculate F.

A = 2*$330 = $660

N = 39 years

i = 10%

$$F = \frac{A\left((1+i)^N - 1\right)}{i} = \frac{660((1.1)^{39} - 1)}{0.1} = \$264{,}956 \quad (3.24)$$

~~Example 3.6 Diversified Portfolio~~

Darlene Shuster was the sole beneficiary of a life insurance benefit payment of $200,000 when her husband died. She invested the face amount as follows:

Stocks	$50,000
Bonds	$50,000
Money Market	$100,000.

The Dow Jones Industrial Average has performed at an average of 10% growth in value every year for the past 100 years, including the Great Depression and the market corrections of 1987 and 2007. The bond funds' yield is about 6% per year. The money market rates are 2.5% for the years 2010–2015; 5.0% for the years 2015–2020; and 8% for the years 2020–2030. After 20 years, what would the future worth of Ms. Shuster's inheritance be?

By Eq. (3.4), the investment in stocks would grow to

$$= (F_s/50{,}000,\ 10\%,\ 20)$$

$$= 50{,}000(1+0.1)^{20} \quad (3.25)$$

$$= \$336{,}375$$

By Eq. (3.4), the investment in bonds would grow to

$$= (F_B/50{,}000,\ 6\%,\ 20)$$

$$= 50{,}000(1+.06)^{20} \quad (3.26)$$

$$= \$160{,}357$$

From the derivation of Eq. (3.4) from Eq. (3.3), it can be seen that the money market funds would grow to

$$100{,}000(1 + 0.025)^5(1 + 0.05)^5(1 + 0.08)^{10} = \$311{,}748 \quad (3.27)$$

Adding the results from Eqs. (3.20–3.22), the future worth of Darlene's investment portfolio would be

$$336{,}375 + 160{,}357 + 311{,}748 = \$808{,}480$$

Example 3.7 George Tataseo's Royalty Payments and Investment

George Tataseo writes books. He receives royalty payments ranging from 8%–15% of the cover price of every book sold in the market. The twice-yearly payment is roughly about \$8000. The expenses incurred every month during the preparation of the book are about \$400. George invests the balance completely into a stock-based mutual fund. The return is about 9.5% on an annualized basis. After 10 years, what would the future worth of George's investments be?

Effective interest rate for six months for return on stock fund

$$= 9.5\%/2 = 4.75\% \quad (3.28)$$

Number of six-month periods in 10 years = 10* 2 = 20

Future Worth = (F/8000, 4.75%, 20) – (F/400,9.5/12,120) (3.29)

$$= 8000^* \frac{(1.0475)^{20} - 1}{0.0475} - 400 * \frac{(1.0475)^{120} - 1}{0.0475}$$

$$= 8000^*32.2056 - 400^*12.4027 \quad (3.30)$$

$$= \$252{,}684$$

The proposed idea of dividing 9.5% by 2 is not accurate.

In order to find the exact interest rate for 6 months, we have to assume that 9.5% is the result of compounding 2 times a year

(from page 28 of the PDF file): $i = \left(1 + \frac{i'}{P}\right)^P - 1$

Hence i=9.5 (0.095) and we want i' for P=2 (two times a year)

Here, i' is calculated as:

[sqrt(0.095 + 1) -1] x 2 = 9.28% so we want: (F/8000 , 9.28/2 , 20)

Likewise for the 12 month yearly expenditures we want:

[rt_{12}(9.5 + 1) – 1] x 12 = 9.1% so we want: (F/400 , 9.1/12 , 120)

So this is the correct future worth:

(F/8000 , 9.28/2 , 20) - (F/400 , 9.1/12 , 120) = 176,855

$$F = \frac{A\left((1+i)^N - 1\right)}{i}$$

~~3.2.3 Given F, I, and A, Find N~~

Current books on the subject refer to numerical solutions or trial-and-error methods for obtaining the pay period N given the future equivalent worth F, interest rate i, and uniform series payment A. Here, a closed-form analytical solution is obtained to calculate the pay period N. Eq. (3.15) can be rearranged as follows:

$$\frac{iF}{A} + 1 = (1 + i)^N \quad (3.31)$$

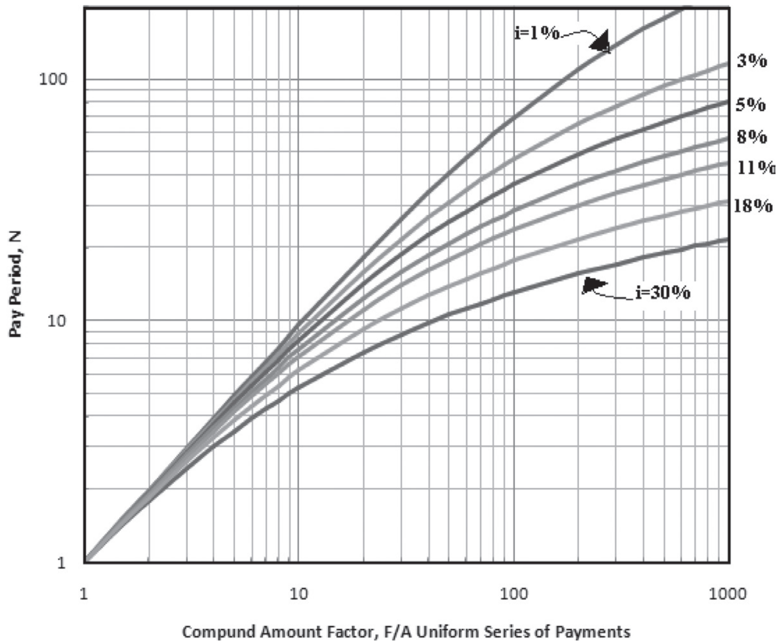


Figure 3.7 Pay Period N vs. Compound Amount Factor, F/A for Interest Rates 1% ≤ i ≤ 30%

$$N = \frac{\log\left(\frac{iF}{A} + 1\right)}{\log(1 + i)}$$

Eq. (3.31) is shown in Figure 3.7 as an easily readable chart. The pay period N needed to achieve a certain investment goal F using a uniform series of payments A for various interest rates ranging from 1% ≤ i ≤ 30% can be read from Figure 3.6. Both the axes are logarithmic. This scale captures the functionality of N with respect to F/A and i as shown in Eq. (3.31).

3.2.4 Given P, I, and A, Find N

Current books on the subject refer to numerical solutions or trial-and-error methods for obtaining the pay period N given the present equivalent worth P, interest rate i, and uniform series payment A. Here, a

closed-form analytical solution is obtained to calculate the pay period N. Eq. (3.19) can be rearranged as follows:

$$1 - \frac{iP}{A} = \frac{1}{(1+i)^N} \tag{3.32}$$

$$N = \frac{\log\left(\dfrac{A}{A - iP}\right)}{\log(1+i)}$$

~~3.2.5 Given F, A, and N, Find I~~

Current books on the subject refer to numerical solutions or trial-and-error methods for obtaining the interest rate i, given the pay period N and given the future equivalent worth F, and uniform series payment A. Here, a closed-form analytical expression is obtained to calculate the interest rate i. It can be noted that the i in the $(1 + i)^N$ is a small number. Given

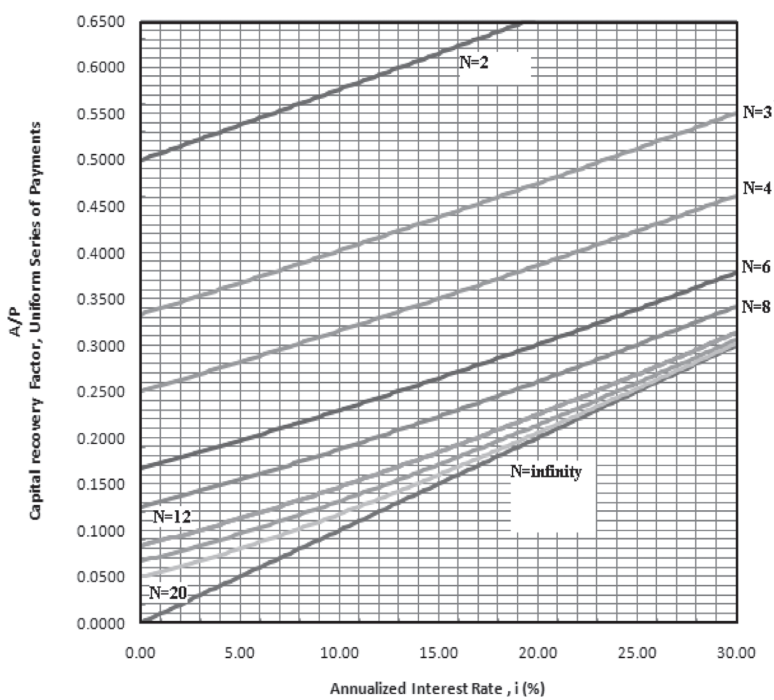


Figure 3.8 Capital Recovery Factor, A/P for Interest Rates $1 \leq i \leq 30\%$ and Pay Period $2 \leq N \leq \infty$

1 is present, the expression is better suited for expansion in the form of a binomial infinite series as follows:

$$(1 + \mathrm{i})^{\mathrm{N}} = 1 + \mathrm{Ni} + \frac{N(N-1)i^2}{2!} + \cdots \tag{3.33}$$

The terms with i^3 and higher order terms are neglected. The truncation error involved is of the order of magnitude of $O(\mathrm{i}^3/3!)$

Combining Eq. (3.33) and Eq. (3.15)

$$\frac{iF}{A} = Ni + \frac{N(N-1)i^2}{2!} \tag{3.34}$$

$$or,\ i = \frac{2}{(N-1)}\left(\frac{F}{AN} - 1\right)$$

~~3.2.6 Given P, A, and N, Find I~~

Numerical solutions or trial-and-error methods are needed for obtaining the interest rate i, given the pay period N, and given the present equivalent worth P, and uniform series payment A. Here, a closed-form analytical expression is obtained to calculate the interest rate i. It can be noted that the i in the $(1 + \mathrm{i})^{\mathrm{N}}$ is a small number. Given 1 is present, the expression is better suited for expansion in the form of a binomial infinite series as follows:

$$(1 + \mathrm{i})\mathrm{N} = 1 + \mathrm{Ni} + \frac{N(N-1)i^2}{2!} + \cdots \tag{3.35}$$

The terms with i^3 and higher order terms are neglected. The truncation error involved is of the order of magnitude of $O(\mathrm{i}^3/3!)$.

Combining Eq. (3.24) and Eq. (3.15)

$$\frac{P}{AN} = \frac{2 + (N-1)i}{2 + 2i + N(N-1)i^2} \tag{3.36}$$

$$\frac{P(N-1)}{A}i^2 + i\left(\frac{2P}{AN} - (N-1)\right) + \frac{2P}{AN} - 2 = 0$$

The interest rate i can be calculated by solution of the quadratic equation arrived at in Eq. (3.25).

Thus,

$$ai^2 + bi + c = 0 \tag{3.37}$$

where,

$$a = \frac{P(N-1)}{A}; \quad b = \frac{2P}{AN} + 1 - N$$

$$c = \frac{2P}{AN} - 2$$

Example 3.8 U-Pay-Now & I-Pay-Later

You are hired by Cowboy Investments upon graduation. They are developing a new investment plan called "U-Pay-Now & I-Pay-Later" for customers. The deal is that the client pays \$500 every month to Cowboy Investments. After a said period in a certain number of years, Cowboy Investments will have the client cease payments; the client then receives \$500 every month forever. At a 10% annualized interest rate (the historic performance of Dow Jones), how many years would you recommend the said period be?

Let the said period of investments = N years (Figure 3.9)

$$(F/A,I,N) = (F/6000,10\%,N)$$

$$F = A(\frac{\left(1+i\right)^N - 1}{i})$$

The future worth after N years is invested until perpetuity

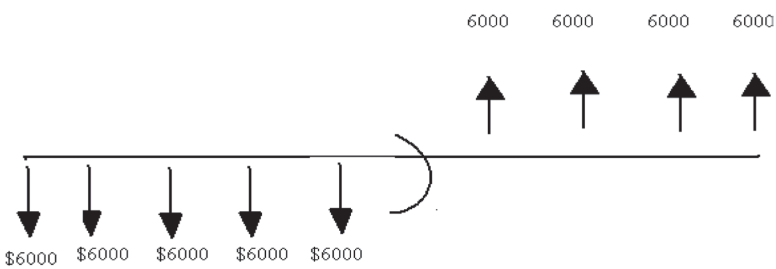


Figure 3.9 Cash Flow Diagram for "U-Pay-Now & I-Pay-Later"

(P/A, 10%, ∞)

$$\frac{P}{A} = \frac{(1+i)^N - 1}{i(1+i)^N}$$

$$\underset{N \to \infty}{Lt} \left(\frac{iP}{A}\right) = 1 - \frac{1}{(1+i)^N} = 1$$

$$P = \frac{A}{i} \tag{3.38}$$

It can be seen that from the "U-Pay-Now & I-Pay-Later" plan

$$F = A\left(\frac{(1+i)^N - 1}{i}\right) = \text{P} = \frac{A}{i}$$

$$(1+i)^N = 2 \tag{3.39}$$

$$N = \frac{\ln(2)}{\ln(1.1)} = 7.3\ \textit{years}$$

~~Example 3.8~~ When Does a Micro-Power Plant Become More Attractive Investment?

K. R. Sridhar, CEO of Bloom Box, was featured on CBS's *60 Minutes*. Bloom Box makes micro power plants using SOFC, solid oxide fuel cell technology. A bloom box can be purchased for \$3000. It consists of miniaturized fuel cells made of silica. The cost of electricity to power up a new home would be 8.0 cents per kWh. Should the existing utility company charge 12.5 cents per kWh, calculate the number of years it would take for a bloom box to be a good investment for new homeowner Jack Tripper. Jack heads a family of six members with four children. Assume that 600 kWh of electricity is consumed every month at the Tripper residence. (i = 10%).

$$N = \frac{\log\left(\frac{A}{A - iP}\right)}{\log(1+i)}$$

$$N = \frac{\log\left(\frac{iF}{A} + 1\right)}{\log(1+i)}$$

At the point when a Bloom Box will become lower cost compared with existing utility company rates, i.e., after N years

$$\$3000 + 600{*}.08{*}12(P/A,10\%,N) = 600{*}0.125{*}12(P/A,10\%,N) \tag{3.40}$$

$$3000 = 324\left(\frac{1.1^N - 1}{0.1(1.1)^N}\right) \tag{3.41}$$

$$0.9259 = 1 - \frac{1}{1.1^N}$$

$$\frac{1}{1.1^N} = 0.07407$$

$$N = \frac{\ln(13.5)}{\ln(1.1)} = 27.3\ \textit{years}$$

3.3 Uniform Gradient Series of Payments

Under certain circumstances, the receipts and expenses of an enterprise are projected to change by a uniform amount each period. A portion of the periodic payment *multiplies* each pay period. These sequences of cash flows are called uniform gradient series of cash flows.

The present equivalent worth of the uniform gradient series of cash payments shown in Figure 3.10 can be calculated as follows:

$$P = \frac{G}{(1+i)} + \frac{G(2)}{(1+i)^2} + \frac{G(3)}{(1+i)^3} + \cdots + \frac{G(N-1)}{(1+i)^{N-1}} + \frac{G(N)}{(1+i)^N} \tag{3.42}$$

$$P = G\sum_{n=1}^{N}\frac{n}{(1+i)^n} \tag{3.43}$$

Figure 3.10 Cash Flow Diagram of Uniform Gradient Series of Payments

Multiply Eq.(3.42) on both sides by $(1 + i)^N$

$$P(1+i)^N = G(1+i)^{N-1} + 2G(1+i)^{N-2} + 3G(1+i)^{N-3} + \cdots + (N-1)G(1+i) + NG \qquad (3.44)$$

The right-hand side, RHS of Eq. (3.44), can be split as follows:

$$RHS = G\left[(1+i)^{N-1} + (1+i)^{N-2} + (1+i)^{N-3} + (1+i) + 1\right] + G\left[(1+i)^{N-2} + (1+i)^{N-3} + \ldots(1+i)) + 1\right] + G\left[(1+i)^{N-3} + \cdots + (1+i) + 1\right] + \cdots G \qquad (3.45)$$

Each of the terms within the square brackets in Eq. (3.45) is a geometric series and can be summed up as

$$RHS = G\frac{(1+i)^N - 1}{i} + G\frac{(1+i)^{N-1} - 1}{i} + G\frac{(1+i)^{N-2} - 1}{i} + G\frac{(1+i)^{N-3} - 1}{i} + \cdots + G\frac{(1+i) - 1}{i} \qquad (3.46)$$

Combining Eq. (3.46) and Eq. (3.44) and simplifying

$$iP(1+i)^N = G\left(\frac{(1+i)^{N+1} - 1}{i} - N\right) \qquad (3.47)$$

Thus, P/G can be seen to be

$$P = \frac{G}{i(1+i)^N}\left(\frac{(1+i)^{N+1} - 1}{i} - N\right) \qquad (3.48)$$

A *gradient to uniform series conversion factor* can also be defined by combining Eq. (3.48) and that for A/P, Eq. (3.20). Thus,

$$A = \frac{G}{(1+i)^N - 1}\left(\frac{(1+i)^{N+1} - 1}{i} - N\right) \qquad (3.49)$$

The future worth of the uniform gradient series of payments can be calculated by combining Eq. (3.48) and Eq. (3.4). Thus,

$$F = \frac{G}{i}\left(\frac{(1+i)^{N+1} - 1}{i} - N\right) \qquad (3.50)$$

3.4 Continuous Compounding

As discussed in Section 3.1.2, compounding periods may vary from daily, weekly, monthly, quarterly, semiannually, yearly, etc. All these are discrete compounding periods. The interest gets calculated after discrete periods of time. Eq. (3.10) gives the relation between the nominal interest rate per compounding period and the effective interest rate per year.

$$i = \left(1 + \frac{i'}{P}\right)^{P} - 1 \tag{3.51}$$

Let P/i′ = m, Eq. (3.51) then becomes

$$i = \left(1 + \frac{1}{m}\right)^{mi'} - 1 \tag{3.52}$$

When the compounding period is changed from discrete to continuous, m in Eq. (3.52) becomes ∞, infinity. Further,

$$\begin{aligned}
\underset{m\to\infty}{Lt}\left(1 + \frac{1}{m}\right)^{m} &= 1^{m} + m(1)^{m-1}\frac{1}{m} + \frac{m(m-1)}{2!m^2}1^{m-2} \\
&\quad + \frac{m(m-1)(m-2)}{3!m^3}1^{m-3} + \cdots \\
&= \underset{m\to\infty}{Lt}\ 1 + 1 + \frac{1}{2!}(1 - \frac{1}{m}) + \frac{1}{3!}(1 - \frac{1}{m})^2 + \cdots \\
&= 1 + 1 + \frac{1}{2!} + \frac{1}{3!} + \cdots
\end{aligned} \tag{3.53}$$

By Taylor series expansion, e^x evaluated at x = 1 can be seen to be

$$e^1 = 1 + 1 + \frac{1^2}{2!} + \frac{1^3}{3!} + \cdots = 2.71828182845905 \tag{3.54}$$

Comparing Eq. (3.53) and Eq. (3.54)

$$\underset{m\to\infty}{Lt}\left(1 + \frac{1}{m}\right)^{\infty} = e \tag{3.55}$$

Combining Eq. (3.52) and Eq. (3.55)

$$i = e^{i'} - 1 \tag{3.56}$$

The effective interest rate for *continuous compounding* is given by Eq. (3.56).

3.4.1 F/P, F/A, P/A, A/F, and A/P for Continuous Compounding

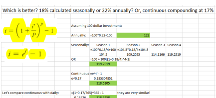
The corresponding formulae for F/P, F/A, and P/A with continuous compounding that were presented for discrete compounding earlier are given side by side in Table 3.3.

Example 3.10 Continuous Compounding

Show the effect of continuous compounding by comparing the future worth of the nest egg of Alice Smith as discussed in Example 3.4. Sketch the future worth of Alice Smith's nest egg as a function of number of years N. Use the same interest rate for discrete as well as continuous compounding.

$$\text{Discrete Compounding: } F_D = \$660^*(F/A,10\%,N)$$

$$= \frac{(1+i)^N - 1}{i} = \frac{1.1^N - 1}{0.1} \quad (3.57)$$


Which is better? 18% calculated seasonally or 22% annually? Or, continuous compounding at 17%

$i = \left(1 + \frac{i'}{P}\right)^P - 1$

$i = e^{i'} - 1$

Assuming 100 dollar investment:					
Annually:	=100*0.22+100	122			
Seasonally:	Season 1	Season 2	Season 3	Season 4	
	=100*0.18/4+100	=104.5*0.18/4+104.5			
	104.5	109.2025	114.1166	119.2519	
OR	=100 + 100[(1+0.18/4)^4-1]				
	119.2519				
Continuous	=e^i' - 1				
e^0.17	0.185304851				
	118.5305				
Let's compare continuous with daily:	=(1+0.17/365)^365 - 1	they are very similar!			
0.18526	118.5258				

Table 3.3 Factors Used to Calculate Discrete and Continuous Compound Interest

To Find	Given	Discrete Factor by Which to Multiply	Compounding Factor Name	Factor Functional Symbol	Continuous Factor by Which to Multiply	Compounding Functional Symbol
F	P	$(1+i)^N$	Single Payment Compound Amount	(F/P,i%,N)	~~ei'~~N	(F/P,i′%,N)
F	A	$\frac{(1+i)^N - 1}{i}$	Uniform Series Compound Amount	(F/A,i%,N)	$\frac{e^{i'N} - 1}{e^{i'} - 1}$	(F/A,i′%,N)
P	A	$\frac{(1+i)^N - 1}{i(1+i)^N}$	Uniform Series Present Worth	(P/A,i%,N)	$\frac{e^{i'N} - 1}{e^{i'N}(e^{i'} - 1)}$	(P/A,i′%,N)
A	F	$\frac{i}{(1+i)^N - 1}$	Sinking Fund	(A/F,i%,N)	$\frac{e^{i'} - 1}{e^{i'N} - 1}$	(A/F,i′%,N)
A	P	$\frac{i(1+i)^N}{(1+i)^N - 1}$	Capital Recovery	(A/P,i%,N)	$\frac{e^{i'N}(e^{i'} - 1)}{e^{i'N} - 1}$	(A/P,i′%,N)

The interest rate used is 10%, which is the average performance of the Dow Jones over the past 100 years.

Continuous Compounding:

The nominal interest rate i' is taken as 10%.

Continuous Compounding: $F_C = \$660*(F/A,i'\%,N)$

$$= \frac{e^{i'N} - 1}{e^{i'} - 1} = \frac{e^{0.1N} - 1}{e^{0.1} - 1} \tag{3.58}$$

Eq. (3.58) and Eq. (3.57) were calculated in an MS Excel spreadsheet and plotted in Figure 3.3. It can be seen from Figure 3.11 that the effect of continuous compounding is only seen after 25 years!

3.4.2 Given F, A, and I′, Find N During Continuous Compounding

Current books on the subject refer to numerical solutions or trial-and error-methods for obtaining the pay period N given the future equivalent worth F, interest rate for continuous compounding, i′% per year

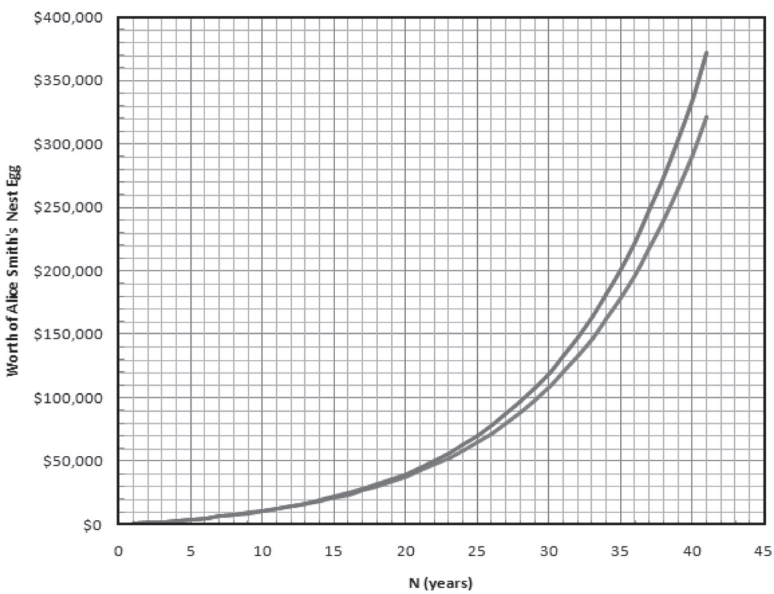


Figure 3.11 Worth of Alice Smith's Nest Egg (F/A) for Continuous & Discrete Compounding

and uniform series payment A. Here, a closed-form analytical solution is obtained to calculate the pay period N. The factor for obtaining F/A for continuous compounding given in Table 3.3 can be seen to be

$$F/A = \frac{e^{i'N} - 1}{e^{i'} - 1} \tag{3.59}$$

Rearranging Eq. (3.50)

$$\left(e^{i'} - 1\right)\frac{F}{A} + 1 = e^{i'N} \tag{3.60}$$

Obtaining the natural logarithms on either sides of Eq. (3.51), the period N can be calculated as

$$N = \frac{1}{i'}\ln\left(1 + \frac{F(e^{i'} - 1)}{A}\right) \tag{3.61}$$

Example 3.11 Time Taken to Achieve Investment Goal

Adam Jackson met with his financial planner and came up with his financial goal of making 1 million dollars. He was willing to deposit ~~$1050~~ every month with the financial planner, who was going to invest Mr. Jackson's monies into stocks, bonds, money market, and gold. The financial planner projected a return of a nominal rate of 12% compounded on a monthly basis. How many years will it take for

(i) Adam to achieve his financial goal using discrete compounding
(ii) Adam to achieve his financial goal using continuous compounding?

$$\text{F/A} = 10^6/1000 = 1000 \tag{3.62}$$

The effective interest rate for a nominal interest rate of 12% per month is 12.68% (3.63)

By Eq. (3.31) for discrete compounding (natural logarithms may be used) the time taken to attain the investment goal may be estimated as

$$N = \frac{\log\left(\frac{iF}{A} + 1\right)}{\log(1 + i)} = \frac{\ln(0.1268{*}1000 + 1)}{\ln(1.1268)} = 40.63\ \mathit{years} \tag{3.64}$$

Nominal	Annual	Semi Annual	Quarterly	Monthly	Daily	Continuous
	1	2	4	12	365	infinity
2	2.000	2.010	2.015	2.018	2.020	2.020
4	4.000	4.040	4.060	4.074	4.081	4.081
6	6.000	6.090	6.136	6.168	6.183	6.184
8	8.000	8.160	8.243	8.300	8.328	8.329
10	10.000	10.250	10.381	10.471	10.516	10.517
12	12.000	12.360	12.551	12.683	12.747	12.750
14	14.000	14.490	14.752	14.934	15.024	15.027
16	16.000	16.640	16.986	17.227	17.347	17.351
18	18.000	18.810	19.252	19.562	19.716	19.722
20	20.000	21.000	21.551	21.939	22.134	22.140
22	22.000	23.210	23.882	24.360	24.599	24.608
24	24.000	25.440	26.248	26.824	27.115	27.125
26	26.000	27.690	28.647	29.333	29.681	29.693
28	28.000	29.960	31.080	31.888	32.299	32.313
30	30.000	32.250	33.547	34.489	34.969	34.986

By Eq. (3.61) for continuous compounding, the time taken to attain the investment goal may be estimated as

$$N = \frac{1}{i'} \ln\left(1 + \frac{F(e^{i'} - 1)}{A}\right) = \frac{1}{0.12\cancel{68}} \ln\left(1 + 1000^*(e^{0.12\cancel{68}} - 1)\right)$$

$$= \cancel{38.75}\ years \tag{3.65}$$

The time taken to achieve the investment goal of Adam Jackson changes only by about 1.88 years, even when continuous compounding is considered.

Example 3.12 Biodiesel Production from Waste Cooking Oil

The study of alternate fuel sources to gasoline and coal is of national importance, given the supply-and-demand characteristics of fuels that are used extensively. Biodiesel is derived from vegetable oil or animal fats. It is recommended for use as a substitute for petroleum-based diesel because it is renewable and biodegradable. The common method of preparation of biodiesel is the transesterification of triacylglycerols in vegetable oil or animal fat with an alcohol such as methanol in the presence of an alkali or acid catalyst. The products are FAME (fatty acid methyl esters) and are called biodiesel. Glycerin is formed as a byproduct. Sodium hydroxide or potassium hydroxide is used as an alkali catalyst. A recent study [2] evaluated the economic feasibility of a plant producing approximately 22 million pounds per year of biodiesel. Tallow was transesterified with methanol in the presence of an alkali catalyst. A second plant is based on canola seed used as the raw material. A by-product of credit can be awarded for glycerin produced from seed crushing. A summary of the capital investment, process cost, and revenue accrued of the three different plants is shown in Table 3.4. The capital cost can be assumed to be paid off over a 30-year period at an interest charge of 6% per year.

Which process is the most profitable?

It is assumed that the capital cost is amortized at an interest rate of 6% per year for 30 years.

Plant I

(A/P, 6%, 30) from Table A-10, the capital recovery factor can be seen to be 0.072649

Table 3.4 Economic Evaluations for Biodiesel Production Plants

	Plant I Alkali-Catalyzed Continuous Process	Plant II Alkali-Catalyzed Batch Process	Plant III Alkali-Catalyzed Continuous Process
Plant Capacity	22 million pounds per year	1.7 million pounds per year	2.2 million pounds per year
Raw Material Used	Beef Tallow	Canola Oilseed	Animal Fats
Total Capital Cost	$12 million	$1 million	$3.12 million
Total Manufacturing Cost	$34 million	$5.95 million	$3.4 million
Glycerin Credit	$6 million	$0.9 million	$1.2 million
Price	$2/lb	$4/lb	$3/lb

Profit = 2*22 – 12*(A/P, 6%,30) – 34 + 6 = 15.128 million

Revenue – capital recovery – cost of production
+ by-product credit = profit (3.66)

Plant II

(A/P, 6%, 30) from Table A-10, the capital recovery factor can be seen to be 0.072649

Profit = 4*1.7 – 1*(A/P, 6%,30) – 5.95 + 0.9 = 1.68 million

Revenue – capital recovery – cost of production
+ by-product credit = profit (3.67)

Plant III

(A/P, 6%, 30) from Table A-10 the capital recovery factor can be seen to be 0.072649

Profit = 3*2.2 – 3.12*(A/P, 6%,30) – 3.4 + 1.2 = 4.17 million

Revenue – capital recovery – cost of production
+ by-product credit = profit (3.68)

Thus, Plant I is the most profitable, followed by Plant III and then Plant II.

Example 3.13 Credit Card Debt of U.S. Households

A recent survey shows there are about 114 million households in the United States. The average credit card debt carried per household is \$15,519. There are about 576.4 million credit cards in circulation. At an average credit card interest rate of 21% per year, what would the debt be if not paid back in another 10 years?

$$P = 576.4*15{,}519 = \$8.9451516 \text{ trillion} \tag{3.69}$$

$$(F/P,21\%,10) = 6.73 \text{ (From Table A-21)} \tag{3.70}$$

$$F = 6.73*8.9451516 = \$60.2 \text{ trillion} \tag{3.71}$$

Example 3.14 Time Taken to Pay Off Credit Card Debt

In Example 3.11, how many years will it take for U.S. households to pay off their current household debts at a rate of 21% per year? Assume that the monthly payment per credit card is \$300 per month.

$$P/A = 15{,}519/3600 = 4.3108 \tag{3.72}$$

$$\text{From Table A-21, N can be seen to be } 12 < N < 14 \tag{3.73}$$

Another way to calculate the number of years to pay off the debt is by applying Eq. (3.32)

$$N = \frac{\log\left(\dfrac{A}{A - iP}\right)}{\log\left(1 + i\right)} = \frac{\ln\left(\dfrac{3600}{3600 - 3259}\right)}{\ln\left(1.21\right)} = 12.36\ years \tag{3.74}$$

Example 3.14 Deferred Annuity

Katy Gerstner, a widow, was planning for her son Joe's college education. What principal amount should Katy set aside now to allow for annual withdrawals of \$4500 each on Joe's 18th, 19th, 20th, and 21st birthdays? The interest rate is taken at 11% per year.

Four withdrawals are to be made of \$4500 each on Joe's 18th, 19th, 20th, and 21st birthdays, as shown in Figure 3.12. The present equivalent worth of this annuity on Joe's 17th birthday is P_{17}.

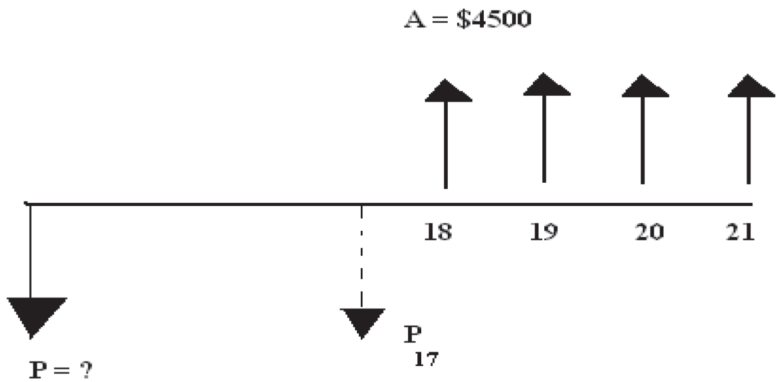


Figure 3.12 Deferred Annuity Payments for Joe Gerstner's College Education

$P_{17} = \$4500(P_{17}/A,11\%,4)$

$(P_{17}/A, 11\%, 4)$ from Table A-15 can be seen to be 3.1024 (3.75)

$P_{17} = \$4500*3.1024 = \$13,961$

The present equivalent worth of P_{17} can be calculated from a single payment compound factor as follows:

$$P = P_{17}(1 + i)^{-17} \tag{3.76}$$

(P/F, 11%, 17) from Table A-15 can be seen to be 0.170557 by interpolating between N= 16 and N = 18

$$P = P_{17}*0.170557 = \$13,961*0.170557 = \$2381.15 \tag{3.77}$$

			4500				
			18	19	20	21	SUM
F/P	1.11		687.6998	619.5494	558.1526	502.8402	2368.242

Expressions to calculate simple interest and compound interest given the principal, P, interest rate i%, and pay period N are provided. The single payment compound amount factor, F/P, as a function of an annualized interest rate, i, is plotted in Figures 3.1 and 3.2 for different pay periods. The characteristics of the curve change for N > 12. Expressions for pay period N and interest rate i% in terms of the F and P are given in Eq. (3.6) and Eq. (3.7). The interest rate vs. F/P for different N is shown

in Figure 3.3. *Nominal interest rates* get compounded on a different basis other than annual. The relation between nominal interest rate and *effective interest rate* is summarized in Table 3.1.

Cash Flow Diagrams are illustrations that show all the monetary transactions during the time of an enterprise. They can be used to calculate the equivalence of a series of uniform payments in terms of its present worth, P, or in terms of its future worth, F. The sum of a geometric series is used to develop an expression for F/A in terms of interest rate, i%, and pay period N for a uniform series of constant payments, A. F/A is called the *uniform series compound amount factor*. A/F is the *sinking fund factor*. Sinking fund factor is plotted vs. effective interest rate for different pay periods in Figures 3.5 and 3.6. In Appendix A, annuity tables are provided to read F/P, P/F, F/A, A/F, P/A, and A/P values for various values of interest rates ranging from 1/8% to 30%, and pay periods N ranging from 1 to 100. In addition to the annuity tables for the uniform series of payments A, annuity tables for uniform gradient series of payments G are also provided in Appendix A. Expressions for *uniform series present worth factor* are also developed. When N becomes infinity, perpetual payment schemes can be developed. Expressions for *capital recovery factor*, A/P are provided. Cash flow diagrams with a uniform gradient series of payments are discussed. Closed-form analytical expressions for P/G, F/G, and A/G are developed.

The inverse problem of finding N given F, A, and i% or P, A and i% is discussed. Closed-form analytical expressions for N are developed using logarithms. Graphical methods and interpolation in annuity tables may also be used. The inverse problem of finding interest rate i% given F, A, and N or P, A, and N are discussed. In addition to the use of annuity tables, graphical methods, a closed-form analytical expression is developed. This is achieved via the use of binomial expansion of $(1 + i)^N$. As many terms in the infinite series as needed for desirable accuracy can be taken.

Worked examples on doubling time of investment, nominal interest rate, perpetual payment for college days prize, 0% interest rate, nest egg estimates, diversification of portfolio, royalty payment and investment, payment and reimbursement schemes, micro power plant, biodiesel production from waste cooking oil, credit card debt for all U.S. households, deferred annuity, time taken to pay off credit card debt, are provided. Exercises on monthly car payments, number of years taken to pay off the

credit card debt, IRR of off-shore drilling, deferred interest rate charges, federal government debt, a Versailles-type home, variable interest rate, deferred payments, payment amount of a uniform series of payments, 0% interest, perpetual payment, design of a social security system for a town, and hydrogen from biomass are developed.

Continuous compounding methods, as opposed to discrete compounding methods, are also developed. The corresponding formulae for F/P, F/A, and P/A with continuous compounding that were presented for discrete compounding earlier are given side by side in Table 3.3. Appendix B provides the annuity tables for continuous compounding.

3.6 References

[1] W. G. Sullivan, E. M. Wicks, and C. P. Koelling, *Engineering Economy*, 14th Edition. Pearson Prentice Hall (2009), Upper Saddle River, NJ.

[2] Y. Zhang, M. A. Dube, D. D. McLean, and M. Kates, "Biodiesel Production from Waste Cooking Oil: Economic Assessment and Sensitivity Analysis," *Bioresource Technology*, Vol. 90, 3 (2003), 229–240.

[3] P. L. Spath, J. M. Lane, M. K. Mann, and W. A. Amos, "Update of Hydrogen from Biomass-Determination of the Delivered Cost of Hydrogen," Milestone Report for U.S. Department of Energy's Hydrogen Program (2001), NREL Laboratory, Golden, CO.

CHAPTER 4

Methods for Evaluation of Capital Projects

Topics Covered

- Present Worth Analysis
- Future Worth Analysis
- Annual Worth Analysis
- Internal Rate of Return, IRR
- External Rate of Return, ERR
- Payback Period

4.1 Overview

The *profitability* of projects can be evaluated in terms of dollars and cents. In this chapter, five different methods are presented that can be used to determine the worthiness of undertaking an enterprise or project. The time taken for recovery of investment can also be calculated. The five different methods are:

(i) Present Worth, PW;
(ii) Future Worth, FW;
(iii) Annual Worth, AW;
(iv) Internal Rate of Return, IRR;
(v) External Rate of Return, ERR.

In addition to these five methods, the *payback period* is a method to estimate the time period over which the monies invested can be recovered.

This is not recommended as a primary decision rule. Hence, it is not listed in the five methods mentioned above. The above methods can be used to make decisions based on dollars and cents.

A MARR, minimum acceptable rate of return, can be specified. In order to be selected as a project, a return from the business enterprise has to be established that exceeds a minimum specified level. This minimum level can be called a MARR. A number of factors can go into the determination of a MARR. These include, but are not limited to:

a) Amount of Monies;
b) Source of Monies;
c) Cost of Monies;
d) Number of Feasible Projects;
e) Purpose of Feasible Projects;
f) Risk of Investment Opportunities;
g) Organizational Ethos.

The cost of monies can be calculated using the methods discussed in Chapter 3.0.

4.2 Present Worth Analysis (PW)

The PW, present worth of a project, can be calculated by the equivalence of all cash inflows (such as receipts) and all cash outflows (such as expenses) to the current time at an interest rate that is a MARR. When the calculated PW is positive, the project is acceptable for further pursuits. The period of time, N, can be obtained suitably from the cash flow diagram. The factors considered during PW analysis are as follows:

(i) Purchase Price of Equipment Needed;
(ii) Estimated Useful Life;
(iii) Operational and Maintenance Costs;
(iv) Energy and Other Utility Costs;
(v) Salvage Value;
(vi) Revenue Accrued;
(vii) Interest Rate, i (%).

The effect of *inflation* and *deflation* on raw materials and energy costs can be taken into account as well, using inflation factors. This will be illustrated in Worked Example 4.4. The inflation factor gives the rate at which the prices go up every year. This information can be obtained from the Bureau of Labor Statistics. Consumer Price Indices are reported and available readily via Internet in the public literature.

When N becomes infinity, PW of such projects can also be called CW, the *capitalized worth* of the projects. The CW method is popular when endowments are established and public projects with indefinite lives are pursued.

Example 4.1 Hydrogen from Fast Pyrolysis and Steam Re-Forming [1]

Hydrogen is seriously considered as an alternative fuel in the transportation sector. In one process, the biomass is pyrolyzed and then steam re-formed to yield hydrogen. Biomass is dried and then converted to oil by very quick exposure to heated particles in a fluidized bed. The char and gases produced are combusted to supply heat to the reactor. The product oils are cooled and condensed. Bio-oil is then shipped by truck from the plant location(s) to the hydrogen production facility. By shipping the bio-oil, which has a higher energy density than the biomass, transportation costs are reduced. On arrival, the bio-oil is fractionated using water extraction into carbohydrate fraction and lignin fractions. The carbohydrate fraction is steam re-formed and the hydrogen that is formed is separated using PSA, pressure swing adsorption. Consider a plant that produces 75,790 kg/day of hydrogen by the pyrolysis process. The biomass feed rate is about 1806 Mg/day. Capital investment fixed costs are \$59.4 million. The selling price of hydrogen is \$10/GJ. The storage and transportation cost of hydrogen through railroad is \$0.94/GJ. The calorific value of hydrogen is 142 MJ/kg. At a MARR of 11%, calculate the PW of the project. The plant ~~has operated~~ for five years. The biomass feedstock cost was \$16.5/Mg. The plant operated at 80% time during the year. The costs of utility and wages of laborers can be lumped into 15c/kg of hydrogen produced.

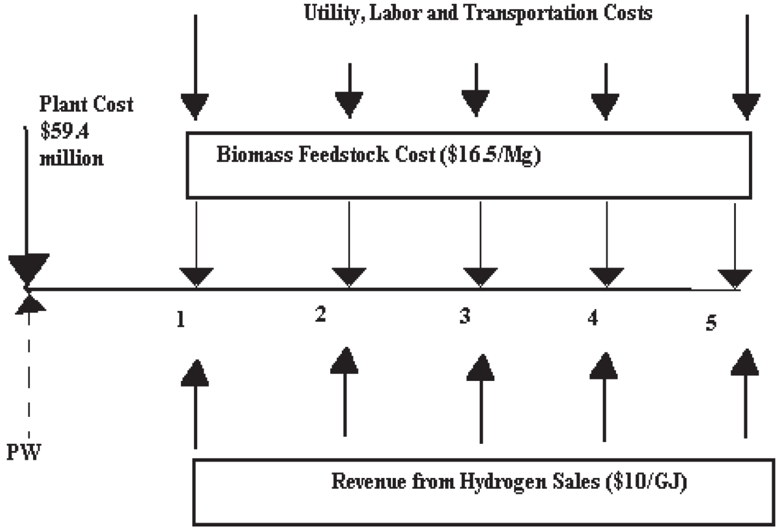


Figure 4.1 Cash Flow Diagram for Revenue and Costs for Hydrogen Production from Pyrolysis and Steam Re-Forming

PW = −$59.4 million (capital cost)
−16.5*1806*365*0.8 (P/A,11%,5) (raw material cost)
−365*0.8*75,790*142/1000*0.94 (P/A,11%,5)
(transportation cost)
$-0.15*75790*365*0.8/10^6$(P/A, 11%,5) (utility and wages)
+10*75,790*365*0.8*142/1000(P/A,11%,5)
(revenue from sales of hydrogen) (4.1)

(P/A,11%,5) = 3.6959 (From Table A-15) (4.2)

PW = −$59.4 million − 3.6959*8.7 − 3.6959*2.9603
− 3.6959*3.31 + 3.6959*31.43
= $1.433405 million = $1,433,405 (4.3)

A present worth of $1.433 million indicates that the project can be selected at 11% MARR. It can be seen that as the interest rate or MARR is increased, at some interest rate the PW will become 0. This interest rate later will be seen to be the IRR, the internal rate of return. For interest rates less than the IRR, the project will be profitable.

Example 4.2 Replacement for the World Trade Center (WTC)

One proposal to replace the World Trade Center (WTC) came from an architect and real estate tycoon named Donald Stone. He proposed another high-rise to replace the WTC, which was destroyed by terrorists on September 11, 2001. In a typical week, 50,000 people are expected to work in these towers and 200,000 visitors are expected to go the top floor to obtain a glimpse of New York City from the 110th floor. The cost of constructing the building is estimated at $5 billion. Assuming a net profit of collecting rent from the occupants is $250 per person per week and a visitor fee of $10, how many years would it take to see a profit? The interest rate can be taken to be 4%.

$$\begin{aligned}\text{Rental Fee collected per year} &= 250*50{,}000*52/10^6 \\ &= \$\ 650 \text{ million}\end{aligned} \tag{4.4}$$

$$\begin{aligned}\text{Visitor Fee collected per year} &= 200{,}000*10*52/10^6 \\ &= \$104 \text{ million}\end{aligned} \tag{4.5}$$

$$A/P = (650+104)/5000 = 0.1508 \tag{4.6}$$

From Table A-8, (4% annuity table) for A/P = 0.1508, N can be seen to be a little less than eight years.

In a little less than eight years, the venture of replacing the WTC will be profitable.

Example 4.3 Capitalized Worth of Stone's Replacement for the WTC

In Example 4.2, calculate the capitalized worth of Stone's proposed replacement for the WTC destroyed by terrorists in the 2001 bombings. The interest rate is 4% and the revenues can be taken as the same in Example 4.2.

$$CW = (P/A, 4\%, \infty) \tag{4.7}$$

By Eq. (3.19)

$$P = \frac{A\left((1+i)^N - 1\right)}{i(1+i)^N} \tag{4.8}$$

When N→∞, Eq. (4.8) becomes

$$P = \frac{A\left(1 - \frac{1}{(1+i)^N}\right)}{i} = \frac{A}{i} \tag{4.9}$$

Thus, CW = ($650 + 104)/0.04 = $18.850 billion dollars.

Thus, the capitalized worth of the replacement for the WTC proposed by architect Stone would be ~~$18.5~~ billion dollars.

4.3 Future Worth Analysis (FW)

The FW (future worth) of a project can be calculated by the equivalence of all cash inflows such as receipts and all cash outflows such as expenses to a future time at an interest rate that is a MARR. When the calculated FW is positive, the project is acceptable for further pursuits. The period of time, N, can be obtained suitably from the cash flow diagram.

Example 4.4 University Apartments

A group of university alumni and professors invests $25 million to build 250 new student apartments near a state university. The money is borrowed at the prime rate of 3.25% calculated on a yearly basis. The period of the loan is for 30 years. The variable expenses that include maintenance and operation run the group $3000 per apartment per year. The rental fees collected are about $12000 per apartment per year. The occupancy rate is about 70%. What is the future worth of the enterprise, should the salvage value of the apartments be $5 million at the end of 30 years?

(F/P,3.25%,30) = Interpolated from Table A-8 and Table A-7 for 4% & 3%

$$= 2.427262 + 0.25*(3.243398 - 2.427262) = 2.6313 \tag{4.10}$$

(F/A,3.25%,~~20~~) = Interpolated from Table A-8 and Table A-7 for 4% & 3%

$$= 47.575416 + 0.25(56.084938 - 47.575516) = 49.7028 \tag{4.11}$$

$$
\begin{aligned}
F\,W = &-25(F/P,3.25\%,30) \text{ (principal invested)}\\
&3000{*}250{*}(F/A,3.25\%,30) \text{ (annual operational expenses)}\\
&+12000{*}250{*}0.70(F/A,3.25\%,30) \text{ (rental revenue)}\\
&+5 \text{ (salvage value)}
\end{aligned} \tag{4.12}
$$

$$
\begin{aligned}
FW &= -65.7825 - 37.2771 + 104.37588 + 5\\
&= \$6.31628 \text{ million.}
\end{aligned} \tag{4.13}
$$

It is profitable to invest in the apartments. The FW of the project would be $6.163 million.

Example 4.5 Present Worth of a Sugar Mill

A business plan was presented to get a sugar mill installed in rural Texas. The cost of raw materials such as sugarcane and utility costs rise every year due to inflation. The revenue from the sale of sugar rises and falls in synchrony with the boom-and-bust cycles of the economy. The EOY (end of year) revenues and costs are shown in Table 4.1.

The capital cost of the sugar mill, including land and all the necessary equipment, is estimated at $8 million. What is the PW of the sugar business at an interest rate of 3%?

Another table is generated using an MS Excel spreadsheet on a desktop computer. Two more columns are added to Table 4.1. One is the

Table 4.1 EOY Receipts and Costs

Year	Receipts Millions $	Cost Millions $
1	2.3	1.00
2	2.5	1.10
3	2	1.21
4	1.5	1.33
5	2	1.46
6	2.5	1.61
7	3	1.77
8	3.5	1.95
9	4	2.14
10	3.5	2.36
11	3	2.59

profit, Pr, obtained each year. This is determined by subtracting the costs from the receipts. The contribution of each cell in the next column to the present worth is obtained using the relation for compound interest given by Eq. (3.5)

$$PW_J = \frac{\mathrm{Pr}_J}{(1 + i)^J} \tag{4.14}$$

where Pr_J is the profit obtained in year J. J varies from 1 to 11 as given in column 1.

$$\begin{aligned} \mathrm{PW} &= -8.0 + \mathrm{SUM}(\mathrm{P/F,3\%,Year}) \\ &= -8.0 + 9.451591 = +1.451591. \end{aligned} \tag{4.15}$$

Thus, the PW of the sugar business is \$1 million and 451,591. It is profitable to start the sugar from sugarcane business.

Example 4.6 Invest in Gold or Stock Market

The gold prices and DJIA (Dow Jones Industrial Average) current, 50, and 100 years ago are given in Table 4.3.

Table 4.2 Present Worth Analysis of EOY Cash Flows (Millions \$) in Table 4.1

Year	Receipts	Cost	Profit	(P/F,3%,Year)
1	2.3	1.00	1.30	1.262136
2	2.5	1.10	1.40	1.319634
3	2	1.21	0.79	0.722962
4	1.5	1.33	0.17	0.150154
5	2	1.46	0.54	0.462272
6	2.5	1.61	0.89	0.744934
7	3	1.77	1.23	0.998833
8	3.5	1.95	1.55	1.224597
9	4	2.14	1.86	1.422785
10	3.5	2.36	1.14	0.849794
11	3	2.59	0.41	0.293489
			Total	9.451591

Table 4.3 Gold and DJIA Current, 50, and 100 Years Ago

Year	Gold (per ounce)	Stock (DJIA Dow Jones Industrial Average)
1909	\$20.67	70
1959	\$35.25	600
2009	\$1087.50	10,000

Which would be a better investment of \$50,000 today in another 30 years, gold or stocks?

Gold has performed at an annualized rate of

$$F/P = 1087.5/20.67 = 52.61 \tag{4.16}$$

For N = 100 the interest rate can be seen from Appendix A as roughly 4%

$$FW\ (\text{gold}) = 50{,}000(F/P,\ 4\%,\ 30) \tag{4.17}$$

From Table A-8, (F/P, 4%, 30) = 3.243398*50,000 = \$162,115

DJIA has performed at an annualized rate of

$$F/P = 10{,}000/70 = 142.86 \tag{4.18}$$

From Appendix A for N = 100, F/P = 142.86 the interest rate can be seen to be 5%

$$\begin{aligned}\text{From Table A-9 } (F/P,5\%,30) &= 4.321942{*}50{,}000 \\ &= \$216{,}097\end{aligned} \tag{4.19}$$

\$219,097 is greater than \$162,115. Hence, it is better to invest in the U.S. stock market rather than in gold. The 5% is the historic performance of DJIA. notwithstanding the Great Depression and stock market corrections in 1987 and 2007. The gold price has increased at 4% despite the government changes in the early 1970s.

Example 4.7 Electric/Gas Hybrid Vehicle

A hybrid electric/gas car such as the 2010 Toyota Prius costs about \$29,000. The achievable fuel economy using the vehicle is 50 miles per

gallon. A comparable all-gasoline vehicle such as the 2010 Toyota Corolla costs \$21,000, and the fuel economy for the Corolla is 32 miles per gallon. At what price of gasoline is it more attractive to purchase the Toyota Prius? It is estimated that the average passenger travels 15,000 miles per year and the life of an automobile is 100,000 miles. The interest rate may be taken as 11%.

The cash flow diagram for Example 4.7 is shown in Figure 4.2.

Let the price of gasoline per gallon be Z \$/gallon. Assume that the lives of both the Toyota Corolla and the Toyota Prius are seven years and the passenger drives either vehicle at 15,000 miles per year.

PW of all payments in order to own the Toyota Prius, PW_{prius}, is;

$$PW_{prius} = -\$29{,}000 - Z^*(15{,}000/50)^*(P/A,11\%,7) \qquad (4.20)$$

From Table A-15

$$(P/A,11\%,7) = ½^*(4.2305+5.1461) = 4.6883$$

$$PW_{prius} = -29{,}000 - 1407Z \qquad (4.21)$$

PW of all payments in order to own the Toyota Corolla, $PW_{corolla}$ is

$$PW_{corolla} = -\$21{,}000 - Z^*(15{,}000/32)^*(P/A,11\%,7) \qquad (4.22)$$

From Table A-15

$$(P/A\ ,11\%,7) = ½^*(4.2305 + 5.1461) = 4.6883$$

$$PW_{corolla} = -21{,}000 - 2198Z \qquad (4.23)$$

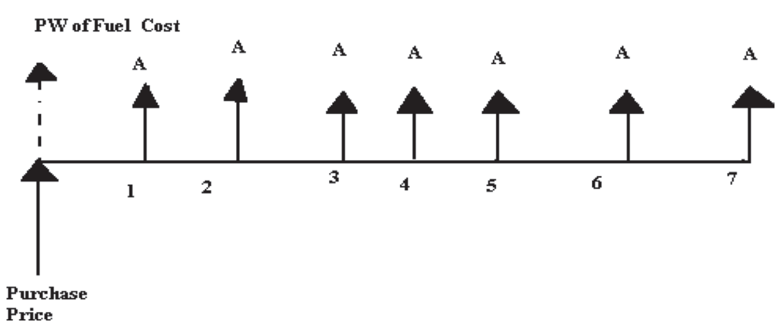


Figure 4.2 Cash Flow Diagram of Fuel Cost for Toyota Prius and Toyota Corolla

At a certain gasoline price, Z, the PW_{prius} will become equal with that of $PW_{corolla}$. Thus,

Eq. (4.23) and Eq. (4.21) can be set equal to each other.

$$-29{,}000 - 1407Z = -21{,}000 - 2198Z \tag{4.24}$$

$$\text{Or, } Z = \frac{(29{,}000 - 21{,}000)}{(2198 - 1407)} = 10.13$$

For gasoline prices greater than \$10.13, the Toyota Prius is a lower-cost alternative compared with the Toyota Corolla. For gasoline prices less than \$10.13, the Corolla is a better alternative compared to the Prius.

Example 4.8 Effect of Inflation on PW

The inflation rates for the past 10 years (1999–2009) are given below in Table 4.4. This table was prepared using the information provided by the Bureau of Labor Statistics.

In 1999 dollars, calculate the PW (present worth) of an ABS (acrylonitrile butadiene styrene) engineering thermoplastic continuous polymerization plant. The capital cost of a new plant in 1999 dollars was \$80 million. The selling price of ABS is \$4 per pound. The raw materials cost rises every

Table 4.4 Inflation Rates for 1999–2009

Year	(%) Inflation
1999	2.19
2000	3.38
2001	2.83
2002	1.59
2003	2.27
2004	2.68
2005	3.39
2006	3.24
2007	2.85
2008	3.85
2009	−0.34

year on account of inflation. The price in year 1998 was $2.8 per pound of ABS sold. The process cost also goes up with inflation on account of increase in utility costs and wages to workers. The process cost can be taken as 30 cents per pound of ABS produced in 1998. The production rate from the two CSTR and falling strand devolatilizer plant is 10,000 lb/hr. The plant can be expected to run about 85% of the time. The selling price of ABS remains flat during the 10-year period. An interest rate of 11% may be assumed.

The inflation rates given in Table 4.4 were used to calculate the materials cost and process cost. This is shown in the column in Table 4.5 under Cost–Materials and Cost–Process. Each row was $(1 + f_i)$ times the previous row, where fi is the inflation rate. Inflation rate was calculated by dividing the values provided under the column (%) inflation by 100. The revenue was calculated at a flat price of $4 per lb. of ABS multiplied with the quantity of ABS produced, i.e., 74.46 million lbs. This was arrived at from the production rate of 10,000 lb/hr multiplied with the operating time of the plant 24*365*0.85, where 0.85 is the utilization factor. The inflation effects on the materials and process costs resulted in loss in the years 2008 and 2009. In other years there was a profit. The profit was obtained by subtracting the total expenses of materials and process from the revenue from sales of ABS. The PW contribution of each cell in the profit column was calculated using the single payment compound amount factor of $\frac{Pr_J}{(1+i)^{Year - 1999}}$ where J varied from 0 to 10 and Pr_J was the profit earned in each year. All the PW contributions from 1999 to 2009 were added using the SUM command in MS Excel spreadsheets on a desktop computer. The capital cost of $80 million was also subtracted as a one-time charge. No salvage value was attributed in 2009. The PW of the ABS plant in 1999 dollars was seen to be $157.07 million.

Example 4.9 Life-Cycle Cost Analysis of HVAC Systems

Life-cycle cost analysis has been found to be advantageous over impressions created from one-time sticker-price numbers. A piece of equipment that may appear initially attractive due to its lower cost may "nickel and dime" you in costs during its operation. Maintenance costs may

Table 4.5 Present Worth Analysis with Inflation Effects Accounted for Effect of Inflation

				(%)	Cost	Cost	Revenue	Profit	PW
			Year	Inflation	Materials	Process	Millions $	Million $	Million $
			1999	2.19	2.861	0.3066	297.84	61.95891	$61.96
Product Rate	10000	lb/hr	2000	3.38	2.958	0.3169	297.84	53.98613	$48.64
ABS Price	4	$/lb	2001	2.83	3.042	0.3259	297.84	47.08507	$38.22
Materials	2.8	$/lb	2002	1.59	3.090	0.3311	297.84	43.09806	$31.51
Process Cost	0.3	$/lb	2003	2.27	3.160	0.3386	297.84	37.31542	$24.58
			2004	2.68	3.245	0.3477	297.84	30.33336	$18.00
Production (lbs)	7E+07		2005	3.39	3.355	0.3595	297.84	21.26489	$11.37
	74.46	million lb	2006	3.24	3.464	0.3711	297.84	12.30385	$5.93
i	11	%	2007	2.85	3.562	0.3817	297.84	4.16607	$1.81
			2008	3.85	3.700	0.3964	297.84	−7.14037	($2.79)
			2009	−0.34	3.687	0.3950	297.84	−6.10344	($2.15)
Capital Cost of Plant									($80)
								Total	$157.07

mushroom and repair costs may make another alternative a better prospect. So how do you arrive at which equipment to select? This can be best accomplished by a life-cycle cost analysis. Here all the relevant costs are considered by the use of PW analysis.

Consider the residence of Sharron Stone. A new HVAC (heating, ventilation, and air conditioning) system was installed in her residence. The estimated useful life is 15 years. A novel high-efficiency gas furnace with add-on air conditioner costs \$11,183. The energy costs per year are \$928 and maintenance cost of \$60. The salvage value of the equipment is \$60. Use PW analysis and calculate its life-cycle cost. An interest rate for funds can be taken as 5%.

$$\text{Life Cycle Cost of HVAC, } C_L = \$11{,}183 - 60^*(P/F,5\%,15) + (\$928+\$60)^*(P/A,5\%,15) \quad (4.25)$$

From Table A-9,

$$(P/F,5\%,15) = ½^*(0.505068 + 0.458112) = 0.48159$$

$$(P/A, 5\%, 15) = 1/2^*(9.8986 + 10.8378) = 10.3682$$

Eq. (4.25) becomes

$$C_L = \$11{,}183 - \$60^*0.48149 + \$988^*10.3682 = \$21{,}397.89 \quad (4.26)$$

The life-cycle costs associated using PW analysis of the HVAC equipment in Stone's residence was found to be \$21,398.

Example 4.10 Municipal Garbage Collection Truck

The cost for the truck used for collecting garbage from residences is about \$81,450. It can be used to serve 1000 households. A weekly charge per household of \$5 can be collected. The gasoline costs run \$150 per month. The labor costs for two drivers are \$3200 per month. A one-time tax credit of \$5000 from the federal government for recycling is given. Is it profitable to run the garbage business at a MARR of 11%. The life of the truck is expected to be eight years.

PW = −\$81,450 (purchase price of truck)
+ \$5000 (tax credit for recycling)

$$+ 1000{*}\$5{*}4{*}12(P/A,11\%,8) \text{ (revenue from garbage collection fees per year)}$$

$$- 3200{*}12(P/A,11\%,8) \text{ (salary for two drivers per year)}$$

$$- \$150{*}12(P/A,11\%,8) \text{ (gasoline cost of truck per year)} \quad (4.27)$$

$$(P/A,11\%,8) = 5.1461 \text{ (from Table A-15)}$$

$$\begin{aligned} PW &= -\$81{,}450 + \$5{,}000 + \$240{,}000{*}(5.1461) \\ &\quad - \$36{,}000{*}(5.1461) - \$1{,}800{*}(5.1461) \\ &= -\$81{,}450 + \$5000 + \$1{,}235{,}064 - \$197{,}610.24 \\ &\quad - \$9262.98 = \$951{,}741 \end{aligned} \quad (4.28)$$

It would be profitable to run the garbage business at a MARR of 11%.

Example 4.11 Hexane Extraction of Rice-Bran Oil

Rice-bran lipids can be extracted using hexane solvent from rice bran at a yield of about 26%. Rice is milled to produce the white long-grain rice sold in supermarkets. The outer layers of the rice kernel are removed. These layers include the hull, the germ, and the bran. The bran includes the testa, peripcarp, nucellus, and aleurone layers. Rice bran becomes rancid and has to be discarded in landfills. Parboiled rice bran has higher lipid levels than unstabilized rice bran. Rice-bran oil consists largely of saponifiable compounds. Hexane costs about \$1.15 per gallon. It is an excellent solvent for nonpolar lipids. Calculate the PW of a hexane extraction unit. The following information was obtained from the *Journal of the American Oil Chemists' Association* [2] and modified suitably for the PW analysis in Example 4.11.

$$\text{Annual Labor Cost} = 8332h{*}\$9.62 = \$80{,}154$$

$$\text{Rice Bran Processed} = 30.96{*}3.7854{*}365/0.26 = 164{,}525 \text{ kg/year}$$

$$\begin{aligned} \text{Annual Revenue} &= 30.96{*}3.7854{*}365{*}\$8.08 \\ &= \$345{,}634 \text{ (from rice-bran oil)} \\ &\quad + \$4.928{*}164525{*}0.146 = \$118{,}374 \end{aligned} \quad (4.29)$$

$$\begin{aligned} \text{Annual Materials \& Labor Cost} &= 164{,}525{*}(5.18+0.22) \\ &= \$888{,}436 \end{aligned} \quad (4.30)$$

Table 4.6 Cost Data for Hexane Extraction Plant for Rice-Bran Oil

Description	Hexane Extraction Unit
Capital Investment	\$220,000
Manufacturing Cost	\$5.18 /year/kg of rice bran
Materials Cost	\$0.22/year/kg of rice bran
Labor Cost	\$9.62/hr
Revenue from Rice-Bran Oil	\$8.08/kg
Revenue from Rice Protein	\$4.928/kg
Production Rate	30.96 gal rice-bran oil/day
Useful Life of Equipment	10 years
Operational Hours per Year	8332 hrs
% Protein in Rice Bran	14.6%
Interest Rate, i	3.0%

$$(P/A,3\%,10) = 8.530203 \text{ (From Table A-7)}$$

$$PW = -\$220{,}000 + (-888{,}436 + 345{,}634 + 118{,}374 - 80{,}154)\ 8.530203 = -\$4.52418 \text{ million} \tag{4.31}$$

The hexane extraction of rice-bran oil is not a profitable venture.

4.4 Annual Worth Analysis (AW)

Another method of evaluating a business can be completed by reducing all cash flows, including the capital investment, and salvage value in terms of annualized cash flow at a certain interest rate, i. Thus, the AW (annual worth) of a project may be estimated as follows:

$$AW = R_k - E_k - \text{Capital Recovery Factor} + \text{Sinking Fund Factor} \tag{4.32}$$

Eq. (4.32) is calculated at a certain annualized interest rate i%. The capital recovery factor can be estimated from Eq. (3.20) and looked up in the annuity tables given in Appendix A or read from Figure 3.8. The capital recovery factor (A/P,i%,N) distributes all the capital investments into an annualized equivalent uniform series of payments. In a similar manner, the salvage value can be distributed using the sinking fund factor (A/F,i%,N) over a uniform series of payments.

Example 4.12 Annual Worth (AW) of a Biodiesel Plant in Taiwan [3]

Biodiesel is produced by transesterification reactions. Triglycerides present in virgin soybean oil are reacted with anhydrous alcohol such as methanol, ethanol, propanal, etc. to form FAME, fatty acid methyl esters, or biodiesel and glycerol. The alkali catalyst used was sodium hydroxide, NaOH. It can potentially be used as an alternative fuel to the ones currently in vogue that tend to be culprits of pollution causing global warming, acid rain, greenhouse gas emissions, etc. Biodiesel is an attractive fuel due to the environmental benefits it has to offer. It is prepared from renewable energy resources such as vegetable oil. Using the information provided in Table 4.7, calculate the AW of the biodiesel plant. The plant is expected to last for 20 years. The interest rate for equivalence calculations can be taken as 3%. The main costs in these plants are the raw material costs.

Table 4.7 Cost and Revenue Data for Biodiesel Production in Taiwan

Biodiesel Production in Taiwan			
#	**Description**	**Cost**	**Quantity**
1	Transesterification Reactor	$335,000	
2	Neutralization Reactor	$25,000	
3	Washing Column	$115,000	
4	FAME Distillation Column	$181,000	
5	Heat Exchangers	$4,000	
6	Pumps	$53,000	
7	Separator, Vacuum Systems	$52,000	
	Production Rate Materials		800,000 kg/year
8	Soybean Oil Feedstock	$6,234,000	
9	Methanol	$196,000	
10	Catalyst and Solvent	$368,000	
	Labor Cost	$564,000	
	Utilities	$124,000	
	Overhead	$431,000	
	Revenue from Biodiesel	$6,845,000	
	Glycerin Credit	$3,038,000	

The AW analysis is completed using an MS Excel spreadsheet as shown in Table 4.8. The capital cost is obtained by using the SUM command from cells D4 to D10. These are the costs of reactors, washing column, distillation column, heat exchangers, pumps, and vacuum

Table 4.8 AW Analysis of Biodiesel Plant in Taiwan

#	Description	Cost	Quantity	
1	Transesterification Reactor	$335,000		
2	Neutralization Reactor	$25,000		
3	Washing Column	$115,000		
4	FAME Distillation Column	$181,000		
5	Heat Exchangers	$4,000		
6	Pumps	$53,000		
7	Separator, Vacuum Systems	$52,000		
	Production Rate Materials		800,000	kg/year
8	Soybean Oil Feedstock	$6,234,000		
9	Methanol	$196,000		
10	Catalyst and Solvent	$368,000		
	Labor Cost	$564,000		
	Utilities	$124,000		
	Overhead	$431,000		
	Revenue from Biodiesel	$6,845,000		
	Glycerin Credit	$3,038,000		
	Capital Cost	$765,000		SUM(D4:D10)
	Materials Cost	$6,798,000	per year	SUM(D13:D15)
	Labor, Utility, and Overhead	$1,119,000		SUM(D16:D18)
	Total Revenue	$9,883,000	per year	SUM(D19:D20)
	(A/P,3%,20)	0.067216	(From Table A-7)	0.067216
	Capital Recovery	$51,420.24	per year	D26*D22
	AW	$1,914,579.76		D25-D24-D23-D27

systems. The capital cost of \$765,000 is amortized over a 20-year period by using the capital recovery factor. The capital recovery factor is obtained from Table A-7 in Appendix A for (A/P,3%,20). This was found to be 0.067216. The materials costs include the soybean oil, methanol, catalyst, and solvent. The annual costs are obtained by adding the materials costs, labor costs, utility costs, and overhead costs. The annual revenue is obtained by adding the sales of biodiesel and glycerin. The AW was calculated as shown by plugging the information available into Eq. (4.32). The AW is about \$1.915 million. Thus, it is profitable to operate the biodiesel plant as described in Taiwan.

Example 4.13 Debt Consolidation

A debt consolidation company worked out a payment plan for Mr. Dan Wardrop. Calculate the monthly payment at an annualized interest rate of 11%, should the loan be repaid in 26 years. Mr. Wardrop's credit card balances carried are shown in Table 4.9.

$$AW = P/12\ (A/P,11\%,26) \tag{4.33}$$

$$(A/P,11\%,26) = 0.117813 \text{ (Table A-)}$$

$$P = \text{SUM(all balances in third Column in Table 4.9)} = \$25{,}700$$

$$AW\ 1/12\ (3027.70) = \$252.32 \tag{4.34}$$

Table 4.9 Credit Card Balances of Mr. Dan Wardrop

#	Credit Card Company	Balance	Annualized Interest Rate (%)
1	Wachovia Platinum Visa	\$11,500	19%
2	Sears MasterCard	\$3,500	21%
3	Amazon Visa	\$2,400	17%
4	Rooms-to-Go Furniture	\$5000	15%
5	Macy's Department Store	\$2000	25%
6	Gap Apparel	\$400	19%
7	JC Penney Retail	\$900	21%

Example 4.14 Solar Panel

Radio Shack has advertised Sunforce 130 W Solar panels with a Sharp module for \$999. Consider purchase of five of these panels for a new home. The monthly utility bill in the new home runs about 750 kWh. The current utility company charges 12 cents for 1 kWh. How many years will it take for the new solar panels to be cost effective if purchased to replace the existing utility? The interest rate is about 11%.

$$\text{Annual Cost of Utilities: } 750{*}0.12{*}12 = \$90{*}12 = \$1008 \tag{4.35}$$

$$\text{Cost of Solar Panels} = 20{*}\$999 = \$4995$$

$$\text{P/A} = 4.9553$$

From Table A-15, N is about eight years.

It would take eight years for the investment in solar panels to be more attractive compared with the current utility company.

Another method to calculate the pay period is from Eq. (3.32)

$$1 - \frac{iP}{A} = \frac{1}{(1+i)^N}$$

$$N = \frac{\log\left(\dfrac{A}{A - iP}\right)}{\log(1+i)} = \frac{\log\left(\dfrac{1008}{1008 - 0.11{*}4995}\right)}{\log(1.11)} = 7.55\ \mathit{years} \tag{4.36}$$

N is plotted as a function of P/A for interest rate, i = 11% in Figure 4.3. It can be seen in Figure 4.3 that there is an abrupt change in curvature at about P/A = 8.0. The significance of this is that the annualized payment must be above a threshold amount to keep the repayment time within a reasonable timeframe. Should A be lowered any further, the time taken for repayment is large.

4.5 IRR (Internal Rate of Return)

The most widely used method of evaluation of a capital project is the Internal Rate of Return Method, or IRR. Other phrases used for the same method are investor's method, discounted cash flow method, profitability interest, etc. An interest rate is calculated, at which point the PW of the

project goes from negative territory to positive territory. This interest rate is called the IRR of the project. In addition, the IRR rate has to be greater than the MARR rate in order for the project to be considered acceptable. The IRR rate is the interest rate at which point the equivalent worth of all cash outflows and equivalent worth of cash inflows are equated to each other. It can be viewed as another *break-even* point. At the IRR

$$\sum_{j=0}^{N} R_j(P/F, i''\%, j) = \sum_{j=0} E_j(P/F, i'\%, j) \qquad (4.37)$$

The IRR can be calculated using graph paper. The PW of the project can be evaluated at various interest rates such as 2%, 4%, 6%, 8%, 10%, 12%, …, etc. The PW is plotted as a function of interest rate. The *x-intercept* or the interest rate at which the PW emerges into the positive territory from negative territory is called the IRR. Software is available to perform the trial-and-error calculations. For example, the MS Excel Spreadsheet has an IRR (range, guess) function call for obtaining the IRR for a set of cash flows.

Although a popular method, some pitfalls of an IRR include the occurrence of multiple values for the IRR. The variation of PW as a function of interest rate, i, is a function of several factors and may vary from

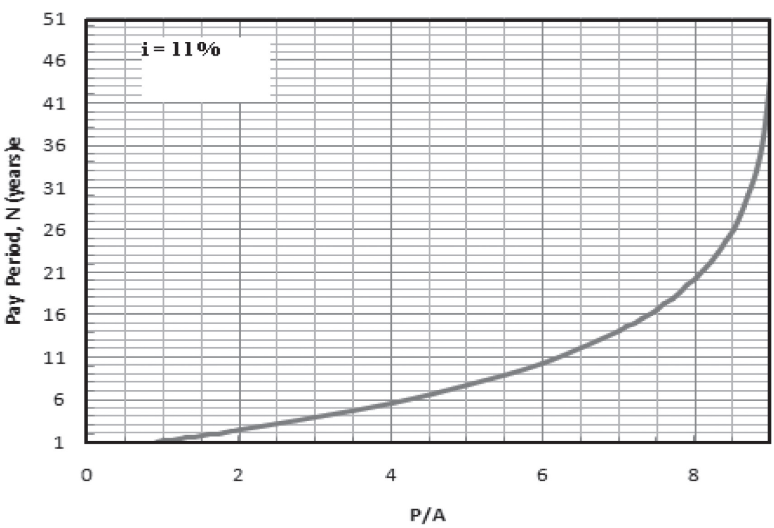


Figure 4.3 Pay Period N for Solar Panels to be Attractive Compared with Current Utility

project to project. When multiple values of an IRR result, one of the other methods of evaluation of the project such as FW, AW, payback period, etc. can be used instead of the IRR method. An IRR method also assumes that the revenues generated are reinvested back into the business. The ERR method does not assume such a thing.

Example 4.15 Profitability of Liquefaction Plants

The conversion of natural gas to liquid (GTL) fuel using the Fischer-Tropsch process was first developed in 1923. Syngas ($CO + H_2$) was converted into synthetic fuels. At a Society of Petroleum Engineers symposium, a novel method of evaluation of the economics of GTL plants was presented [4]. The capital expenses (CAPEX) are pegged to the production of one barrel of hydrocarbon liquid per day (BLPD). Annual operational expenses (OPEX) are expressed as a certain percentage of CAPEX. It is assumed that the overall thermal efficiency of GTL plants is 60% and the plant operates 330 days per annum. Consider one instance of CAPEX of $40,000/BLPD and OPEX of 7% of CAPEX. Plant is considered to be in existence for eight years. The price of crude oil is $37.65/bbl. Calculate the IRR for the project.

Basis of Production: 1 bbl a day of liquid fuel/oil

Annual Revenue = $37.65*330*1 = $12,424.5

Capital Expenses = $40,000

Annual Expenses = 0.07*$40,000 = $2800

PW = −$40,000 + $12,424.5*(P/A,i%,8) − 2800*(P/A,i%,8)

The PW for different interest rates was calculated using an MS Excel Spreadsheet on a desktop computer. The P/A values for the interest rates used in the simulation were read from the annuity tables given in Appendix A. The values are given in Table 4.10 below. The PW as a function of interest rate is plotted in Figure 4.4. The IRR can be read from Figure 4.4 as 17.5%. Should the MARR be less than the IRR, the project is considered *profitable*.

Table 4.10 IRR Analysis for GTL Plant at 1 Bbl per Day

N	8		CAPEX	−$40,000
P/A	i (%)	PW	Revenue	$12,424.50
7.325481	2	$30,504.09	OPEX	−$2800
6.732745	4	$24,799.30		
6.2098	6	$19,766.22		
5.7466	8	$15,308.15		
5.3349	10	$11,345.75		
4.9676	12	$7,810.67		
4.4873	15	$3,188.02		
3.9544	19	−$1,940.88		
3.7256	21	−$4,142.96		
3.3289	25	−$7,961.00		
2.9247	30	−$11,851.22		

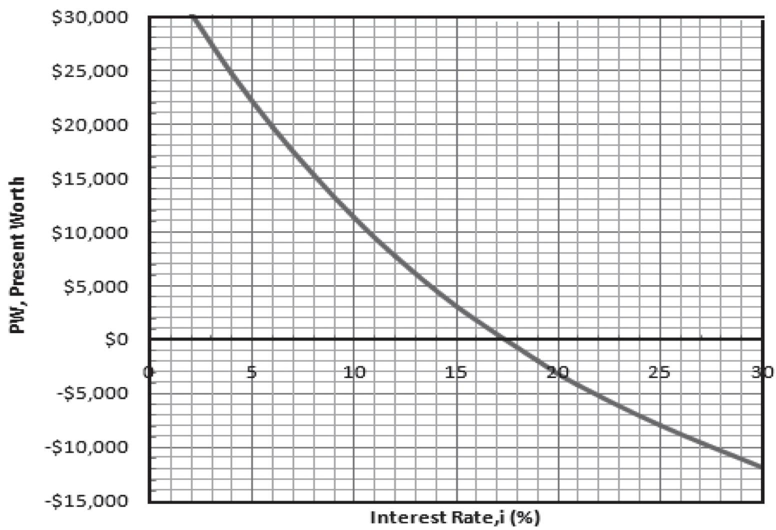


Figure 4.4 IRR Calculations for GTL Plant

Example 4.16 Ethanol Production by Fermentation

Bioethanol can be produced by the fermentation of sugars obtained from saccharine biomass such as sugarcane or sugar beets, starchy biomass such

Table 4.11 Cost Information for Six Methods

(M = million)							
#	**Description**	**Steam Stripping Process**	**Flash Fermentation**	**Single Column Distillation**	**Two Column Distillation**	**Distillation with Heat Pump**	**Flash and Vacuum Distillation**
1	Capital Equipment Cost	\$6.0M	\$18.0M	\$14M	\$6M	\$10M	\$25M
2	Utility Cost	\$5.5M/year	\$7.5M/year	\$6.5M/year	\$3.5M/year	\$3.6M/year	\$5.5M/year
3	Feedstock Cost	\$10.6M	\$14M	\$39.1M	\$41.7M	\$7.9M	\$25.6M
4	Revenue	\$50M	\$50M	\$50M	\$50M	\$50M	\$50M
5	Plant Life	10 years	10 years	10 years	10 years	10 years	10 years
6	IRR	565%	157%	28%	77%	385%	77%

as corn, or cellulosic biomass such as agricultural wastes. Bioethanol, which comes from renewable resources, can be used as an alternative to fossil fuel resources such as coal and petroleum. Six different methods have been identified for recovering ethanol from fermentation broth. These methods are:

(i) Steam Stripping and Distillation;
(ii) Flash Fermentation and Distillation;
(iii) Single Column Distillation;
(iv) Two Column Distillation;
(v) Distillation with Heat Pump;
(vi) Flash and Vacuum Distillation.

The ethanol production rate was set at 100 million liters/year in the process simulation study [5]. The cost data for cases where the ethanol concentration in the fermenter was 40 g/l is shown in Table 4.11. At an ethanol price of 50 cents per gallon, calculate the IRR for the six methods.

The IRR in row 6 of Table 4.11 was calculated using an MS Excel spreadsheet on a desktop computer. The PW as a function of interest rate for the six methods of recovery of ethanol from biofeedstock is shown in Figure 4.5.

4.6 ERR (External Rate of Return)

Some of the pitfalls encountered in the IRR method of evaluation of the capital projects are overcome in the ERR method. In the IRR method, the revenues generated are assumed to be reinvested at the IRR. An external interest rate, e, external to the project at which net cash flows required by a project over its life can be borrowed is identified. This is MARR, minimum acceptable rate of return. Identical results are generated from the methods of IRR and ERR when the ERR happens to equal the project's IRR.

During the ERR procedure, all net cash outflows to time 0 at ε% per compounding period are discounted. Then all net cash inflows are compounded to period N at e%. The ERR is obtained by establishment of equivalence of the two quantities.

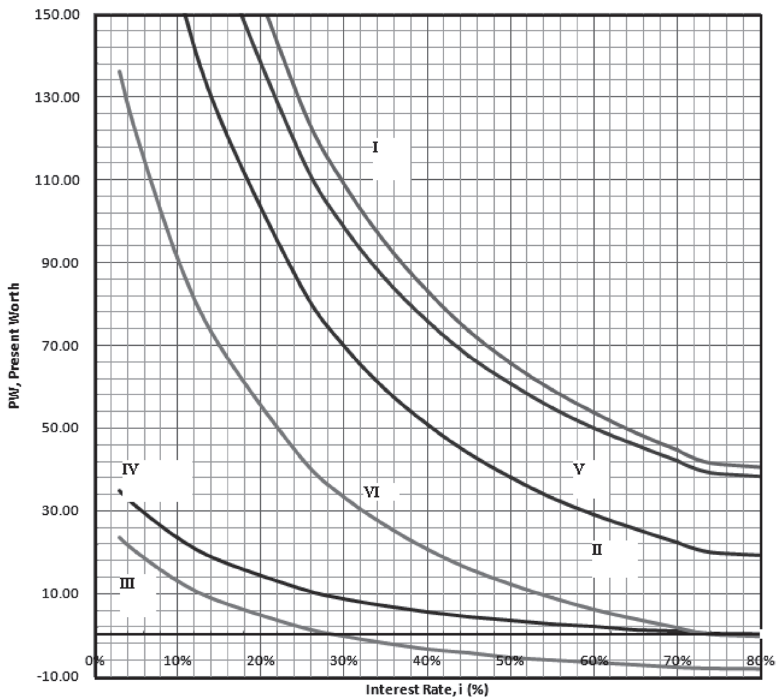


Figure 4.5 IRR Analysis for Six Methods of Recovery of Ethanol from Biomass

$$\sum_{k=0}^{N} E_k(P/F,\varepsilon\%,k)\,(F/P,i''\%,N) = \sum_{k=0}^{N} R_k(F/P,\varepsilon\%,N-k)$$

Where R_k = excess of receipts over expensed in period k

E_k – excess of expensed over receipts in period k

N – project period

ε - external reinvestment rate per period

4.7 Payback Period

Payback period is the number of years for the cash inflows to equal the cash outflows. It is a measure of the liquidity. It is a simple method. A number of years for break-even point attainment is obtained. But the cash flows occurring after the payback period q is not available using this method. The project desirability is not clear. The speed at which the investment

can be recovered is calculated. It can be used as supplemental information to one of the other five methods discussed earlier in this chapter.

Example 4.17 Copper-Chlorine Thermochemical Cycles for Hydrogen Production

The hybrid copper-chlorine (CuCl) cycle is one promising method of a thermochemical cycle that can be used for hydrogen production with nuclear or solar heat. A CuCl cycle plant producing 125 MT H_2/day required 210 MW of thermal energy and 87.8 MW of electrical energy. The total capital investment for the CuCl plant using Turton et al.'s software package [6] can be estimated at $131 million. Estimated price of hydrogen is $5.0/kg. Calculate the payback period at an interest rate of 5%. The cost of electricity can be taken as $60 per 1000 kWh. Bulk CuCl costs $7200 per pound and bulk HCl costs $241/metric ton. A 50% yield can be assumed for the three major reactions that are needed in the process: (i) electrolysis reaction, by which cupric chloride, $CuCl_2$, is produced at the anode and H_2 at the cathode; (ii) hydrolysis of cupric chloride leads to the formation of copper oxychloride; and (iii) molten cuprous chloride is produced from the decomposition reaction. Thermal utility is $20 per 1000 kWh. Assume 1% loss of HCl and 1 ppm loss of CuCl during the operation of the cycle.

$$2CuCl + 2HCl + 4H_2O \rightarrow 2CuCl_2.2H_2O + H_2$$

$$2CuCl_2 + H_2O \rightarrow CuOCl_2 + 2HCl$$

$$CuOCl_2 \rightarrow ½\, O_2 + 2CuCl$$

$$\text{Revenues from } H_2 \text{ Sales per Year} = (125)*(100)*(\$5)*365/10^6 = \$22.8125 \text{ million}$$

$$\text{Material Cost–HCl per Year} = (125)*(\$241)*(365)*(18.25)*(0.01)/10^6 = \$2.0067 \text{ million}$$

$$\text{Material Cost–CuCl per Year} = (125)*(100)*(\$7200*2.2)*(99)*(365)*(10^{-12}) = \$7.1547 \text{ million}$$

$$\text{Thermal Energy Cost per Year} = (\$20)*(210)*(365)/10^6 = \$1.533 \text{ million}$$

Electric Energy Cost per Year = (\$60)*(87.8)*(365)/10^6
= \$1.92282 million

(Revenue–Expenses) per year = \$22.8125 − \$2.0067−\$7.1547
−\$1.533-\$1.92282

= \$10.1953 million

Capital Cost = \$131 million

$$\frac{P}{A} = \frac{\$131}{\$10.1952} = 12.8491$$

From Table A-9, N for P/A = 12.8491 1t 5% interest rate can be seen to be 21 years.

4.6 Summary

There are five methods to evaluate a single project. These are: (i) PW (present worth) analysis; (ii) FW (future worth) analysis; (iii) AW (annual worth) analysis; (iv) IRR (internal rate of return) analysis; and (v) ERR (external rate of return) analysis. In addition to these five methods, a *payback period* can be calculated that denotes the time taken over which the monies invested can be recovered back. The MARR is the minimum acceptable rate of return. For a project to be selected, it has to have a rate of return (ROR) greater than that of a MARR.

The PW of a project can be estimated by finding out the equivalence of all the cash inflows (such as receipts) and cash outflows (such as expenses) to the present time at a specified interest rate that is usually greater than the MARR. When PW falls in positive territory, the project is considered to be acceptable. The project study period, N, may be determined after establishing the useful lives of critical equipment used in the execution of the project. The factors considered during PW analysis are capital costs incurred in the purchase of equipment, estimated useful life, variable costs of labor, materials, and utilities, salvage value, revenues accrued from sales of product and the interest rate. Inflation factors may be used to account for the variability in materials costs incurred. When the study period reached infinity, the PW becomes the CW, capitalized worth

of the project. Examples of PW analysis were developed using hydrogen production from fast pyrolysis of biomass and steam re-forming, replacement for the World Trade Center (WTC), sugar mill, hybrid gas/electric vehicle, inflation rates seen in the U.S. economy, life-cycle cost analysis of HVAC systems, municipal garbage collection truck, and hexane extraction of rice-bran oil.

The FW of a project can be estimated by finding out the equivalence of all the cash inflows, such as receipts and all the cash outflows, such as expenses to a future time. Worked examples were developed on a university apartment business and investment in gold or stock.

The AW of a project can be estimated by reducing all cash flows including the capital investment, salvage value in terms of annualized cash flows at an interest rate i. Worked examples on AW analysis were implemented from case studies of the manufacture of a biodiesel plant in Taiwan and debt consolidation.

Worked examples on calculation of payback period were created for solar panels for homes and hydrogen from the thermochemical cycle of copper-chlorine.

The IRR is the most widely used method of evaluation of capital projects. The PW of the project is calculated as a function of interest rate i (%). The point in the graph of PW vs. interest rate where the PW goes from positive territory to negative territory is called the IRR. Worked examples on IRR analysis have been constructed for GTL (gas to liquefaction) plants and ethanol production via fermentation by six different methods.

There are some deficiencies in the IRR method of analysis. Results of multiple IRR is one of them. The ERR method is developed to overcome these deficiencies. The revenues generated are not assumed to be reinvested in this method. An external interest rate external to the project at which net cash flows required by a project over its life can be borrowed is used.

End-of-chapter exercises have been developed on the life-cycle costs of HVAC systems, hybrid electric vehicle, PEV (plug-in-electric vehicle), time taken to repay credit card balances, capitalized worth of a vacation condominium in Galveston, Texas, the AW of energy efficient windows, deferred interest charges, profitability of outpatient drug treatment center, PW of a convenience store and gas station complex, AW of public

Internet, video and printing services, bonds, profitability of an olive oil business, IRR for the short path distillation method of extraction of paraffin wax, car rental business, payback period of a frozen yogurt parlor, FW of a coffeehouse, turnaround business, PW of an onshore oil well, a custom foam fabricator, polypropylene bag manufacture, activated carbon from bamboo, meglumine antimonite drug manufacture, ABS engineering, thermoplastic manufacture, hydrogen from biomass, use of additional wells in an oil field, long-term parking lot, recycling of plastics using closed-loop granulation, solar hats, savings from using PFR (plug flow reactor) over CSTRs, and the use of gradient series.

4.7 References

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CHAPTER 5

Decision Analysis and Comparison of Alternates

Topics Covered

- Develop Alternatives by Brainstorming
- Study Period—Don't Compare an Apple and an Orange
- Useful Lives—Repeatability and Co-terminated Basis
- Rate of Return and Project Ranking
- EW, Equivalent Worth Method
- Optimal EW
- Household Finances

5.1 Overview

As discussed in Chapter 1.0, engineering economy is about the development of alternatives for a given problem. Brainstorming may be used to develop alternatives. All the alternatives selected have to be viable. Decision analysis is to rank the alternatives with the most profitable or lowest cost alternative being given the highest rank. Different alternatives can have different cash flow diagrams. The revenues may be different: costs incurred such as materials, labor, and utility may vary and the interest rate and periods may not be the same. The five methods of evaluation of capital projects discussed in Chapter 4.0 shall be used here to select the best possible solution. These are the PW, present worth analysis, FW, future worth analysis, IRR, internal rate of return, AW, annual worth analysis, ERR, external rate of return. A variation of the IRR method is the payback period analysis. Factors are introduced to take inflation into account.

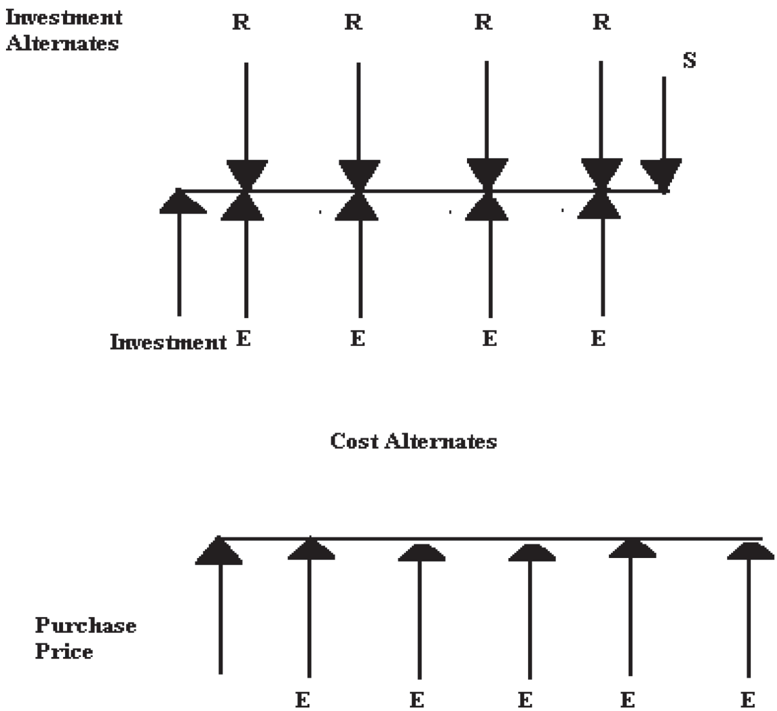

Figure 5.1 Cash Flow Diagrams of Investment and Cost Alternatives

The right questions must be posed: (i) what are the differences among the alternatives; (ii) what is a common unit of measure; (iii) are all the relevant criteria met; (iv) what is the risk-reward ratio; (v) who pays for the capital investments—government or private companies; (vi) what are the local, state, and federal regulations; (vii) is the product for domestic consumption or for export; (viii) what are the exchange rates for the $; (ix) what are the inflation rates and deflation rates; (x) what are the government incentives given; (xii) is the business capital intensive like a nuclear power plant or cash flow intensive such as the gas station and convenience store complex studied in the end of Chapter 4.0 exercises; (xiii) what is the role of assets and liabilities of the enterprise; and (xiv) what is the MARR (%) (minimum acceptable rate of return).

Public sector projects are evaluated slightly differently compared with private sector projects. A benefit-cost analysis will be unveiled at a later chapter. In addition to costs and benefits, the disbenefits are also taken into account in cases of public sector projects.

(i) Comparison of alternatives can be undertaken for the purpose of:
(ii) Invest in the more profitable process—Profit Based;
(iii) The cash flow diagrams have a capital investment and have net cash inflows over the study period as shown in Figure 5.1;
(iv) Select the lower life-cycle cost alternative—Cost Based;
(v) The cash flow diagrams have cash outflows only as shown in Figure 5.1;
(vi) Quantitating the savings accrued from implementation of a process improvement;
(vii) Assessment of environmental impact of different chemical processes—Other Criteria.

Alternatives can be different kinds:

Mutually Exclusive Alternatives: In this category, only one viable alternative can be selected from all the options available. For example, an engineer has to select the right equipment such as CSTR (continuous stirred tank reactor) or PFR (plug flow reactor) in order to accomplish a given process. When one is selected, the other is rejected.

Independent Projects: Here one or more of the available projects may be selected for implementation. The projects can be evaluated using PW, FW, IRR, AW, or ERR methods and rank ordered according to the profitability. For example, a savvy investor with $1 million dollars may put $300,000 in stocks, $200,000 in bonds; $100,000 in FDIC-insured CDs; $100,000 in money market funds; and $300,000 in gold. Here, the different options were rank ordered and monies allocated according to the rank.

Screening by Other Criteria: In some cases, dollars and cents cannot be the sole determining criteria to evaluate alternatives and come to the most profitable decision. All the relevant criteria as discussed in the seven principles in chapter 1.0 have to be met. (Criteria such as all humans are created equally and laborers cannot be discriminated against based on the color of their skin, or national origin, age, gender, etc.) The environmental impact is also rapidly becoming an added criterion for selection of the best possible alternative.

The PW of the identified alternatives are calculated from their respective cash flow diagrams. The project with the highest PW in dollar

amount is selected as the best possible alternative. Special cases can be identified when there are two alternatives, A and B, considered.

Case A: PW_A, present worth of Project A is in positive territory and PW_B, present worth of Project B is negative territory. $PW_A > 0$ & $PW_B < 0$. This means that Project A would fetch a profit and Project B would result in a loss when undertaken. In such cases, Project A may be selected.

Case B: Both PW_A and PW_B are in positive territory. Both projects can result in a profit. In such cases, the projects may be *rank ordered*. Sometimes the project with the maximum PW may be selected. $PW_A > 0$ & $PW_B > 0$.

Case C: The dollar value of PW_A and PW_B are within 20% of each other. Such cases may be determined as *too close to call*. This is so because of the uncertainty involved in the estimates of revenues, costs, capital investment and salvage values, interest rates, lives of equipment, inflation rates, etc. In a later chapter, sensitivity analysis and spider plots will be discussed in greater detail.

Any of the five methods such as PW, FW, AW, IRR, and ERR may be used in order to evaluate the alternatives prior to arriving at the decision. In cases of PW, FW, AW, the alternative with the higher dollar value can be selected. In the case of IRR, the project with the higher IRR is not selected. IRR between projects should not be compared. The IRR of a project is compared with MARR. If IRR > MARR, the project is acceptable. The business significance of comparison of IRRs from different projects is low.

Example 5.1 Best Process to Manufacture CNTs (Carbon Nanotubes)

CNTs are one of the four allotropes of carbon: (i) diamond; (ii) graphite; (iii) C_{60} fullerenes; and (iv) CNTs, carbon nanotubes. Graphene sheets of carbon rolled around the needle axis were discovered by Ijima in 1991. The diameter of these tubes is 1–2 nm and length is a few microns. CNTs offer attractive properties ranging from 1 TPa of modulus of elasticity to 2000 w/m/k thermal conductivity, etc. They can be used as a blend substituent to make novel nanocomposites.

There are two processes used to manufacture CNTs. Find out which is more profitable at 12% effective interest rate.

Process A: The HiPCO reactor is used. HiPCO is the high-pressure carbon monoxide process to manufacture CNT using the Boudard reaction [1]. A PFR (plug flow reactor) designed at Rice University in Houston is used at high pressures of 30–50 bar and at temperatures 1273–1473 K. CNTs and CO_2 are produced from CO, carbon monoxide, and iron pent carbonyl catalyst precursor. The conversion was 20%.

Process B: The CoMoCAT reactor is used. CO disproportionation is effected over a mixed cobalt-molybdenum catalyst on silica support in a fluidized bed reactor developed at the University of Oklahoma. The reaction forms CNTs and CO_2–at temperatures between 973–1223 K and total pressure ranging from 15–50 psia. The CO selectivity is 80% and conversion about 20%.

The information needed for economic analysis on the processes is given in Table 5.1 from Agboola et al. [2].

$$\text{Annual Materials Expenses for Process A} = \$140 \text{ million} \tag{5.1}$$

$$\text{Annual Electricity Expenses for Process A}$$
$$= (1{,}056)^*(24)^*(330)^*(\$0.125) = 1.05 \text{ million} \tag{5.2}$$

$$\text{Annual Gas Expenses for Process A}$$
$$= (486)^*(24)^*(330)^*(\$0.50)/1\text{E}6 = \$1.925 \text{ million} \tag{5.3}$$

$$\text{Annual Steam Expenses for Process A}$$
$$= (2{,}565)^*(24)^*(330)^*(\$0.125)/1\text{E}6 = \$2.54 \text{ million} \tag{5.4}$$

$$\text{Annual Sales of CNTs from Process A}$$
$$= (595)^*(24)^*(330)^*(\$38)/1\text{E}6 = \$179.1 \text{ million} \tag{5.5}$$

$$\text{Annual Profit (R} - \text{E)} = (\$179.1) - (\$140)$$
$$-\text{SUM(Gas, Steam, Electricity)} = \$33.562 \text{ million} \tag{5.6}$$

$$\text{Present Worth Factor, (P/A,12\%,10)} = 5.6502 \tag{5.7}$$

$$PW_A = -(\$4.6) + (\text{R} - \text{E})^*(\text{P/A,12\%,10})$$
$$= -\$4.6 + 5.6502^*\$33.56185 = \$189.63 \text{ million} \tag{5.8}$$

Table 5.1 Two Processes Used to Manufacture CNTs

Item	Process A (Amount)	Process A (Description)	Process B (Amount)	Process B (Description)
Catalyst		Fe		Co-Mo
Reactants		CO & $Fe(CO)_5$		CO
Reactor		Plug Flow Reactor		Fluidized Bed
Temperature		1050°C		950°C
Pressure		450 psia		150 psia
Selectivity		90%		80%
Purification		Oxidation, Filtration		Leaching, Froth Flotation
Product Rate		595 kg/h		595 kg/h
Steam	12.5 cents/kg	2,565 kg/h		2,885 kg/h
Gas	50 cents /kg	486 kg/h		486 kg/h
Electricity	12.5 cents/kWh	1,056 kW		387 kW
Capital Cost of Plant	$4.6 million		$4.4 million	
Price	$38/kg		$25/kg	
Materials Cost	$140 million		$84 million	
Plant Life		10 years		10 years
Operation		330 days/year		330 days/year

$$\text{Annual Materials Expenses for Process B} = \$84 \text{ million} \tag{5.9}$$

$$\text{Annual Electricity Expenses for Process B} \\ = (387)*(24)*(330)*(\$0.125) = 0.383 \text{ million} \tag{5.10}$$

$$\text{Annual Gas Expenses for Process B} \\ = (486)*(24)*(330)*(\$0.50) = \$1.925 \text{ million} \tag{5.11}$$

$$\text{Annual Steam Expenses - Process B} \\ = (2{,}885)*(24)*(330)*(\$0.125) = \$2.856 \text{ million} \tag{5.12}$$

$$\text{Annual Sales of CNTs from Process B} \\ = (595)*(24)*(330)*(\$25) = \$117.81 \text{ million} \tag{5.13}$$

$$\text{Annual Profit } (R - E) = (\$117.81) - (\$84) \\ -\text{SUM(Gas, Steam, Electricity)} = \$28.646 \text{ million} \tag{5.14}$$

$$\text{Present Worth Factor, } (P/A, 12\%, 10) = 5.6502 \tag{5.15}$$

$$PW_B = -(\$4.4) + (R - E)^*(P/A,12\%,10)$$
$$= -\$4.4 + 5.6502^*\$28.64616 = \$157.4565 \text{ million} \quad (5.16)$$

Comparing Eq. (5.8) and Eq. (5.16), Process A has a higher present worth compared with Process B. Hence, Process A is the more profitable route to prepare CNTs.

Example 5.2 Cost Savings by Using Microfiltration Pretreatment During SWRO (Sea Water Reverse Osmosis)

Pretreatment has been used to improve the performance and lower the overall cost of obtaining water using SWRO (sea water reverse osmosis plants). Control of membrane fouling is agreed upon as a problem encountered during the operation of the SWRO plants. The conventional pretreatment method controls and minimizes the plugging potential of the feed water by coagulating, flocculating, and filtering out colloidal and suspended solids and by achievement of reduction in SDI (silt density index) of feed water. MF (microfiltration) pretreatment appears to be an attractive method. Turbidities of water can be lowered less than 0.1 NTU regardless of inflow turbidity level. UF pretreatment results in removal of TC (total coli) from bacteria *Giardia* and *Cryptosporidium.* Viruses are also removed. The novel pretreatment strategy leads to reduction in capital costs, reduction in operating costs due to lower membrane replacement, less chemical consumption, elimination of cartridge filters, less downtime, less maintenance costs, less labor costs, etc. The MF unit is skid mounted and impurities larger than 200 nm are removed using this device. The machine consists of polypropylene filtration modules, circulation pump, associated valves, piping, and instrumentation and control systems.

The data needed to perform comparison of alternatives is given below in Table 5.2 and taken from [3]. The capital expenses such as treatment plants, pumps, etc. are financed over a 30-year period at 5% interest rate.

Cost of Water (Conventional)

$$\text{Annual Variable Expenses} = \$9.648 + \$4.662 + \$3.478$$
$$= \$17.788 \text{ million} \quad (5.17)$$

$$\text{Capital Recovery Factor } (A/P,5\%,30) = 0.065051 \quad (5.18)$$

Table 5.2 Economic Data for Evaluation of MF Pretreatment

	Conventional			Microfiltration	
Description	**\$ millions/ year**	**\$ (millions)**		**\$ millions/ year**	**\$ (millions)**
Plant		145.3777			40.64489
Pumps		1.829328			1.829328
Dosing Plant		0.230588			0
Structural		11.89832			11.89832
Subtotal		159.336			54.37253
Electricity	9.648			7.09	
Chemicals	4.662			1.637	
Filters	3.478				
Water Rate			27,276 m^3/day		

$$\text{Capital Recovery Amount} = \$159.336{*}(0.065051) = \$10.365 \tag{5.19}$$

Plant operates 330 days in the year

$$\text{Cost of Water} = \frac{(\$17.788 + \$10.365)}{27{,}276{*}330} = \$3.13/m^3 = 0.313c/liter \tag{5.20}$$

Cost of Water (MF Pretreated)

$$\text{Annual Variable Expenses} = \$7.09 + \$1.637 = \$8.727 \text{ million} \tag{5.21}$$

$$\text{Capital Recovery Factor (A/P,5\%,30)} = 0.065051 \tag{5.22}$$

$$\text{Capital Recovery Amount} = \$54.37253{*}(0.065051) = \$3.537 \tag{5.23}$$

Plant operates 330 days in the year

$$\text{Cost of Water} = \frac{(\$8.727 + \$3.537)}{27{,}276{*}330} = \$1.363/m^3 = 0.1363c/liter \tag{5.24}$$

Comparing Eq. (5.20) and (5.24) the microfiltration pretreatment process modification would lower the cost of producing water by \$1.77/m^3. Hence, the microfiltration pretreatment alternative is preferred over the conventional method of pretreatment.

Example 5.3 Bioethanol from Sugarcane Bagasse

Renewable energy sources are sought the world over in order to provide energy security for our nation and the world. According to a recent Goldman Sachs report, Brazil, Russia, India, and China, BRIC nations, are expected to emerge as the world's largest economies. In Brazil, 387 million tons of sugarcane was produced in 2005/2006. Additionally, 104 million tons of bagasse came from the sugar mills. From a ton of sugarcane, 73 kg of fermentable sugars and 27 kg of dry bagasse can be produced. The composition of bagasse is as follows:

Cellulose	43%
Xylans	25%
Lignin	23%

Hemicellulose is a polymer of xylose and cellulose is a polymer of glucose. The sugarcane bagasse can be burned in boilers to generate electricity. Sugarcane bagasse can be converted to bioethanol and furfural by a dilute acid hydrolysis process. The acid hydrolysis process is a two-stage operation. In the first-stage, hemicellulose is converted into sugar using dilute acid and steam. In the second stage, cellulose is converted to sugar by hydrolysis. Upon hydrolysis, the sugar can be processed in order to produce ethanol, furfural, and other products. Furfural can either be used alone or combined with phenol, acetone, or urea to make solid resins. Such resins are used to make fiberglass, aircraft components, and automotive brakes. It can also be used as a hydrocarbon solvent. Coproduction of ethanol and furfural can make the process more profitable. The acid hydrolysis plant consists of the following equipment:

Pump and Heater;
Reactor to Produce Furfural and Glucose;
Purification and Separation.

After passing through the heater, the feedstock is treated with dilute sulfuric acid solution (0.5 wt%) at high temperature for a short time. Cellulose and pentose are converted to glucose and furfural, respectively. The hydrolyzed stream is then separated using a distillation column. The hydrolyzed material is then neutralized with lime. The upper stream is passed through a condenser leading to the formation of furfural. The unreacted cellulosic residue and lignin from stream can be burned to generate electricity or sold as a by-product. The total capital investment needed for the acid hydrolysis plant is about $4.016 million [4]. The furfural can be sold for $1.91/kg and ethanol can be sold for $1.37/kg. The bagasse suppliers are paid at 18 cents per kg of feedstock. A dilute acid hydrolysis plant can operate at 4000 kg/h of sugarcane bagasse. The utilities and labor costs were estimated and found to be $0.80 per kg of ethanol and $1.11/kg of furfural. The plant utilization factor is 80%. The yield of ethanol is 15% and that of furfural is 7% from the bagasse.

Which would be a better alternative: (i) start a dilute acid hydrolysis plant with an estimated life of 15 years; or (ii) generate electricity from bagasse using a power plant? The cost of the steam boiler and turbine is about $1 million each. The electricity can be sold at 12.5 cents per kWh. The calorific value of bagasse is 8.0 MJ/kg. The power plant thermal efficiency can be taken as 30%. The interest rate is 6%. For purposes of analysis, assume that the power plant life is equal to that of the acid hydrolysis plant and is 15 years. The operating expenses of the power plant are about 4 cents per kWh.

Dilute Acid Hydrolysis Process

Annual Sales of Furfural

$$= (4000)*(24*365*0.8)*(0.07)*(\$1.91) = \$3.748 \text{ million} \qquad (5.25)$$

Annual Sales of Ethanol

$$= (4000)*(24*365*0.8)*(0.15)*(\$1.37) = \$5.761 \text{ million} \qquad (5.26)$$

Annual Expenses of Furfural

$$= (4000)*(24*365*0.8)*(0.07)*(\$1.11) = \$2.178 \text{ million} \qquad (5.27)$$

Annual Expenses of Ethanol

$$= (4000)*(24*365*0.8)*(0.15)*(\$0.80) = \$3.364 \text{ million} \qquad (5.28)$$

$$(P/A,6\%,15) = 0.5^*(9.295 + 10.1059) = 9.723 \text{ (from Table A-10)}$$

$$PW_{acid} = -4.016 + P/A^*(R - E) = -4.016$$
$$+ 9.723^*(\$3.748 + \$5.761 - \$2.178 - \$3.364)$$
$$= \$34.55 \text{ million} \quad (5.29)$$

Power Plant Option

Annual Sales of Electricity

$$= (4000)^*(0.3)^*(8\text{ E6})^*(0.8)^*(\$0.125) = \$0.96 \text{ million} \quad (5.30)$$

Annual Cost of Power Plant Operation

$$= (4000)^*(0.3)^*(17\text{ E6})^*(0.8)^*(\$0.04) = \$0.6528 \text{ million}$$

$$PW_{plant} = -2 + 9.723^*(\$0.96 - 0.6528) = \$0.987 \text{ million} \quad (5.31)$$

Comparing Eq. (5.29) and Eq. (5.31), the present worth of the acid hydrolysis plant is at $34.55 million and is greater than the present worth of the power plant at $0.987 million. Hence, it is better to invest in the acid hydrolysis plant compared with the steam power plant.

Example 5.4 Sequestration by Dimethyl Carbonate Formation

Two routes for chemical reuse of CO_2, carbon dioxide, are considered. Interest in sequestration projects is increasing due to the concerns about global warming [5]. Studies have shown that a 10% dimethyl carbonate (DMC) addition to diesel leads to 20% smoke reduction. Both plants operate at 7,200 h/year. CO_2 is available at no cost. Each ton of CO_2 sequestered leads to carbon credits of $19.02. Electric utilities cost 43 cents per kWh. Which is a better alternative at an interest rate of 10% and a plant life of 14 years?

Route A: Production from Ethylene Oxide: Ethylene oxide and CO_2 are allowed to react to form ethylene carbonate. Ethylene carbonate produced is reacted with methanol in a second PFR (plug flow reactor), leading to the production of DMC and ethylene glycol. The capital cost can be taken as $80 million; 36,732 tons/year of CO_2 are sequestered. The energy used in this process is 216 TJ/year. Revenue from the plant is $2500 per ton of CO_2 sequestered.

Route B: Production from Urea: Urea is produced by reacting ammonia and CO_2. DMC is produced by urea methanolysis. The capital cost for

this route is given as \$100 million; 27,129 tons/year of CO_2 are sequestered. The energy used in this process is 415 TJ/year. The revenue from the plant is \$3800 per ton of CO_2 sequestered.

$$(P/A, 10\%, 15) = 7.3667 \text{ (from Table A-14)}$$

$$\begin{aligned}\text{Annual Revenue—Route A} &= (36{,}732)^*(2{,}500)\\ &= \$91.83 \text{ million}\end{aligned} \tag{5.32}$$

$$\begin{aligned}\text{Annual Carbon Credit—Route A} &= (36{,}732)^*(\$19.02)\\ &= \$0.698{,}643 \text{ million}\end{aligned} \tag{5.33}$$

$$\begin{aligned}&\text{Annual Energy Expenses—Route A} =\\ &(216\ E12)(\$0.43)/1000/7200 = \$12.94 \text{ million}\end{aligned} \tag{5.34}$$

$$\begin{aligned}PW_A &= -\$80 + 7.3367^*(\$91.83 + \$0.698{,}643 - \$12.94)\\ &= \$506.292 \text{ million}\end{aligned} \tag{5.35}$$

$$\begin{aligned}\text{Annual Revenue—Route B} &= (27{,}129)^*(3{,}800)\\ &= \$103.09 \text{ million}\end{aligned} \tag{5.36}$$

$$\begin{aligned}\text{Annual Carbon Credit—Route A} &= (27{,}129)^*(\$19.02)\\ &= \$0.515{,}994 \text{ million}\end{aligned} \tag{5.37}$$

$$\begin{aligned}&\text{Annual Energy Expenses—Route A}\\ &= (414.99\ E12)(\$0.43)/1000/7200 = \$24.78 \text{ million}\end{aligned} \tag{5.38}$$

$$\begin{aligned}PW_B = -\$100 + 7.3367^*(\$91.83\\ + \$0.698{,}643 - \$12.94) = \$480.66 \text{ million}\end{aligned} \tag{5.39}$$

Comparing Eq. (5.35) and Eq. (5.39), it can be seen that the present worth (PW) of the plant A for sequestration is \$506.292 million and the PW of the plant B is \$480.66 million. In this case, it is too close to call one better than the other. However, Route A can be ranked higher than Route B, and Route A can be selected as the better alternative.

5.2 Study Period—Don't Compare an Apple and an Orange

When two alternatives are compared, the life of the equipment used by either process may be different from each other. When this happens,

comparisons have to be made on a comparable basis. In order to make more meaningful comparisons, the following quantities are defined:

Study Period: The study period is the duration over which the comparisons are drawn. This can be lower, equal to, or greater than the useful life of the equipment used.

Useful Life: The duration over which the equipment is functioning is called the useful life of the equipment.

Repeatability Assumption: This can be used when the useful lives of equipment in Alternative A and Alternative B are different from each other. The study period is selected as the lowest common multiple of the two useful lives. For example, say the decision analysis is to select the best photocopier for the department. Copier A is imported from Japan and has a useful life of nine years. Copier B is from a domestic supplier with a useful life of six years. Then the study period is set at 18 years; 18 is the LCM of (6,9). Two A copiers are allowed to function in series in time, thus lasting 2*9 = 18 years. In a similar manner, three copiers of B are allowed to function one after the next, thus lasting 3*6 = 18 years. The study period of 18 years is now the same for both copiers A and B. The purchase price, annual supplies cost, maintenance cost, utility cost, labor cost, and salvage value obtained at the end of use of the machine can all be accounted for two copiers of A and three copiers of B. In some cases, the quantity may have to be multiplied by the number of copiers—it depends on the given application. Sometimes the LCM may have to be the multiple of the two useful lives. For example, should copier A have a life of five years and copier B have a life of three years, the study period can be selected as LCM(3,5) = 15 years. In this case, five copiers A have to be considered as a basis and three copiers of B have to be considered as a basis. In such cases, projections into the future may cause errors such as changes in interest rates, inflation rates, etc. This method can also be used when the study period tends to infinity or is indefinitely long.

Co-terminated Assumption: The study period is truncated from the useful life of equipment in one alternative in order to have equal

the useful life of the equipment in the second alternative. This can be used when the useful life of alternative A, NA is greater than useful life of alternative B, NB—i.e., NA > NB. The study period can be selected as NB. All the revenues and expenses for alternative A are then reevaluated for a co-terminated period of NB. This is feasible because NB < NA.

Example 5.5 Life-Cycle Cost of Photocopier

The Xerox Work Center can handle up to 200,000 pages per month and has a copy speed of up to 45 pages per minute. Its costs about $2,200 and weighs 80 lbs. The toner cartridge costs about $90 per cartridge. It can be used to print 2000 pages. This machine has a useful life of four years. Toshiba's e-STUDIO 455 can copy up to 45 pages per minute and the base unit costs about $10,800. The toner yield is 30,000 pages and cost is $190 each. The useful life of the e-STUDIO 455 is about 16 years. Which copier has the lower life cycle cost: the Xerox Work Center or the Toshiba e-STUDIO? The interest rate is 8.0%. Assume that 1 million pages are copied every year.

The two machines have unequal lives. The repeatability assumption is used. Four Xerox Work Centers are needed for a common study period of 16 years. They are assumed to work one after the other as shown in the cash flow diagram in Figure 5.2. The PW analysis is used in order to calculate the life-cycle cost of the two different photocopiers.

$$\text{Annual Toner Cost for Xerox Work Center} = 1\text{ E6}/2000^*(\$90) = \$45{,}000 \quad (5.40)$$

$$\text{Annual Toner Cost for Toshiba's e-STUDIO} = 1\text{E6}/30{,}000^*(\$190) = \$6333.33 \quad (5.41)$$

$$(P/A,8\%,16) = 8.8514 \text{ (From Table A-12)} \quad (5.42)$$

$$(P/F,8\%,4) = 0.735030 \text{ (From Table A-12)} \quad (5.43)$$

$$(P/F,8\%,8) = 0.540269 \text{ (From Table A-12)} \quad (5.44)$$

$$(P/F,8\%,12) = 0.397114 \text{ (From Table A-12)} \quad (5.45)$$

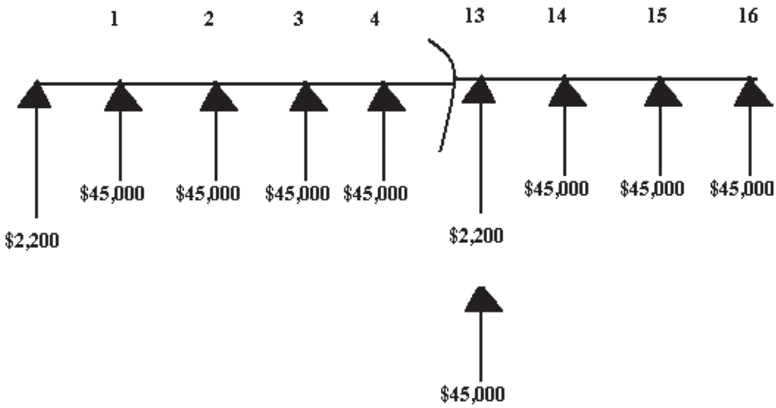

Figure 5.2 Cash Flow Diagram Adjusted for Repeatability Assumption for Xerox Work Center with Useful Life of 4 Years

Life-Cycle Cost of Xerox Work Center

$$= CX + (P/A,8\%,16)*(-\$45{,}000) + (P/F,8\%,4)*C_X + (P/F,8\%,8)*CX + (P/F,8\%,12)*C_X \quad (5.46)$$

$$= -\$2{,}200 - (8.8514)(\$45{,}000) - 0.735030*(\$2{,}200) - 0.540269*(\$2{,}200) - 0.397114*(\$2{,}200) = \$404{,}192 \quad (5.47)$$

The e-STUDIO was purchased at a one-time charge of $10,800. Over 16 years, the toner costs were found to be $6,333.33 every year. The cash flow diagram for expenses for Toshiba's e-STUDIO is shown in Figure 5.3.

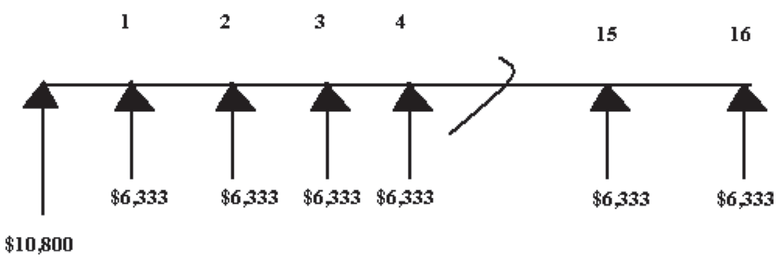

Figure 5.3 Cash Flow Diagram for Toshiba's e-STUDIO with a Useful Life = 16 years

Life-Cycle Cost of Toshiba e-STUDIO

$$= -\$10{,}800 - 8.8514^*(\$6333.33) = \$66{,}859 \quad (5.48)$$

It can be seen that by comparison of Eq. (5.47) and Eq. (5.48), the Toshiba e-STUDIO has the lowest life-cycle cost. This is contrary to the purchase price. The purchase price of the Xerox Work Center is lower at $2,200, compared with Toshiba's $10,800. When taking into account the toner expenses, the Toshiba e-STUDIO turns out to be the lower total cost at $66,859 to that of the Xerox Work Center's $404,192. In both cases, the basis was 1 million copies per year made.

Example 5.6 ED Lighting

Light-emitting diode works on the principle of *electroluminescence*, which is the release of photon energy from electron-hole recombinations within the device. The color of the light depends on the wavelength of the light corresponding to the energy released. LEDs have lower energy consumption and longer lives compared with incandescent lamps. Boeing wants you to calculate the life-cycle cost of LEDs and incandescent lamps for its new aircraft. They need 420 lights at 100 W each. The incandescent lamps have 100 hrs/life and cost $1.36 each. The illumination received is 1680 lumens per bulb. Mercury LED lamps of 400 W cost $32.73 per lamp and can last for 24,000 hours at an illumination of 36,000 lumens [6]. Assume that the Boeing flight is in operation for 200 days in the year. The cost of electricity is 25 cents per kWh. The interest rate is 5%.

$$(P/A,5\%,5) = 4.3295 \quad (5.49)$$

Number of hours of in-flight service per year

$$= 200^*24 = 4800 \text{ hrs} \quad (5.50)$$

The LED lamps and incandescent bulbs have unequal lives. The repeatability assumption can be used. The study period is selected as five years.

The number of incandescent bulbs needed for a five-year period $= (5)^*(4800/100)^*420 = 100{,}800$ (5.51)

The cost of purchasing the incandescent bulbs

$= 100{,}800^*(\$1.36) = \$137{,}88$

Annual Electric Utility costs
$= (100)*(420)*(4800)/1000*(\$0.25) = \$50{,}400/\text{year}$ (5.52)

Total cost of purchase and operation of incandescent bulbs
$= -\$137{,}88 - (4.3295)*(\$50{,}400) = \$319{,}007$ (5.53)

Number of LED (mercury lamps) needed for a five-year period at the same illumination

$= (4800/24{,}000*420*5)*(1680/36{,}000) = 19.6 = 20 \text{ lamps}$ (5.54)

Cost of purchasing the mercury lamps
$= 20*(\$32.73) = \$654.60$ (5.55)

Annual Electric Utility Costs
$= (400)*(4)/1000((\$0.25) = \$0.40$ (5.56)

Total cost of purchase and operation of mercury lamps
$= -\$654.60 - 4.3295*(\$0.40) = -\$656.33$ (5.57)

It can be seen that by comparisons of Eq. (5.53) and Eq. (5.57), it is lower in total cost to choose LED mercury lamps as aviation lighting compared with incandescent lamps.

5.3 Equivalent Worth Method

When two alternatives are evaluated and when the critical equipment has unequal lives, other methods of evaluation are considered. One is the *rate of return* method. The rate of return can be calculated as

$$ROR = \frac{(R - E)}{C_I} \tag{5.58}$$

Where R is the revenues generated in a prototypical year, E is the expenses generated in a prototypical year and C_I is the capital investment made less the salvage value received to start the business. The project with the higher rate of return can be selected. The IRRs of two projects should not be compared. The IRR has to be compared with the MARR. IRR comparisons are meaningless because the PW changes with the interest rate depending on the project, particularly the revenue function, cost

Example of Cotermination (Case 2)

Suppose the study period had been stated to be 20 years. Which boiler would you recommend?

	Boiler A	Boiler B
Investment cost	$50,000	$120,000
Useful life	20 yrs.	40 yrs.
Salvage value @ end of useful life	10,000	20,000
Annual costs	9,000	6,000

Useful life of A = 20 years = study period

Useful life of B = 40 years > study period

Assume MV_B @ EOY 20 = $50,000

The MARR is 10% per year.

Example Continued (Cotermination)

$AW_A(10\%) = -14,700$

$AW_B(10\%) = -6,000 - [20,000(A|P, 10, 20) - 50,000(A|F, 10, 20)]$

$= -19,225$

Still select Boiler A to minimize costs.

What would the market value of Boiler B @ EOY 20 have to be in order to select Boiler B instead of A?

Example Continued (Cotermination)

Market Value of Boiler B to reverse the decision = ?

Set $AW_A = AW_B$, let X be the unknown market value

$$-14{,}700 = -6{,}000 - \{120{,}000(A|P, 10, 20) - X(A|F, 10, 20)\}$$

$$-8{,}700 = -\{14{,}100 - 0.0175X\}$$

$$5{,}400 = 0.0175X$$

X = \$308,571 therefore, MV_B > \$308,571 to favor B

Such a value is very unlikely because X is more than the initial cost of Boiler B.

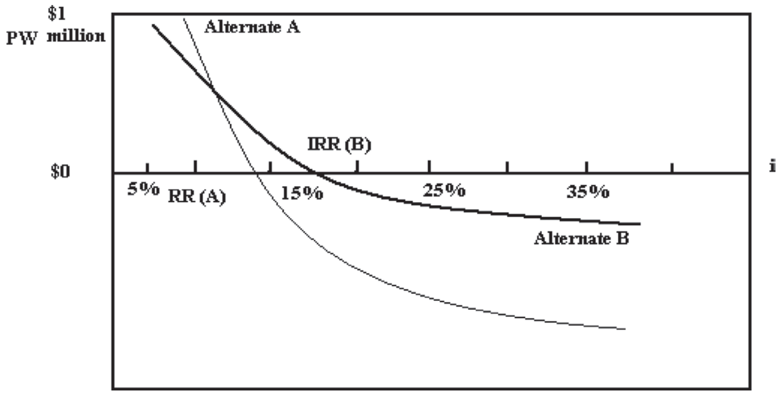

Figure 5.4 Comparison of IRRs Can Lead to Erroneous Conclusions

function changes with the interest rate. Under some conditions, the PW of Project A may be greater than the PW of Project B, although the IRR of Project B may be greater than the IRR of Project A. Under those conditions, Project A has to be selected. Selection is based on the value of PW and not IRR. This is shown in Figure 5.4. As can be seen in Figure 5.4, for interest rates less than 12% $PW_A > PW_B$ and can be selected although the $IRR_B > IRR_A$. Only when the interest rates are high does Project B become more attractive compared with Project A.

AW method analysis may also produce good results. This can be the case when the study period is not equal to useful life.

Example 5.7 Anti-Allergic Cream

A pharmaceutical plant is proposed to manufacture anti-allergic cream. The plant consists of six unit operations: (i) melting; (ii) blending; (iii) filling; (iv) cartoning; and (v) packaging, both shrink wrapping and for shipment. Batches of emulsifying wax and foam stable are independently heated in a jacketed heater. Then emulsifier and ointment are added. Excipient 3 is dissolved in hot deionized water DI and then blended with the other ingredients. The active ingredient is finally added. Cooling procedures are instituted that last for 19 hours. The blended product is filled in tubes of 15 g each. The filling machine can fill 30 tubes per minute. Five sealed boxes per minute are prepared.

Table 5.3 Cost and Revenue Information for Pharmaceutical Plant

	Plan A Manual Cartoning	Plan B Automatic Cartoning
Batches	72/year	195/year
Production	959,976 tubes	2,599,936
Capital Cost (Equipment)	–	$900,000
Operating Cost	$1,437,631	$3,847,157
Revenue	$4,001,400	$10,835,500

In a second method, a new multifunctional blending tank and automatic cartoning machine with line speed of 60 tubes per minute are introduced. The cost is about $900,000. The data needed to evaluate the two alternatives is given in Table 5.3. The interest rate may be taken as 8%. A 20-year life of the modification can be assumed.

The AW method may be used to evaluate the advantages of using automatic cartoning.

The planning horizon is taken as 20 years. The study period is thus the same for Plan A and Plan B.

Plan A—Manual Cartoning

$$AW_A = R_k - E_k = \$4{,}001{,}400 - \$1{,}437{,}631 = \$2{,}563{,}769 \quad (5.59)$$

Plan B—Automatic Cartoning

$$(A/P, 8\%,20) = 0.101852$$

$$AW_B = \$10{,}835{,}500 - \$3{,}847{,}500 - (A/P,8\%,20)*\$900{,}000 = \$6{,}896{,}333 \quad (5.60)$$

It can be seen from comparisons of Eq. (5.59) and (5.60) that Plan B with automatic cartoning has a higher AW compared with Plan A. Plan B with automatic cartoning has been selected.

5.4 Optimization

Often the PW of a business can be *optimized*. There is a trade-off between alternatives with different variable costs and fixed costs. For example, a

number of unit operations in chemical processes and the biotechnology industry are operated stagewise and in a continuous manner. Consider a distillation column that can be used to accomplish the given task, for instance the separation of ethyl alcohol from water. The use of an overhead condenser and reflux can improve the performance of the distillation column. At minimum reflux, the number of plates or stages needed to achieve the desired level of purity in the product would be larger. At a larger reflux ratio, the number of stages required to accomplish the given task would be lower. Consider the two alternatives:

Alternative A: Distillation column with lower reflux ratio and larger number of plates needed to achieve the desired level of consistency of alcohol. This route will require a larger capital cost and lower energy or variable cost.

Alternative B: Distillation column with higher reflux ratio and lower number of plates needed to achieve the desired level of consistency of alcohol. This route will require larger energy or variable costs, but should result in lower capital cost.

Which of the two alternatives should be selected? When capital is lower in cost, a low reflux ratio with a higher number of stages can be selected. When energy is cheap, a higher reflux ratio and lower number of stages can be selected. Let the useful life of the distillation column be N years and the interest rate be i(%). The PW for any distillation column can be written as

$$PW = -N_P C_F' - RC_V' * \ (P/A, i\%, N) \tag{5.61}$$

Where the fixed costs increases linearly with the number of stages and the annual utility costs increase linearly with the reflux ratio needed R. The variable costs will increase with the reflux ratio used. The number of stages and reflux ratio can be related using expressions such as the *Kremser Equation*, which is more applicable when the thermodynamic equilibrium relation between the components separated is linear. The Kremser Equation can be written as

$$N_p = \frac{\ln (Purity)}{\ln \left(\dfrac{R}{m(R+1)} \right)} \tag{5.62}$$

An optimal PW of the distillation column can be identified when

$$\frac{d(PW)}{dN_p} = 0 \tag{5.63}$$

At the optimal value, the total life-cycle cost of operation of the distillation column in order to achieve a desired level of purity can be *minimized.*

There is increased interest in recent years to perform what is called a *techno-economic analysis*. Here, the technical feasibility of the process and the EW of the process are evaluated together. Ramifications of such analysis can be interesting. For example, there is identified an optimal number of fermenters that would result in a maximum present worth. A battery of fermenters can be used to generate biotechnology products. The batch fill and emptying times are such that the PW of the plant reaches a maximum at a certain number of fermenters.

At a certain number of effects in multiple effect evaporators, the total cost of the process reaches a minimum. The energy cost and additional capital cost as a function of number of effects are calculated. With more effects, more efficiency is reached in the evaporation, and hence the revenue generated from more effects would be greater. It is not clear at what number of effects maximum profit can be generated. The techno-economic analysis can be performed to arrive at a maximum profitable process. This may work out to be three or four. It depends on the additional efficiency that can be generated in the technology used for more effects. It also depends on the interest rates and revenue that can be accrued for better quality.

A number of discussions in the literature are about the golden triangle of *production, quality, and profit*. When quality required goes up, usually production goes down. More care is needed to produce a product with better quality. The product with better quality can be *priced* higher. Usually this results in better revenue. A techno-economic analysis may be conducted and conditions where profitability can be maximized can be identified.

Example 5.8 Optimal Number of Effects in Multiple Effect Evaporator

One of the critical unit operations used to manufacture *tomato paste* is the use of a multiple effect evaporator. With the use of more effects,

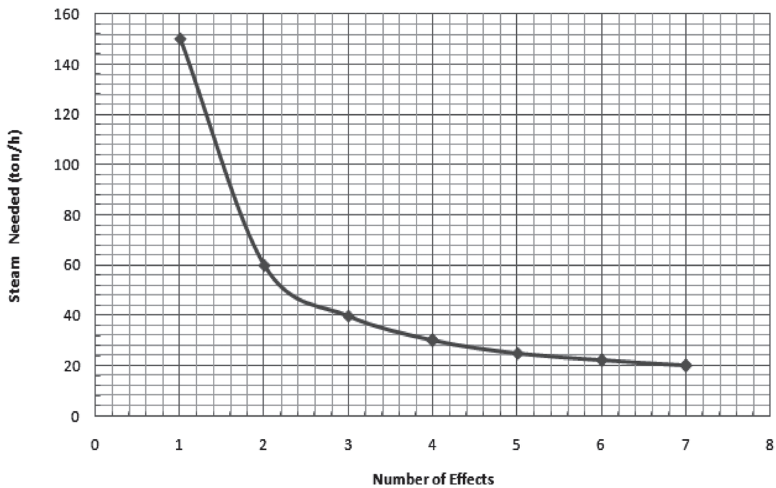

Figure 5.5 Steam Needed as a Function of Number of Effects in a Multiple Effect Evaporator

the capital cost increases linearly and the steam needed decreases in a nonlinear manner. This is shown for a multiple effect evaporator in New Zealand [7] in Figure 5.5 (Steam Flow Rate vs. Number of Effects) and Figure 5.6 (Heat Exchange Area vs. Number of Effects). The cost and economic data to evaluate the multiple effect evaporator needed to concentrate tomato paste is provided in Table 5.4. Calculate the number of effects of the evaporator when the PW of the operation is maximized. The interest rate is 10% and the study period is 13 years. Assume that the plant is operational 225 days per year at 10 tons/h tomato paste production rate.

The quality of the tomato paste from evaporators increases with the number of effects used. Hence, the price of tomato paste from a multiple effect evaporator can be charged higher compared with the product from a single pass evaporator. It can be seen from Figures 5.4 and 5.5 that with more numbers of effects, the capital cost increased but the steam requirement decreased. An *optimal* number of effects can be expected when the PW of the operation is maximized.

The PW of manufacturing tomato paste as a function of number of effects of the evaporator was calculated using an MS Excel spreadsheet using the data given in Table 5.4. The results are shown in Figure 5.7.

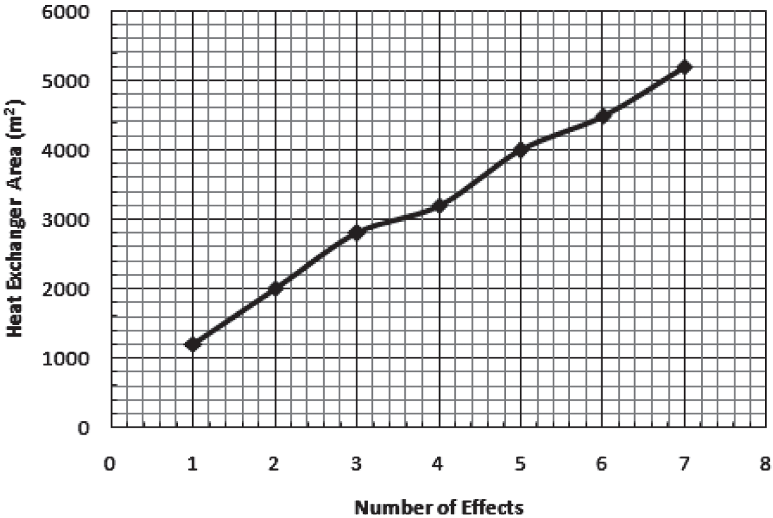

Figure 5.6 Heat Exchanger Area vs. Number of Effects

It can be seen from Figure 5.7 that the optimal number of effects is about 3 for a maximum PW of the manufacture of tomato paste. The results are shown in Table 5.5.

5.5 Household Finances

The head of a household in the 21st century must make a variety of financial decisions. Sometimes the value of money principles and decision analyses explained in the earlier chapters may be applicable to personal

Table 5.4 Cost and Economic Data for Tomato Paste Factory

Number of Effects, N_E	Capital Cost (Millions, $)	Steam Utilities Cost (Millions, $ per year)	Price $ per kg
1	92	8.1	4
2	108	3.24	4.07
3	115	2.16	4.14
4	122	1.62	4.21
5	129	1.35	4.28
6	136	1.188	4.35
7	143	1.08	4.42

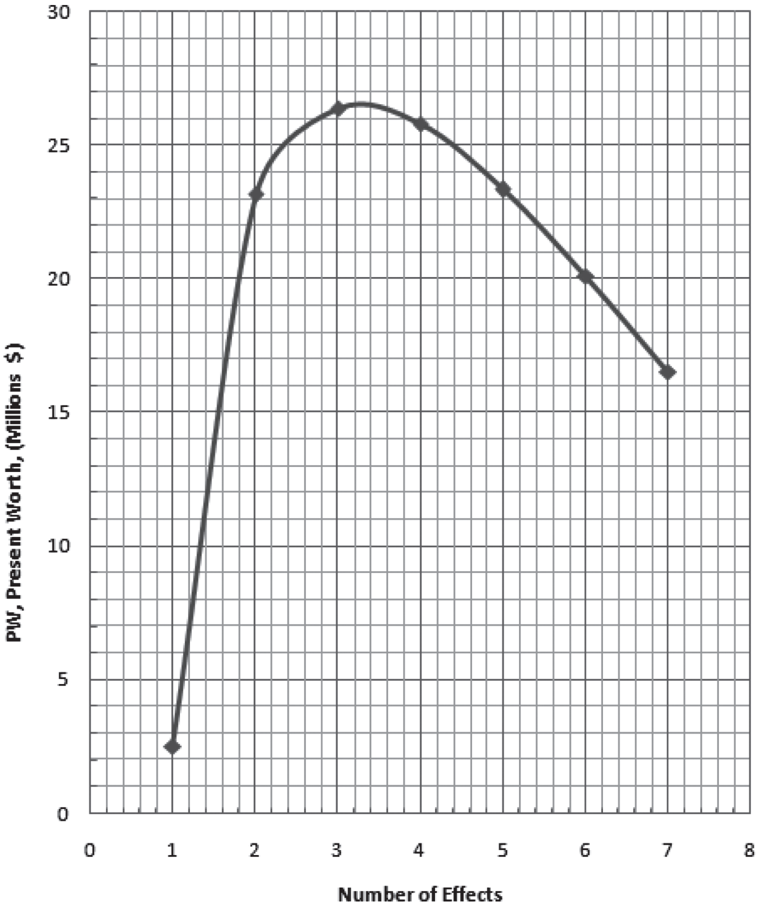

Figure 5.7 PW of the Multiple Effect Evaporator Operation as a Function of Number of Effects

finances. The five largest financial decisions that every individual can be expected to make during their earning years are as follows:

Home Ownership and Mortgage;
Car Ownership and Financing;
Insurance and Taxes;
Credit Card Application and Credit Score;
Retirement Planning—Nest Egg.

The time period associated with home mortgages is usually 30 years. New cars are financed over 60 or 72 months. Income tax to the federal

Table 5.5 PW Analysis for Concentration of Tomato Paste

Number of Effects, N_E	Capital Cost (Millions, $)	Steam Utilities Cost (Millions, $ per year)	Price $ per kg	Revenue (Millions, $)	PW (Millions, $)
1	92	8.1	4	21.6	2.5
2	108	3.24	4.07	21.98	23.2
3	115	2.16	4.14	22.4	26.4
4	122	1.62	4.21	22.7	25.8
5	129	1.35	4.28	23.1	23.3
6	136	1.188	4.35	23.5	20.1
7	143	1.08	4.42	23.9	16.5

government, state government, and city government must be paid every year. Life insurance, health insurance, auto collision and liability insurance, and home owners' or renters' insurance payments are expected once every six months or per quarter. A college graduate starting his career in this day and age would be looking to establish his or her line of credit. Credit card companies loan the principal when one of their customers wants to make a purchase anywhere, anyplace, anytime in the world. In return, the customer pays them every month at an interest rate a lot higher than the prime lending rate to banks and other financial institutions. In the new millennium, social security payments are not sufficient for the elderly after their retirement. Inflation eats into their peace of mind during retirement. IRA, Keogh accounts, and 401(k) savings plan have to be opened and maintained for a secure retirement by middle-class citizens.

The total household wealth in our nation is about $54.2 trillion. According to a recent survey, there are 114 million households in the United States. The number of millionaires has tripled in recent decades.

Home ownership is a means for less-wealthy individuals to attain net worth that will place them in the top quartile of the wealthiest people in America. New homes are built in neighborhoods within commuting distance from major cities. The purchase price of a home ranges from $50,000–$1 million. The purchase price is paid for first by banks that let the individual take out a mortgage. Home mortgages are repaid over a time period of 30 years. As can be seen from the annuity tables in

Appendix A, the multipliers for 30 years are not all that different from the multipliers at infinite time. This is a good break for the borrowers and serves as an incentive to purchase mortgages. Banks borrow money from the Federal Bank Reserve at the prime lending rate. Part of these monies is given out as home mortgages at a slightly higher rate compared with the prime rate. The banks operate within the mortgage and prime rates. All their employees' salaries, profits, and building costs come from these two numbers. For example, in 2010 at the time of this writing, the prime rate is 3.25%. The home mortgage rates from leading lenders are about 4.5%. Mortgage rates can be fixed or a mortgage can be an ARM (adjustable rate mortgage). The interest rates over a 30-year period may change. For example, on December 19, 1980, the prime rate was at a record high of 21.5%. During the course of the time period of a 30-year mortgage, a home owner can do what is called *refinancing*. Here the balance of the loan is paid off by the bank in full and a fresh loan is issued at the new refinanced rate.

When the amount owed on the house is close to the value of the house, leading banks would expect payment of a PMI (mortgage insurance). The monthly payment the individual makes every month consists of three or more components: (i) principal and interest; (ii) real estate taxes; and (iii) mortgage insurance. The principal and interest go toward repayment of the loan at the agreed-upon interest rate. The taxes are paid to the state and local governments. They use these monies for several things, including running schools, etc. Often, politicians run and win elections and lower taxes. Some of the leaders feel that government is the problem. Others, equally popular, feel that government can do more. According to them, there is nothing wrong about America that cannot be corrected with what is right about America. The monthly mortgage payment for an individual depends on the price of the home and the individual's income. The three C's of finance are character, capacity, and collateral. The character is the individual's past credit history. Have they issued bounced checks? Have they paid all their bills and credit card payments on time? Capacity is the individual's income. Is he a physician drawing \$200,000 in annual salary? Is he a rock star? Collateral is the other assets that the individual has against which the bank can issue the loan. Rules of thumb such as no more than 28% of monthly income can

go toward mortgage payment is common in parts of Texas. Some banks in New England require the purchase price to be within two times the borrower's annual salary.

A short sale occurs when the selling price of the house is lower than the amount owed on the house. Foreclosure happens after the individual defaults on mortgage payments and the bank takes possession of the house (and usually sells it). Reverse mortgage is applicable to the elderly who have gained equity on their homes. They can take back their equity in the form of uniform monthly payments as fixed income. This is usually used for expenses incurred during retirement.

Example 5.8 Refinancing o.f Steve Gandhi

Steve Gandhi purchased a home in New England in 1980 at a mortgage rate of 21% for 30 years. The price of the home was \$150,000. In 1992 the mortgage rate fell to 6.5%. He went for refinancing. With closing costs of \$3000, by how much did the monthly payment come down for Steve after the refinancing?

(A/P,21%,30) = 0.210692 (from Appendix A)

Monthly Principal and Interest Payment at 21% mortgage rate
= \$150,000*0.210692/12 = \$2,633.65

In 1992 the balance owed by Steve = X

~~X = PW(\$2633.65,21%,18) = (P/A,21%,18)*2,633.65*12~~
~~= 4.6079*\$2,633.5*12~~ = \$145,627.2

Number of years left in the 30 year mortgage
= 30 – 12 = 18 years

(A/P,6.5%,18) = 0.092357 (from Appendix A)

Monthly Payment after Refinancing for Steve
= (145,627.2+3000)*0.092357/12 = \$1,143.90

Reduction in Monthly Payment of Principal and Interest
= \$2,633 − \$1,143.90 = \$1,489.75

Example 5.9 Toyota National Clearance

The 2010 Toyota Corolla can be purchased for $18,890. There is a rebate of $1,500 or 0% financing for 60 months. The finance rate, should the rebate be selected, is 7.0%. Which option has a lower monthly payment, the cash rebate or 0% financing for 60 months? The life of a 4-cylinder-engine car is around 105,000 miles. With an average driving of 15,000 miles per commuter, this can be taken as seven years.

Option A—0% financing for 60 months

Let the annual payment for 7 years be uniform and = X

For the first 5 years the interest rate is 0%

Hence the amount owed at end of year 5 = $18,890 – 5X

For the 6th and 7th year the interest rate = 7.0%

(A/P,7%,2) = 0.553092

X = 0.553092*($18,890 – 5X)

$$X = \frac{0.553092*\$18{,}89}{(1 + 0.553092*5)} = \$2774.67$$

Monthly Payment = X/12 = $231.22

Option B—$1500 Rebate

(A/P,7%,7) = 0.5*(0.20976 + 0.167468) = 0.188632

Monthly Payment = (A/P,7%,7)*($18,890 − $1,500)/12 = $273.36

The option with 0% finance rate for 60 months will result in a lower monthly payment compared with the $1500 cash rebate option.

~~The wealth of households has to come from savings at some point. Japan, the European Union countries, and China have higher savings rates compared to us. Wealth building can come from saving money and investing in assets such as~~

~~Stocks;~~
~~Bonds;~~

Money Market;
CDs;
Gold.

Individuals can purchase shares from companies listed in the NYSE (New York Stock Exchange), NASDAQ (National Association of Securities Dealers Automated Quotations), etc. A broker is needed. Leading brokerage firms such as Quick and Reilly allow clients to trade online and brokerage fees are charged as a percentage of the value of the trade. Often individuals are busy making money and do not have time to follow the stock markets and budget deficits of the federal government on a day-to-day basis. There are investment bankers who are willing to take care of the savings of an individual. They operate through *mutual funds*. The front-end and back-end fees of the mutual fund must be considered prior to the individual investing in mutual funds. These days, the mutual fund units are transferrable. Investment bankers let the principle of *diversification* work for their clients. There is a certain *risk* involved in buying stocks or bonds. Rather than the individual assuming the entire risk, the burden of risk is shared by several mutual fund investors. The investment banker may allocate a certain percentage of the funds to stocks, bonds, and money market accounts. He may trade in foreign markets such as Tokyo, the European Union, Bombay, etc. Examples of mutual funds are given in Table 5.4. The banker may buy more stocks and bonds when the interest rates are *low* and buy more money market funds and gold when interest rates are *high*.

The number of credit cards has increased in the last three decades. There are 576.4 million credit cards with an average balance of $15,519 active today, per a recent survey among U.S. households. There is a website, www.creditcards.com, where comparison shopping of credit cards can be done. There are different types of credit cards. These are

Low-Interest Credit Cards;
Instant Approval Credit Cards;
Balance Transfer Cards;
Cash Back Credit Cards;
Travel and Airline Credit Cards;
Business Credit Cards;

Student Credit Cards;
Prepaid and Debit Cards.

Some of these credit cards can be used at locations all over the world. Many hotels and car rental companies will not transact business with customers without a credit card. Credit cards are used as deposits at several places of business. Some of these cards, such as a Macy's Department Store card, can be used only within the retail chain. An Exxon gasoline card can be used to buy gasoline at Exxon gas stations. Most general-purpose cards

Table 5.6 Top Mutual Funds (US News & World Report)

Name of Mutual Fund	Value of Fund	Last 1-Year Return (August 2010)	Stocks, Bonds, Money Market Mix of Holdings
Invesco van Kampen Equity and Income Fund	$12.07 billion	14.8%	Stocks—68.45% Bonds—15.4%
Yachtman Fund	$2.1 billion	22.5%	Stocks—85.11% Cash—14.5%
FMI Large Cap Fund	$2.7 billion	17.0%	Stocks—88.96% Cash—11.04%
Templeton Global Bond Fund	$34.6 billion	14.0%	Bonds—80.42% Cash—16.68%
Eaton Vance Tax Managed Emerging Markets Fund	$1.8 billion	24.7%	Stocks—92.74% Bonds—0.01% Cash—0.85%
TCW Total Return Bond Fund	$5.0 billion	16.1%	Bonds—94.3% Cash—5.68%
Appleseed Fund	$139.6 million	20.3%	Stocks—71.1% Cash—14.1%
Monetta Young Investor	$6.7 million	26.3%	Stocks—85.1% Cash—14.9%
Marsico Flexible Capital Fund	$61.7 million	33.7%	Stocks—84.9% Bonds—6.9% Cash—8.2%
Boston Trust Small Cap Fund	$156.2 million	24.5%	Stocks—97.6% Cash—2.4%
Tilson Dividend Fund	$16.6 million	25.4%	Stocks—86.5% Cash—9.1%

are unsecured. The credit line is extended based on credit history. Secured cards are issued after backed up by opening a savings account. Secured cards cater to a market of individuals with challenged credit or little or no credit history. Some universities have a course on *consumer finance*. The average consumer has nine credit cards.

Credit card interest rates can change dramatically over the pay period of a purchase. It can range from 0% limited balance transfer offers to as high as 30%. The prime rate these days is low at 3.25%. The record high prime rate was 21.5% in the last quarter of 1980. The credit card companies are allowed to charge 20% above the prime rate. So should the prime rate reach 21.5% again, the credit card interest rate would be as high as 41.5%. This will increase the minimum monthly payment and the time taken to pay the credit card back. The APR set for a customer by the credit card issuing bank depends on several factors such as credit score, income, assets, current debt load, credit inquiries, payment history, and economic conditions. The lowest rates are given to customers with positive and proven credit histories.

Banks, credit unions, retailers, and credit card companies are allowed to issue credit cards. Someone who wants to issue a credit card may do so after getting the necessary approvals from state officials. The department that issues the sole proprietorship, partnership, and C corporation can permit people who want to issue credit cards. Visa and MasterCard are companies that help process payments. American Express started like Diners Club as a restaurant card. It used to be that all charges had to be paid at the end of the month when using an American Express card. Discover Card pays some cash back. A small percentage of the purchase amounts are refunded to the customer. Every credit card issued has a signed agreement that has to be followed. The contract is binding. The agreement discloses the credit limit available, APR (annual percentage rate), interest calculation method—such as by daily balance, fixed or variable APR—grace period, fees, etc.

The Credit Card Act of 2009 mandated that credit card bills contain warnings about the consequences of paying only the minimum balance. This act, signed into law by President Obama, limits when issuers of consumer credit cards can increase interest rates and bans billing and payment practices that the Federal Reserve calls "unfair or deceptive."

5.6 Summary

Brainstorming can be used to develop alternatives for a given problem. The five methods of analysis, PW, FW, AW, IRR, and ERR can be used to evaluate the alternatives. Evaluation of alternatives can be used to invest in the more profitable process, selection of the lower cost alternative, quantitate the savings accrued from implementation of a process improvement, and assessment of environmental impact of chemical processes. Alternatives can be mutually exclusive, independent projects, and screening by other criteria.

The PW (present worth) of identified alternatives can be calculated and compared with each other. Three cases are identified. When one PW is greater than zero and the second PW is less than zero, the one greater than zero can be selected over the one that is less than zero. When both PWs are greater than zero, the alternatives are rank ordered. When the PWs of both alternatives are within 20% of each other, the projects can be adjudged as too close to call within the sensitivity of the estimates of the capital and operating expenses and expected revenues.

Worked examples have been developed to select the better process to manufacture CNTs (carbon nanotubes), cost savings by using microfiltration in pretreatment during SWRO (sea water reverse osmosis), evaluation of the process to manufacture bioethanol from sugarcane bagasse, and better routes for sequestration.

When the useful lives of critical equipment are different for different alternatives, two methods for comparison analysis are developed. In the repeatability assumption the study period is selected as a lowest common multiple between the two useful lives. In the co-terminated assumption, the alternative with greater useful life is truncated at the useful life of the second alternative. Expenses and revenues are reevaluated after the truncation. For example, the salvage value can be moved up. Worked examples on comparison on fair basis were discussed on life-cycle costs of copiers with different initial fixed costs and variable costs; life-cycle costs of aviation lighting using LEDs or incandescent bulbs; and evaluation of automatic cartoning vs. manual cartoning in a pharmaceutical plant. The rate of return concept was introduced.

PW can be optimized. This is especially so when there exists a trade-off between variable and incremental capital costs. For example, when tomato paste is manufactured with more effects in a multiple effect evaporator, the energy costs decrease and the capital costs increase. The PW of the plant to manufacture tomato paste reaches a maximum value at an intermediate value of number of effects as the number of effects is increased. There exists a golden triangle of production, quality, and profit. A worked example on optimal number of effects in a multiple effect evaporator was created.

The rudiments of household finances are discussed. These include home mortgage, car ownership and financing, insurance and taxes, credit card application and credit score, and retirement planning and nest eggs. Home mortgages are loans provided for home purchase with a 30-year payback period. During that period, the lender holds the lien on the house. The payback period is nearly infinity. Auto financing is a loan given to the buyer with a six- or seven-year payback period. During that time, the title of the car belongs to the financial service provider. The nest eggs of individuals for retirement can be created with a diversified portfolio of stocks, bonds, money market accounts, and gold. There are many different kinds of credit cards.

End-of-chapter exercises based on case studies were developed. These include evaluation of manufacturing processes for ABS engineering thermoplastic; polystyrene production; snack factory; transcontinental oil pipeline; Stirling dish solar power plant; refinancing scheme; analysis for renting or buying a home; business plan for a software development service; life-cycle costs of a PEV (plug-in electric vehicle); largest solar thermal power plant; and wind energy production in Saudi Arabia. Other examples explored include printing books in hard copy form or e-book form; blow-molding press; life insurance by direct marketing vs. life insurance via the Internet; emissions reduction in an oil refinery; improvements to combined cycle power plants; four different cooling tower designs; rechargeable batteries; and heap leaching and agitation leaching processes for gold recovery.

Engineering Economics

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10 Replacement Analysis

This chapter introduces *replacement analysis*, an engineering economic analysis technique for determining when to replace assets. Definitions for terms related to replacement analysis are discussed including replacements, augmentation, retirement, challengers, defenders, physical life, accounting life, service period, sunk cost, block replacement, reduced performance, alternative requirements, and obsolescence. Methods are presented for determining when to replace equipment or assets, and part of the chapter covers analyzing mutually exclusive alternatives.

One of the challenges of operating a firm that owns equipment or other assets is to determine when to replace them with newer equipment or assets. Before the techniques for determining when to replace equipment or assets are discussed, Section 10.1 introduces replacement analysis terms.

10.1 DEFINITIONS USED IN REPLACEMENT ANALYSIS

This section provides definitions for terms used in engineering economic analysis related to the replacement of equipment and assets.

10.1.1 Replacements

The term *replacement* refers to new equipment or assets purchased and put into service in place of existing equipment or assets. The equipment or assets removed from service are used for other purposes, sold for a salvage value, or disposed of permanently.

10.1.2 Augmentation

Augmentation is the term for equipment or assets purchased and installed to increase the capacity, or to alter the capabilities of, existing equipment. In order to augment existing equipment or other assets, the existing equipment or assets are kept in service.

10.1.3 Retirement

Retirement of equipment or assets occurs when they are removed from service and either repurposed to perform other operations, left idle and only used if other similar equipment or assets are temporarily removed from service for repairs or replacement, or disposed of without a new piece of equipment or other asset being purchased to replace them.

10.1.4 Challengers

When an engineering economic analysis is being performed to evaluate the potential replacements of equipment or other assets, the label of *challenger* is applied to potential new equipment or assets when they are being considered as an alternative.

10.1.5 Defenders

In engineering economic analysis, when existing equipment or assets are being considered for replacement, they are *defenders*.

10.1.6 Economic Life

The *economic life* of equipment or an asset is the time period used for the evaluation process. The economic life is usually different from the physical life of equipment or assets since it is only used for engineering economic evaluations.

10.1.7 Physical Life

The *physical life* of equipment or assets is the amount of time that transpires from when the equipment or asset is created until it is either disposed of or repurposed and used in another application.

10.1.8 Accounting Life

In addition to the economic and physical life of equipment and assets, there is also an *accounting life*. The accounting life is based on the length of time equipment or assets are depreciated for tax purposes. Depreciation is covered in Chapter 13.

10.1.9 Ownership Life

The *ownership life* of equipment and assets includes the length of time from when the equipment or asset is purchased until it is sold.

10.1.10 Service Period

The *service period* of equipment and assets is the time the equipment or asset is available for use within a company.

10.1.11 Sunk Costs

Sunk costs, as described in Sections 2.2.4 and 10.1.11, are expenses already incurred or spent and it is not possible to retrieve them or to be reimbursed for them.

10.1.12 Block Replacements

A *block replacement* occurs when all of the same types of units of equipment are replaced at the same time regardless of whether all of the units are no longer operational.

10.1.13 Reduced Performance

Equipment or assets physically deteriorated, and the deterioration impairs the functioning of the equipment or assets, are considered to be experiencing *reduced performance.*

10.1.14 Alternative Requirements

In some cases, equipment or assets are replaced because of new requirements for items such as speed or accuracy of the equipment or assets, and these are *alternative requirements.*

10.1.15 Obsolescence

Obsolescence of equipment or assets occurs when changing technology creates new equipment or assets that perform more efficiently than existing equipment or assets. An example of obsolescence is when computers with faster processors are developed and being sold in the marketplace.

The existing computers still perform their function but the new ones with faster processors are desired to help increase productivity.

10.2 DETERMINING WHEN TO REPLACE EQUIPMENT OR ASSETS

The process for determining when to replace equipment or assets is unique to each firm. Some firms replace their equipment or assets based on obsolescence, especially if they operate in a cutting-edge profession. Other firms may only keep their equipment or assets as long as they are able to depreciate it and deduct the depreciation from their taxable income. Automobile fleets and office furniture are two examples of assets usually sold at the end of their depreciable life so replacement assets may be purchased to restart the cycle of depreciation. Some construction firms may keep their heavy construction equipment for decades, as long as it is still functioning efficiently, even though the firm is no longer able to depreciate the equipment.

The time when equipment or assets should be replaced occurs when new assets will generate a higher net present worth or equivalent uniform annual worth than the existing equipment or assets or the net present cost or equivalent uniform annual cost of the proposed replacement is less than the existing facility. Engineering economic analysis techniques are used to calculate the point in time where this occurs, which indicates the equipment or assets should be replaced with new ones. In some firms, there may not be any employees who are capable of performing engineering economic analysis and if this is the case, managers may use other criteria to determine when they will replace their equipment or assets.

In order to determine when to replace equipment or assets, net present worth or equivalent uniform annual worth analysis techniques are used to calculate the point in time where a new replacement will generate more income than the existing equipment or asset. The next section provides techniques for analyzing when to replace equipment or assets.

10.3 ANALYZING MUTUALLY EXCLUSIVE ALTERNATIVES FOR REPLACEMENT: SOLVED EXAMPLE PROBLEMS

This section provides example problems that evaluate equipment and assets to determine whether they should be replaced and when they should be replaced with new equipment or assets.

Example 10.1 is a problem addressing the replacement of an existing piece of equipment using equivalent uniform annual worth analysis to determine when the most cost effective time is to replace the equipment.

Example 10.1

A mechanical engineering firm is determining when to replace a piece of equipment bought and put into service four years ago. The original cost of the equipment was $26,000.00. The equipment has a current salvage value of $13,000.00, and the salvage value will decline each year to $10,000.00 at the end of the first year, $8,125.00 at the end of the second year, $7,000.00 at the end of the third year, and $6,250.00 at the end of the fourth year. The operating and maintenance costs are $3,000.00 the first year and increase by $1,000.00 each year until the end of the four years. The interest rate for this analysis is 10%. Out of all of the challengers, the one with the lowest equivalent uniform annual cost is $7,100.00. Calculate when the existing equipment should be replaced by the challenger using equivalent uniform annual cost analysis.

Solution

The salvage value of $13,000.00 is considered to be the first cost for the replacement system and it declines over each of the four years by the salvage values listed in the problem statement. The operating and maintenance costs are $3,000.00 per year increasing by a gradient of $1,000.00

each year. Therefore, the formula for calculating the equivalent uniform annual cost for each year is the following:

$$\begin{aligned}\text{EUAC} &= \text{salvage value}(A/P,i,n) + \text{salvage value at time}(t)(A/F,i,t) + A \\ &\quad + G(A/G,i,n) \\ &= -\$13{,}000(A/P,i,n) + \text{salvage value at time}(t)(A/F,i,t) \\ &\quad + \$3{,}000.00 + \$1{,}000.00(A/G,i,n)\end{aligned}$$

Using this formula, the equivalent uniform annual cost is calculated for years one through four and compared to the equivalent uniform annual cost of the challenger of $7,100.00 to determine which year the equipment should be replaced with the new equipment. Table 10.1 provides the values used in the formula for each year.

TABLE 10.1
Data for Replacement Analysis Calculations

Year	Salvage Value	Salvage Value at Time (t)	Annuity	Gradient
1	−$13,000.00	$10,000.00	$3,000.00	—
2	−$13,000.00	$8,125.00	$3,000.00	$1,000.00
3	−$13,000.00	$7,000.00	$3,000.00	$1,000.00
4	−$13,000.00	$6,125.00	$3,000.00	$1,000.00

Calculate the equivalent uniform annual cost for each of the four years in the analysis period.

Year 1

$$\begin{aligned}\text{EUAC}_1 &= -\$13{,}000.00(A/P,i,n) + \text{salvage value at time}(t)(A/F,i,t) \\ &\quad - \$3{,}000.00 \\ &= -\$13{,}000.00(A/P,10,1) + \$10{,}000.00(A/F,10,1) - \$3{,}000.00 \\ &= -\$13{,}000.00(1.1000) + \$10{,}000.00(1.0000) - \$3{,}000.00 \\ &= -\$14{,}300.00 + \$10{,}000.00 - \$3{,}000.00 \\ &= -\$7{,}300.00\end{aligned}$$

Year 2

$$\begin{aligned}\text{EUAC}_2 &= -\$13{,}000.00(A/P,i,n) + \text{salvage value at time}(t)(A/F,i,t) \\ &\quad - \$3{,}000.00 - \$1{,}000.00(A/G,i,n) \\ &= -\$13{,}000.00(A/P,10,2) + \$8{,}125.00(A/F,10,2) - \$3{,}000.00 \\ &\quad - \$1{,}000.00(A/G,10,2) \\ &= -\$13{,}000.00(0.57619) + \$8{,}125.00(0.47619) - \$3{,}000.00 \\ &\quad - \$1{,}000.00(0.4761) \\ &= -\$7{,}490.47 + \$3{,}869.05 - \$3{,}000.00 - \$476.10 \\ &= -\$7{,}097.52\end{aligned}$$

Year 3

$$\begin{aligned}\text{EUAC}_3 &= -\$13{,}000.00(A/P,i,n) + \text{salvage value at time}(t)(A/F,i,t)\\ &\quad -\$3{,}000.00 - \$1{,}000.00(A/G,i,n)\\ &= -\$13{,}000.00(A/P,10,3) + 7{,}000.00(A/F,10,3) - \$3{,}000.00\\ &\quad -\$1{,}000.00(A/G,10,3)\\ &= -\$13{,}000.00(0.40211) + \$7{,}000.00(0.30211) - \$3{,}000.00\\ &\quad -\$1{,}000.00(0.9365)\\ &= -\$5{,}227.43 + \$2{,}114.77 - \$3{,}000.00 - \$963.50\\ &= -\$7{,}076.16\end{aligned}$$

Year 4

$$\begin{aligned}\text{EUAC}_4 &= -\$13{,}000.00(A/P,i,n) + \text{salvage value at time}(t)(A/F,i,t)\\ &\quad -\$3{,}000.00 - \$1{,}000.00(A/G,i,n)\\ &= -\$13{,}000.00(A/P,10,4) + \$6{,}125.00(A/F,10,4) - \$3{,}000.00\\ &\quad -\$1{,}000.00(A/G,10,4)\\ &= -\$13{,}000.00(0.31547) + \$6{,}125.00(0.21547) - \$3{,}000.00\\ &\quad -\$1{,}000.00(1.3811)\\ &= -\$4{,}101.11 + \$1{,}319.75 - \$3{,}000.00 - \$1{,}381.10\\ &= -\$7{,}162.46\end{aligned}$$

Therefore, the yearly equivalent uniform annual cost is more than the challenger after the first year so the challenger should replace the existing piece of equipment after the first year.

Example 10.1 demonstrates determining when to replace an existing piece of equipment by calculating the equivalent uniform annual cost for each of the four years over which the equipment could be replaced at the end of each year. But in some circumstances, the equipment may be replaced earlier than what is indicated by the engineering economic analysis because the new equipment could be faster or more efficient.

The next example, Example 10.2, provides a problem demonstrating determining whether to replace existing vehicles with leased vehicles.

~~Example 10.2~~

In the past, an executive from a nuclear power company has purchased vehicles for all of the executives. The executive has been approached by an automobile firm about leasing new vehicles to replace the existing fleet of vehicles. Table 10.2 includes the cost and salvage values associated with the options of purchasing vehicles versus leasing new ones. Using equivalent uniform annual cost analysis, an interest rate of 12%, and a life of 10 years determine whether the firm should replace their existing fleet of vehicles with purchased or leased vehicles. There are eight vehicles in the existing fleet of vehicles. Figures 10.1 and 10.2 are the cash flow diagrams for the purchased and leased vehicles.

TABLE 10.2
Data for Purchasing Vehicles versus Leasing Vehicles

Costs and Disbursements	Purchasing Vehicles (Defender)	Leasing Vehicles (Challenger)
Initial cost	$21,000.00	—
Yearly lease cost	—	$4,500.00 per year
Annual operating cost	$6,000.00	$7,000.00
Salvage value	$4,000.00	—

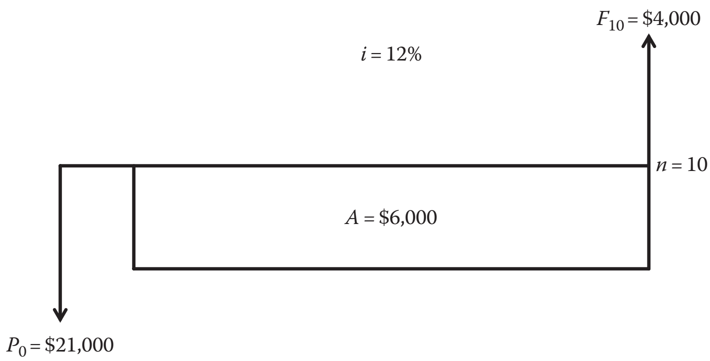


FIGURE 10.1 Cash flow diagram for purchasing vehicles for Example 10.2.

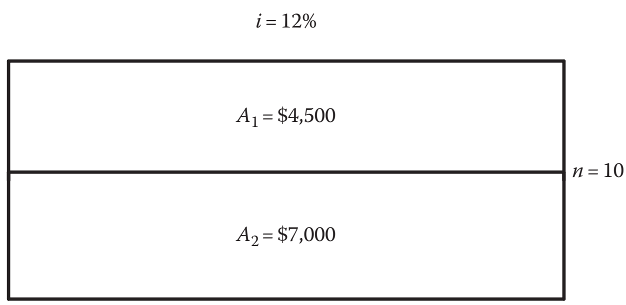


FIGURE 10.2 Cash flow diagram for leasing vehicles for Example 10.2.

Solution

Purchasing vehicles (defender)

$$
\begin{aligned}
\text{EUAC}_D &= P_0\left(A/P,i,n\right)+F_{10}\left(A/F,i,n\right)+A \\
&= -\$21{,}000.00\left(A/P,12,10\right)+\$4{,}000.00\left(A/F,12,10\right)-\$6{,}000.00 \\
&= -\$21{,}000.00\left(0.17698\right)+\$4{,}000.00\left(0.05698\right)-\$6{,}000.00 \\
&= -\$3{,}716.58+\$227.92-\$6{,}000.00 \\
&= -\$9{,}488.66 \text{ per vehicle}
\end{aligned}
$$

$$
\text{Total} = \frac{-\$9{,}488.66}{\text{Vehicle}} \times 8 \text{ vehicles} = -\$75{,}909.28
$$

Leasing vehicles (challenger)

$$\text{EUAC}_C = -\$4,500.00 - \$7,000.00 = -\$11,400.00 \text{ per vehicle}$$

$$\text{Total} = \frac{\$11,400.00}{\text{Vehicle}} \times 8 \text{ vehicles} = -\$92,000.00$$

Therefore, the firm should continue to purchase vehicles rather than switching to leasing vehicles since the equivalent uniform annual cost for purcasing the vehicles is less than the equivalent uniform annual cost for leasing the vehicles −\$75,909.28 < −\$92,000.00.

Example 10.3 is a problem using replacement analysis techniques to determine whether to replace an existing pipeline.

Example 10.3

An oil pipeline in East Texas has been in service for five years. A petroleum engineer working for the oil company is evaluating whether to replace the pipeline with a new one. The data for the existing pipeline and the potential new pipeline are listed in Table 10.3. Using an interest rate of 6%, and net present worth analysis and then equivalent uniform annual worth analysis, the petroleum engineer determines whether the existing pipeline should be replaced with the proposed new pipeline. Figures 10.3 and 10.4 are the cash flow diagrams for the existing pipeline and the proposed new pipeline.

TABLE 10.3
Data for Pipeline Alternatives

Costs	Existing Pipeline	Proposed Pipeline
Initial cost	—	~~\$300,000.00~~
Salvage value at time zero	\$1,000,000.00 ~~increases each replacement every 10 years by \$500,000.00 per year~~	—
Yearly maintenance	\$100,000.00 per year increasing by \$50,000.00 per year for 10 years	\$100,000.00 per year increasing by \$10,000.00 per year
Life in years	10	40

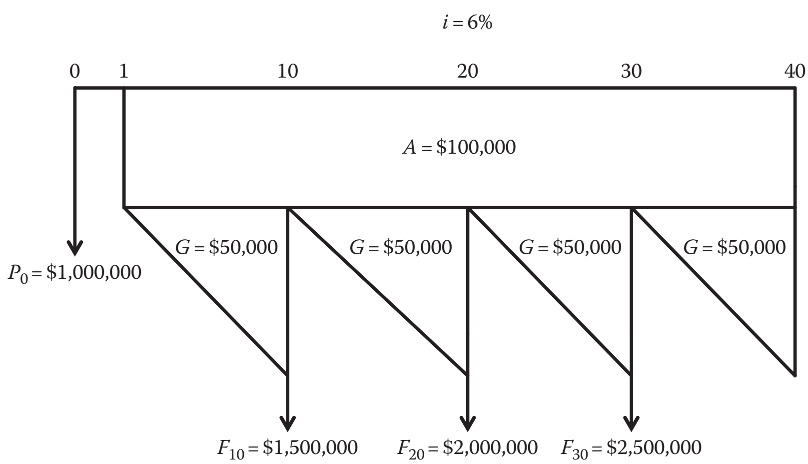


FIGURE 10.3 Cash flow diagram for the existing pipeline in Example 10.3.

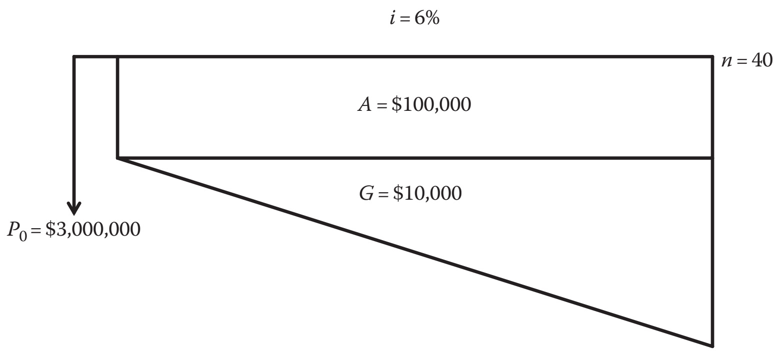


FIGURE 10.4 Cash flow diagram the proposed new pipeline in Example 10.3.

Solution

Existing pipeline

The salvage value for the existing pipeline is considered to be a cost of keeping the pipeline in service when analyzing the existing pipeline.

Calculate the net present worth of the existing pipeline and then convert the net present worth into an equivalent uniform annual cost:

$$
\begin{aligned}
\text{NPW}_D &= P_0 + F_{10}\left(P/F,i,n\right) + F_{20}\left(P/F,i,n\right) + F_{30}\left(P/F,i,n\right) + A\left(P/A,i,n\right)\\
&\quad + G\left(A/G,i,n\right)\left(P/A,i,n\right)\\
&= -\$1{,}000{,}000.00 - \$1{,}500{,}000.00\left(P/F,6,10\right)\\
&\quad - \$2{,}000{,}000.00\left(P/F,6,20\right) - \$2{,}500{,}000.0\cancel{0}\\
&\quad - \$100{,}000.00\left(P/A,i,40\right) - \$50{,}000.00\left(A/G,6,10\right)\left(P/A,i,40\right)\\
&= -\$1{,}000{,}000.00 - \$1{,}500{,}000.00\left(0.55839\right)\\
&\quad - \$2{,}000{,}000.00\left(0.31180\right) - \$2{,}500{,}000.00\left(0.17411\right)\\
&\quad - \$100{,}000.00\left(15.046\right) - \$50{,}000.00\left(4.0220\right)\left(15.046\right)\\
&= -\$1{,}000{,}000.00 - \$837{,}585.00 - \$623{,}600.00 - \$435{,}275.00\\
&\quad - \$1{,}504{,}600.00 - \$3{,}025{,}750.60\\
&= -\$7{,}426{,}810.60
\end{aligned}
$$

$$
\begin{aligned}
\text{EUAC}_D &= -\$7{,}426{,}810.60\left(A/P,6,40\right) = -\$7{,}426{,}810.60\left(0.06646\right)\\
&= -\$493{,}585.83
\end{aligned}
$$

Proposed pipeline

Calculate the net present worth of the proposed pipeline and then convert the net present worth into an equivalent uniform annual worth:

$$
\begin{aligned}
\text{NPW}_C &= P_0 + A\left(P/A,i,n\right) + G\left(P/G,i,n\right)\\
&= -\$3{,}000{,}000.00 - \$100{,}000.00\left(P/A,6,40\right) - \$10{,}000.00\left(P/G,6,40\right)\\
&= -\$3{,}000{,}000.00 - \$100{,}000.00\left(15.046\right) - \$10{,}000.00\left(185.95\right)\\
&= -\$3{,}000{,}000.00 - \$1{,}504{,}600.00 - \$1{,}859{,}500.00\\
&= -\$6{,}364{,}100.00
\end{aligned}
$$

$$\text{EUAC}_D = -\$6,364,100.00(A/P,6,40) = -\$6,364,100.00(0.06646)$$
$$= -\$422,958.09$$

Therefore, the new pipeline should be installed since it has a lower equivalent uniform annual cost than the existing pipeline −\$422,958.09 < −\$493,585.83.

Example 10.4 provides another problem where equivalent uniform annual cost analysis techniques are used to compare two facilities to determine whether to replace the existing facility with a proposed new facility.

Example 10.4

An engineering office building has a current value of \$3,000,000.00. The building requires \$200,000.00 a year to operate and maintain the building. If the office building is retained, it will have an estimated resale value of \$15,000,000.00 in 40 years. Construction of a new building is being investigated to replace the existing building. The new building under consideration would cost \$4,000,000.00 and have maintenance and operating costs of \$100,000.00 per year. The estimated resale value of the proposed building would be \$10,000,000.00 in 40 years. The interest rate for financing the new building is 6%. Determine whether the existing building should be retained or whether it should be replaced by the proposed building using equivalent uniform annual cost analysis. Figures 10.5 and 10.6 are the cash flow diagrams for the existing and proposed buildings.

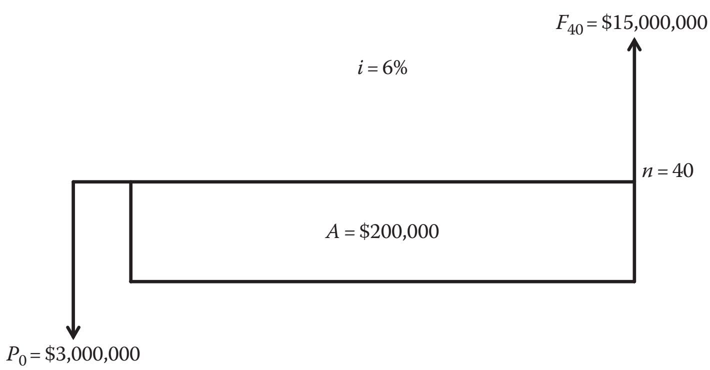


FIGURE 10.5 Cash flow diagram for the existing building in Example 10.4.

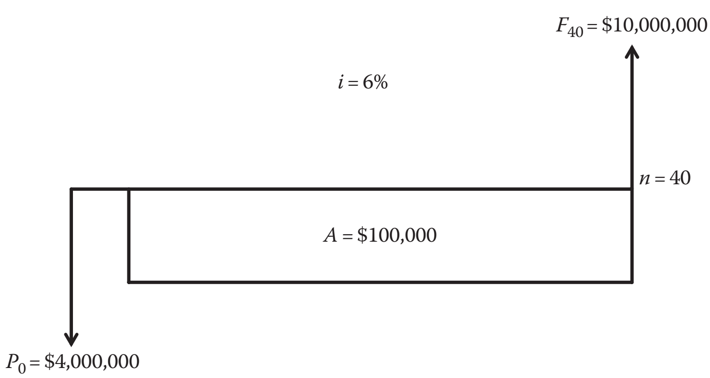


FIGURE 10.6 Cash flow diagram for the proposed building in Example 10.4.

Solution

Existing building

$$\begin{aligned}
\text{EUAC}_D &= \left[P_0 + A\left(P/A,i,n\right) + F_{40}\left(P/F,i,n\right)\right]\left(A/P,i,n\right)\\
&= \big[-\$3{,}000{,}000.00 - \$200{,}000.00\left(P/A,6,40\right)\\
&\quad + \$15{,}000{,}000.00\left(P/F,6,40\right)\big]\left(A/P,6,40\right)\\
&= \big[-\$3{,}000{,}000.00 - \$200{,}000.00\left(15.046\right)\\
&\quad + \$15{,}000{,}000.00(0.09722)\big]\left(0.06646\right)\\
&= \left[-\$3{,}000{,}000.00 - \$3{,}009{,}200.00 + \$1{,}458{,}300.00\right]\left(0.06646\right)\\
&= -\$4{,}550{,}900.00\left(0.06646\right)\\
&= -\$302{,}452.81
\end{aligned}$$

Proposed building

$$\begin{aligned}
\text{EUAC}_C &= \left[P_0 + A\left(P/A,i,n\right) + F_{40}\left(P/F,i,n\right)\right]\left(A/P,i,n\right)\\
&= \big[-\$4{,}000{,}000.00 - \$100{,}000.00\left(P/A,6,40\right)\\
&\quad + \$10{,}000{,}000.00\left(P/F,6,40\right)\big]\left(A/P,6,40\right)\\
&= \big[-\$4{,}000{,}000.00 - \$100{,}000.00\left(15.046\right)\\
&\quad + \$10{,}000{,}000.00(0.09722)\big]\left(0.06646\right)\\
&= \left[-\$4{,}000{,}000.00 - \$1{,}504{,}600.00 + \$972{,}200.00\right]\left(0.06646\right)\\
&= -\$4{,}532{,}400.00\left(0.06646\right)\\
&= -\$301{,}223.30
\end{aligned}$$

The new building has a slightly lower equivalent uniform annual cost than the existing building −\$301,223.30 < −\$302,452.81; therefore, the new building should be selected by the firm.

10.4 SUMMARY

This chapter provided an introduction to replacement analysis and the methods for determining whether to replace an existing facility, equipment, or asset with a proposed new facility, equipment, or asset. This chapter included definitions for terms used in conjunction with replacement analysis including replacements, augmentation, retirement, challengers, defenders, physical life, accounting life, service period, sunk cost, block replacement, reduced performance, alternate requirements, and obsolescence. Methods for determining when to replace a facility, equipment, or asset were included and the last part of this chapter provided example problems demonstrating the analysis process for mutually exclusive alternatives to determine when to replace an existing facility, equipment, or asset with a new asset.

12 Benefit/Cost Ratio Economic Evaluations

Another engineering economic technique for evaluating potential projects is *benefit/cost ratio economic analysis* also referred to as *discounted profitability index, profitability index, profit investment ratio,* or *value investment ratio.* Benefit/cost ratio economic analysis is used in conjunction with present worth and equivalent uniform annual worth analysis techniques since the costs and benefits for each alternative need to be in equivalent terms before they are compared to determine which alternative has the highest benefit/cost ratio.

This chapter defines the terms used in benefit/cost ratio economic evaluations and explains the procedures for conducting benefit/cost ratio economic evaluations by including specific steps for performing a benefit/cost ratio economic evaluation. Example problems are provided to help illustrate the process for calculating benefit/cost ratios.

12.1 DEFINITIONS AND TERMS USED IN BENEFIT/COST RATIO ECONOMIC EVALUATIONS

This section defines the terms used in benefit/cost ratio economic evaluations and explains how they are integrated into the formulas for calculating *benefit/cost ratios*. There are two types of costs to owners associated with projects: (1) capital costs and (2) maintenance costs. Sections 12.1.1 and 12.1.2 explain these two types of costs in relation to benefit/cost ratio economic evaluations, and Sections 12.1.3 and 12.1.4 cover the benefits associated with benefit/cost ratio economic evaluations.

12.1.1 Benefit/Cost Ratio Costs

In the formulas for calculating benefit/cost ratios, there are three different terms representing different types of costs and they are the following:

1. C_f—Equivalent capital cost (construction, acquisition, or other costs) of a proposed future facility provided as an equivalent uniform annual cost
2. C_p—Equivalent capital worth of an existing facility provided as an equivalent uniform annual worth (This is also the current salvage value of an existing facility.)
3. $C_n = C_f - C_p$—Net capital cost required if a new facility replaces an existing facility

12.1.2 Benefit/Cost Ratio Maintenance Costs

There are three types of maintenance costs in benefit/cost ratios and they are the following:

1. M_f—Equivalent annual operating and maintenance costs for a proposed future facility
2. M_p—Equivalent annual operating and maintenance costs of an existing facility
3. $M_n = M_f - M_p$—Net operating and maintenance costs of a proposed (future) facility minus the existing (present) facility operating and maintenance costs

12.1.3 Benefit/Cost Ratio Benefits

The following are the types of benefits included in benefit/cost ratios:

1. B_n or U_n—Net annual benefits or savings in cost occurring due to improvements in safety procedures and decreased expenses
2. $U_n = U_f - U_p$—Where U_p is the user benefits of a present facility and U_f is the user benefits of the future facility. The net user benefit U_n is the annual equivalent benefit to an owner.
3. $U_n = U_p - U_f$—Where the benefits are derived from a reduction in the yearly cost U_p to the owner through a new facility U_f

12.1.4 Benefits and Disbursements

When evaluating projects using benefit/cost ratios, one of the first steps is to determine what are the benefits and the disbenefits as defined by the following:

1. *Benefits* are advantages accruing to an owner. For public projects, the owner is considered to be the public. Benefits on public projects include cost savings, less wear and tear on vehicles, lower fuel consumption, safer roadways, time reductions, and so forth.
2. *Disbenefits* are disadvantages to the owner and they are subtracted from the benefits, not added to the costs.

12.2 BENEFIT/COST RATIO ECONOMIC ANALYSIS

Benefit/cost ratio economic analysis techniques are mainly for evaluating public sector projects, but they are also sometimes used by private owners and investors when they are considering investments. Benefit/cost ratios provide a comparison of the benefits and costs associated with a project and they are used to determine whether the benefits will be greater than the costs, which would result in a benefit/cost ratio greater than one. Since public projects are not built to generate profits, the benefits are usually in the form of reduced costs to users of facilities. One example is providing a road that reduces the distance drivers have to travel to a particular destination. Other examples of public projects providing benefits to citizens are bridges, public buildings, tunnels, upgrading technologies such as fiber optic cables, and water treatment facilities.

If only one alternative is being evaluated using benefit/cost ratio analysis techniques, the alternative is compared to the do nothing alternative. When there are two alternatives, they are compared to each other. Before benefit/cost ratio comparisons are performed, all of the costs and benefits need to be converted to either a present worth or an equivalent uniform annual worth so all of the values being compared are being analyzed based on equivalent terms. The basic benefit/cost ratio formula is Equation 12.1.

$$B/C = \frac{\text{Benefits} - \text{Disbenefits}}{\text{Costs}} \tag{12.1}$$

Equation 12.1 is used when the B/C ratio of one facility is being calculated to determine if the benefits are greater than the costs, as shown in Equations 12.2 and 12.3.

$$B/C \geq 1.0 \text{ project is justified} \tag{12.2}$$

$$B/C \leq 1.0 \text{ project is not justified} \tag{12.3}$$

In addition to the basic B/C ratio, there are two other B/C ratios used when comparing alternatives and they are the *conventional B/C* and *modified B/C*.

12.2.1 Conventional Benefit/Cost Ratio

The conventional *B/C* ratio is Equation 12.4.

$$\text{Conventional } B/C = \frac{\text{Net savings to users}}{\text{Owner's net capital cost} + \text{Owner's net operating and maintenance costs}} \tag{12.4}$$

Equation 12.5 is the formula for the conventional *B/C* ratio written using the terms introduced in Section 12.1.

$$\text{Conventional } B/C = \frac{U_n}{C_n + M_n} \quad \text{or} \quad = \frac{B_n}{C_n + M_n} = \frac{(U_f - U_p)}{(C_f - C_p) + (M_f - M_p)} \tag{12.5}$$

12.2.2 Modified Benefit/Cost Ratio

The modified *B/C* uses the same data as the conventional *B/C*, but the net operating and maintenance costs M_n are considered to be disbenefits and not costs, as they are in the conventional *B/C* ratio. The modified *B/C* ratio is Equation 12.6.

$$\text{Modified } B/C = \frac{U_n - M_n}{C_n} \quad \text{or} \quad = \frac{B_n - M_n}{C_n} = \frac{(U_f - U_p) - (M_f - M_p)}{C_f - C_p} \tag{12.6}$$

12.2.3 Incremental Benefit/Cost Ratio

If there are two or more alternatives being evaluated, then the incremental $\Delta B/\Delta C$ ratio is calculated using Equation 12.7 and the difference between the two alternatives to determine whether the $\Delta B/\Delta C \geq 1.0$:

$$\Delta B/\Delta C = \frac{B_f - B_p}{C_f - C_p} \tag{12.7}$$

If the $\Delta B/\Delta C \geq 1.0$, then the higher initial cost alternative is selected as the most beneficial alternative. If the $\Delta B/\Delta C \leq 1.0$, then the lower initial cost alternative is selected rather than the higher initial cost alternative because the incremental increase in the cost of the higher initial cost alternative is not justified in economic terms.

12.3.4 Steps for Performing Benefit/Cost Ratio Economic Analysis

The following steps outline the process for performing a benefit/cost ratio economic analysis:

1. Determine which of the elements being analyzed are benefits:
   a. Benefits are advantageous elements expressed in dollars accruing to the owner (public).
   b. Disadvantages are disbenefits and they are not included with the costs but are subtracted from the benefits.
2. Determine which of the elements being analyzed are costs:
   a. Costs include items such as construction costs and operating and maintenance expenses.
   b. If a government project is being evaluated, the expenses are incurred by the appropriate government agency.

3. Before calculating the benefit/cost ratio, convert all dollar values to equivalent amounts:
   a. Using the present worth formula or
   b. Using the equivalent uniform annual worth formula
4. If only one proposal is being considered, compare it to the do nothing alternative. If there are two or more alternatives being considered, compare them in order of the lowest to highest initial cost:
   a. If the $\Delta B/\Delta C \geq 1.0$, the extra benefits of the higher initial cost alternative justify the selection of the higher initial cost alternative.
   b. If the $\Delta B/\Delta C \leq 1.0$, the extra benefits of the higher initial cost alternative do not justify the selection of the higher initial cost alternative.
5. The capitalized cost of an existing facility is its salvage value for comparison purposes:
   a. The salvage value is the amount a firm would receive if the facility were purchased or demolished and its contents and materials are sold.
   b. Do not subtract the salvage value from the proposed facility since it is a cost of the existing facility because it is the amount not realized if the facility is retained and continues in service.
6. Benefit/cost ratios will be in one of the two forms:
   a. $B/C = \dfrac{\text{Present worth of benefits}}{\text{Present worth of costs}}$
   b. $B/C = \dfrac{\text{Equivalent uniform annual benefit (EUAB)}}{\text{Equivalent uniform annual cost (EUAC)}}$

12.3 SOLVED EXAMPLE PROBLEMS

This section provides several example problems that demonstrate calculating benefit/cost ratios.

Example 12.1 compares two alternatives using the basic B/C ratio.

Example 12.1

A systems engineer is considering the purchase of a new device that will save his firm money on an annual basis by speeding up the production process. There are two devices being considered and both would save the firm money. Both devices cost \$100,000.00, but the first one will save the firm \$30,000.00 per year for five years and the second one will save \$40,000.00 in the first year and the savings will decline by \$5,000.00 each year until year five. If the firm uses an interest rate of 7%, determine which device should be purchased by calculating the benefit/cost ratio of each alternative using present worth analysis techniques. Figures 12.1 and 12.2 are the cash flow diagrams for the two devices.

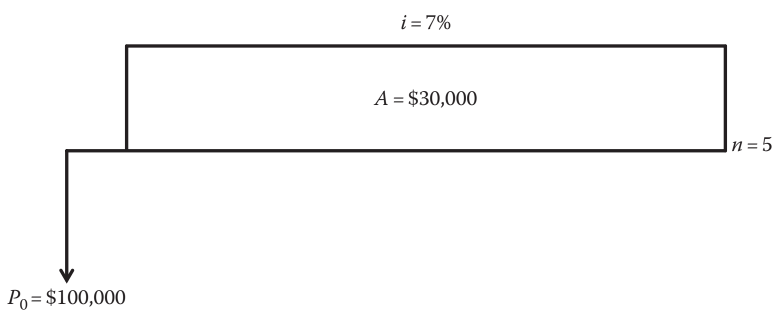


FIGURE 12.1 Cash flow diagram for the systems engineering device 1 in Example 12.1.

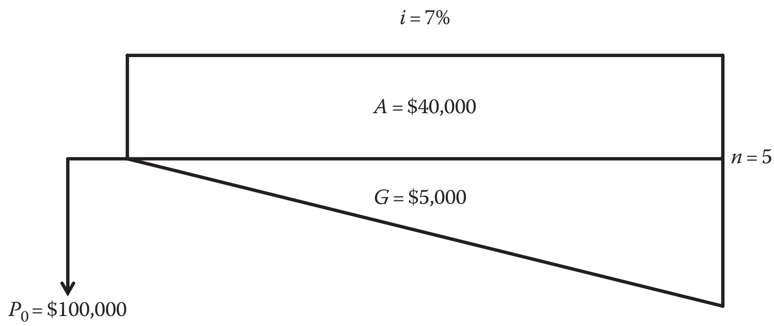


FIGURE 12.2 Cash flow diagram for the systems engineering device 2 in Example 12.1.

Solution

$$\text{Present worth of costs for device 1} = \$100,000.00$$

$$\begin{aligned}\text{Present worth of benefits for device 1} &= \$30,000.00(P/A,7,5)\\ &= \$30,000.00(4.1002)\\ &= \$123,006.00\end{aligned}$$

$$B/C = \frac{\$123,006.00}{\$100,000.00} = 1.23$$

$$\text{Present worth of costs for device 2} = \$100,000.00$$

$$\begin{aligned}\text{Present worth of benefits for device 2} &= \$40,000.00(P/A,7,5)\\ &\quad - \$5,000.00(P/G,7,5)\\ &= \$40,000.00(4.1002)\\ &\quad - \$5,000.00(7.6466)\\ &= \$164,008.00 - \$38,233.00\\ &= \$125,775.00\end{aligned}$$

$$B/C = \frac{\$125,775.00}{\$100,000.00} = 1.26$$

Therefore, since device 2 has a higher B/C ratio than device 1 select device 2, $1.26 > 1.23$

Note*:* Since both of the devices under consideration have the same initial cost, it is not possible to compare them using the $\Delta B/\Delta C$ ratio.

Example 12.2 demonstrates calculating the $\Delta B/\Delta C$ ratio for two alternatives.

Example 12.2

Two metal punching machines are being evaluated for purchase by an industrial engineer. The first punching machine has an initial cost of $200,000.00 and a salvage value at the end of six years of $50,000.00. This machine would provide the firm with an annual benefit of $95,000.00. The second machine would cost $700,000.00 and have a salvage value of $150,000.00 at the end of 12 years. The second machine provides an annual benefit of $120,000.00. The interest rate is 10%.

Calculate the benefit/cost ratio for each individual punching machine, and then using incremental benefit/cost ratio analysis techniques and equivalent uniform annual cost analysis, determine which punching machine should be selected by the firm. Figures 12.3 and 12.4 are the cash flow diagrams for the punching machines.

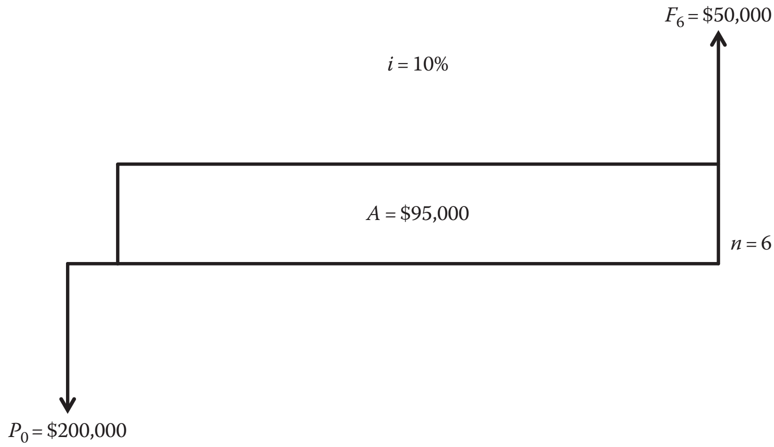


FIGURE 12.3 Cash flow diagram for punching machine 1 in Example 12.2.

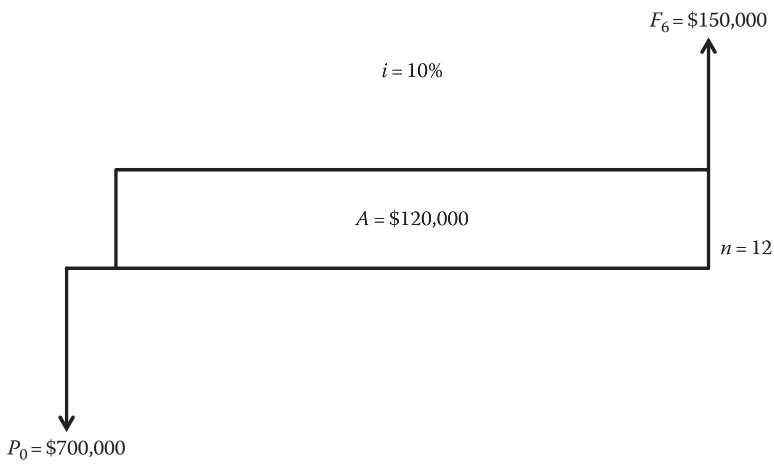


FIGURE 12.4 Cash flow diagram for punching machine 2 in Example 12.2.

Solution

The salvage value is considered a reduction in costs not a benefit.

Punching machine 1

$$\cancel{\text{EUAC}}_1 = P_0(A/P,10,6) - F_6(A/F,10,6)$$
$$= -\$200,000.00(A/P,10,6) - \$50,000.00(A/F,10,6)$$
$$= -\$200,000.00(0.22961) - \$50,000.00(0.12961)$$
$$= -\$45,922.00 - \$6,480.50$$
$$= -\$52,402.50$$
$$\text{EUAB}_1 = \$95,000.00$$
$$B/C = \frac{\$95,000.00}{\$52,402.50} = 1.81$$

Punching machine 2

$$\text{EUAC}_{\cancel{2}} = P_0(A/P,10,6) - F_{12}(A/F,10,6)$$
$$= -\$700,000.00(A/P,10,12) - \$150,000.00(A/F,10,12)$$
$$= -\$700,000.00(0.14676) - \$150,000.00(0.04676)$$
$$= -\$102,732.00 - \$7,014.00$$
$$= -\$109,746.00.00$$
$$\text{EUAB}_2 = \$120,000.00$$
$$B/C = \frac{\$120,000.00}{\$109,746.00} = 1.09$$

Calculate the incremental benefit/cost ratio for the difference between the two metal punching machines

$$\Delta B/\Delta C = \frac{\text{benefits of machine 2} - \text{benefits of machine 1}}{\text{cost of machine 2} - \text{cost of machine 1}} = \frac{\$120,000.00 - \$95,000.00}{\$109,746.00 - \$52,402.50}$$
$$= \frac{\$25,000.00}{\$57,343.50} = 0.44$$

Therefore, since the $\Delta B/\Delta C$ of $0.44 \le 1.0$ choose punching machine 1

Example 12.3 uses benefit/cost ratios and incremental benefit/cost ratios to compare three alternatives.

Example 12.3

A county is considering three locations for a new dam. The three alternatives for the dams for the three locations cost \$25,000,000.00, \$30,000,000.00, and \$32,000,000.00. Currently flood damage amounts to \$20,000,000.00 per year. If the new dams are built, the flood damage will be reduced to \$17,000,000.00; \$16,000,000.00; and \$15,500,000.00 per year, respectively. Determine which dam should be built based on benefit/cost ratio analysis by determining the individual benefit/cost ratios using an interest rate of 5% and a life of 10 years and then calculate the incremental benefit/cost ratios.

Solution

First, calculate the equivalent uniform annual cost of the initial cost for each of the three dam alternatives:

$$\text{EUAC}_{1\,of\ initial\ cost} = \$25,000,000.00(A/P,5,10) = \$25,000,000.00(0.12950)$$
$$= \$3,327,500.00$$

$$\text{EUAC}_{2\,of\ initial\ cost} = \$30,000,000.00(A/P,5,10) = \$30,000,000.00(0.12950)$$
$$= \$3,885,000.00$$

$$\text{EUAC}_{3\,of\ initial\ cost} = \$32,000,000.00(A/P,5,10) = \$32,000,000.00(0.12950)$$
$$= \$4,144,000.00$$

Second, calculate the benefit for each alternative:

$$\text{Benefit}_1 = \$20,000,000.00 - \$17,000,000.00 = \$3,000,000.00$$

$$\text{Benefit}_2 = \$20,000,000.00 - \$16,000,000.00 = \$4,000,000.00$$

$$\text{Benefit}_3 = \$20,000,000.00 - \$15,500,000.00 = \$4,500,000.00$$

Third, calculate the benefit/cost ratios for each alternative:

$$B/C_1 = \frac{\$3,000,000.00}{\$3,237,500.00} = 0.93$$

$$B/C_2 = \frac{\$4,000,000.00}{\$3,885,000.00} = 1.03$$

$$B/C_3 = \frac{\$4,500,000.00}{\$4,144,000.00} = 1.09$$

Finally, calculate the incremental benefit/cost ratios:

$$\Delta B/\Delta C_{2-1} = \frac{\$4,000,000.00 - \$3,000,000.00}{\$3,885,000.00 - \$3,237,500.00} = \frac{\$1,000,000.00}{\$647,500.00} = 1.54$$

Since the $\Delta B/\Delta C_{2-1} \geq 1.0$ retain alternative 2 and compare it to alternative 3

$$\Delta B/\Delta C_{3-2} = \frac{\$4,500,000.00 - \$4,000,000.00}{\$4,144,000.00 - \$3,885,000.00} = \frac{\$500,000.00}{\$259,000.00} = 1.93$$

Therefore, since the $\Delta B/\Delta C_{3-2} \geq 1.0$ select alternative 3.

Example 12.4 uses the conventional benefit/cost ratio to determine which of two alternatives should be built.

Example 12.4

For Example 12.3, if the maintenance costs for alternative 2 are $200,000.00 per year and $250,000.00 per year for alternative 3, calculate the conventional benefit/cost ratio for the difference between alternatives 3 and 2.

Solution

$$\text{Conventional } \Delta B/\Delta C_{3-2} = \frac{(U_f - U_p)}{(C_f - C_p) + (M_f - M_p)}$$

$$= \frac{(\$4{,}500{,}000.00 - \$4{,}000{,}000.00)}{(\$4{,}144{,}000.00 - \$3{,}885{,}000.00) + (\$250{,}000.00 - \$200{,}000.00)}$$

$$= \frac{\$500{,}000.00}{\$259{,}000.00 + \$50{,}000.00}$$

$$= \frac{\$500{,}000.00}{\$309{,}000.00} = 1.62$$

Therefore, since the $\Delta B/\Delta C_{3-2} \geq 1.0$ select alternative 3.

Example 12.5 demonstrates the process for using the modified benefit/cost ratio to analyze two alternatives.

Example 12.5

Calculate the modified benefit/cost ratio of alternative 3 compared to alternative 2 using the data from Examples 12.3 and 12.4.

Solution

$$\text{Modified } \Delta B/\Delta C = \frac{U_n - M_n}{C_n} \quad \text{or} \quad = \frac{B_n - M_n}{C_n} = \frac{(U_f - U_p) - (M_f - M_p)}{C_f - C_p}$$

$$= \frac{(\$4{,}500{,}000.00 - \$4{,}000{,}000.00) - (\$250{,}000.00 - \$200{,}000.00)}{\$4{,}144{,}000.00 - \$3{,}885{,}000.00}$$

$$= \frac{\$500{,}000.00 - \$50{,}000.00}{\$259{,}000.00}$$

$$= \frac{\$450{,}000.00}{\$259{,}000.00} = 1.74$$

Therefore, since the $\Delta B/\Delta C_{3-2} \geq 1.0$ select alternative 3.

Example 12.6

A city is planning to build a public swimming pool costing $3,000,000.00 that will have a life of 25 years. The annual maintenance cost for the swimming pool will be $1,000.00 per year increasing by $200.00 per year. There will be no salvage value. An average benefit is assigned of $1.50 per use and there will be an average of 75 users per hour, 8 hours per day, 316 days per year. Calculate the modified benefit/cost ratio using a rate of return of 6%.

Solution

First, calculate the total yearly benefit:

$$\text{Yearly benefit} = \frac{\$1.50}{\text{Use}} \times \frac{75 \text{ users}}{\text{Hour}} \times \frac{8 \text{ hours}}{\text{Day}} \times \frac{316 \text{ days}}{\text{Year}} = \$284,400.00$$

Second, calculate the equivalent uniform annual cost of the initial cost of the swimming pool:

$$\text{EUAC} = \$3,000,000.00(A/P,6,25) = \$3,000,000.00(0.07823) = \$224,690.00$$

Third, calculate the equivalent uniform annual cost of the gradient:

$$\text{EUAC}_G = \$200.00(A/G,6,25) = \$200.00(9.0722) = \$1,814.44$$

Fourth, add the equivalent uniform annual cost of the gradient to the yearly uniform series maintenance cost:

$$\text{EUAC}_M = \$1,000.00 + \$1,814.44 = \$2,814.44$$

Finally, calculate the modified benefit/cost ratio:

$$\text{Modified } B/C = \frac{U_n - M_n}{C_n} = \frac{\$284,400.00 - \$2,814.44}{\$224,690.00} = \frac{\$281,585.56}{\$224,690.00} = 1.25$$

Case Study 12.1 demonstrates calculating conventional and modified *B*/*C* ratios.

~~Case Study 12.1 Conventional and Modified *B*/*C* Ratios~~

A city is trying to determine whether it should install a new traffic signal at a major intersection. Table 12.1 provides the data collected on the costs and benefits of the existing traffic signal and the proposed traffic signal. Using an interest rate of 6% and a life of five years for both traffic signals, determine whether the proposed traffic signal should be installed by the city using both the conventional and modified incremental benefit/cost ratios.

TABLE 12.1
Data for Traffic Signal Alternatives

Costs and Benefits	Existing Traffic Signal	Proposed Traffic Signal
Initial cost	—	$68,670.00
Maintenance costs	$3,000.00 per month	$60,000 per year
User costs for accidents	$95,280.00 per year	$3,000.00 per year
Acceleration and deceleration costs (average cost to stop)	$0.025	$0.025
Number of vehicles	80,000 per day	40,000 per day

SOLUTION

$$\text{Cost to stop for existing signal} = \frac{\$0.025}{\text{Vehicle}} \times \frac{80,000 \text{ vehicles}}{\text{Day}} \times \frac{365 \text{ days}}{\text{Year}}$$

$$= \$730,000 \text{ per year}$$

$$\text{Cost to stop for proposed signal} = \frac{\$0.025}{\text{Vehicle}} \times \frac{40{,}000 \text{ vehicles}}{\text{Day}} \times \frac{365 \text{ days}}{\text{Year}}$$

$$= \$365{,}000 \text{ per year}$$

$$\text{EUAC}_E \text{ of existing signal initial cost} = 0$$

$$\text{EUAC}_P \text{ of proposed signal initial cost} = \$68{,}670.00(A/P, 6, 5)$$

$$= \$68{,}670.00(0.23740)$$

$$= \$16{,}302.26$$

$$\text{Total benefits for existing signal} = \$730{,}000.00 + \$95{,}280.00 = \$825{,}280.00$$

$$\text{Total benefits for proposed signal} = \$365{,}000.00 + \$3{,}000.00 = \$368{,}000.00$$

$$\text{Total mainteance cost for existing signal} = \frac{\$3{,}000.00}{\text{Month}} \times \frac{12 \text{ months}}{\text{Year}} = \$36{,}000.00$$

$$\text{Total mainteance cost for proposed signal} = \$60{,}000.00$$

$$\text{Conventional } \Delta B/\Delta C = \frac{U_n}{C_n + M_n} = \frac{U_f - U_p}{(C_f - C_p) + (M_f - M_p)}$$

$$= \frac{\$825{,}280.00 - \$368{,}000.00}{(\$16{,}302.26 - 0) + (\$60{,}000.00 - \$36{,}000.00)}$$

$$= \frac{\$457{,}280.00}{\$16{,}302.26 + \$24{,}000.00} = \frac{\$457{,}280.00}{\$40{,}302.26} = 11.35$$

$$\text{Modified } \Delta B/\Delta C = \frac{U_n - M_n}{C_n} = \frac{(U_f - U_p) - (M_f - M_p)}{C_f - C_p}$$

$$= \frac{(\$825{,}280.00 - \$368{,}000.00) - (\$60{,}000.00 - \$36{,}000.00)}{(\$16{,}302.46.00 - 0)}$$

$$= \frac{\$427{,}280.00 - \$24{,}000.00}{\$16{,}302.26} = \frac{\$403{,}280.00}{\$16{,}302.26} = 24.74$$

Therefore, since the $\Delta B/\Delta C \geq 1.0$ for both methods the new traffic signal should be installed by the city.

12.4 SUMMARY

This chapter defined the terms related to benefit/cost ratio economic evaluations and explained the procedures for calculating benefit/cost ratios. It also described specific steps for performing a benefit/cost ratio economic evaluation. Example problems and a case study were included to illustrate the procedures for calculating benefit/cost ratios.

13 Depreciation

This chapter introduces depreciation and the formulas for calculating the depreciation that is deducted from taxable income when calculating U.S. federal income taxes. This chapter provides definitions for depreciation and other related terms, discusses the components considered when calculating deprecation, and covers the methods for calculating the four most prevalent types of depreciation—production, straight line, declining balance, and sum-of-the-years digits.

13.1 DEFINITIONS FOR DEPRECIATION TERMS

Depreciation only exists as a means of reducing the income taxes businesses are obligated to pay to the U.S. federal and state governments since income taxes are paid on the net income of a firm minus business expenses and depreciation. Individuals owning rental property are able to depreciate it and deduct the yearly depreciation from the income earned on the rental property or other income since rental property is considered to be a business asset by the Internal Revenue Service (IRS).

Sections 13.1.1 through 13.1.6 provide definitions for terms related to depreciation.

13.1.1 Depreciation

Depreciation is defined by the IRS as a decrease in value of the assets of a business. The IRS allows businesses to calculate yearly depreciation for their assets and deduct the depreciation from their gross income. Depreciation is only calculated on business assets, not personal assets.

13.1.2 Deterioration

Assets deteriorate over time when they are being used for their intended purpose. When an asset wears out, or it no longer performs its intended function as well as when it was first purchased, then this is *deterioration* in economic terms.

13.1.3 Obsolescence

Assets still functional, but the function they perform could be performed in a more efficient manner by other assets, are considered to be obsolete. The *obsolescence* of existing assets may be caused by technological improvements in new products that supersede existing products.

13.1.4 Book Value

The *book value* of an asset represents the current value of the asset as determined by the IRS. Book value is the original cost of the asset minus the depreciation to date. Book values are calculated at the end of each year and they are the book value at the beginning of the year minus the depreciation for the year. The book value of an asset at a particular year is the original cost of the asset minus depreciation to date. Book values only apply when after-tax engineering economic analysis techniques are being used in evaluations.

13.1.5 Market Value

Market value is the amount of money realized if an asset is sold on the open market. The market value of an asset could be different from its book value since some assets sell for more or less on the open market than the book value determined by the techniques prescribed by the IRS for calculating book value.

13.1.6 Land Value

It is important to note that land is not a depreciable asset since the IRS considers land to not decrease in value. Therefore, before calculating the value of the depreciation for facilities, the value of the land is subtracted from the value of the facility.

13.2 COMPONENTS CONSIDERED WHEN CALCULATING DEPRECIATION

The three major components considered when calculating the yearly depreciation of assets are covered in Sections 13.2.1 through 13.2.3.

13.2.1 Allowable Depreciation

The IRS sets the type of assets subject to depreciation and this information is available in IRS publication 534. Form 4552—Depreciation and Amortization—is the IRS form that explains the process for depreciating assets.

13.2.2 Useful Life of Assets

The IRS determines the *useful life* of assets, which is the number of years over which an asset may be depreciated by a business. Examples of useful lives are listed in Table 13.1.

TABLE 13.1
Examples of the Useful Life of Assets as set by the IRS (2015)

Asset	Useful Life (Years)
Office furniture	10
Automobiles	3
Trucks	4
Apartments	40
Houses	18

13.2.3 Depreciation of Real Property

Houses and apartments are considered to be *real property* by the IRS and they may only be depreciated if they are business property. Tenants, or owners who occupy their homes, are not able to depreciate them since depreciation only applies to business property. No personal property may be depreciated for tax purposes.

13.3 METHODS FOR CALCULATING DEPRECIATION

There are four main methods for calculating depreciation on a yearly basis. Once depreciation is determined for a particular year using one of these four methods, it is deducted from the gross income of a firm, along with other business expenses, to arrive at the taxable income. Members of firms use one of the four methods for calculating depreciation and the selection of which method to use is based on the type of firm, the business performed, the amount of yearly income, the type of asset, and whether a firm wants to deduct more depreciation during the first few years of owning an asset while the asset is generating more income than in future years.

The following are the four methods for calculating depreciation discussed in Sections 13.3.1 through 13.3.4:

1. Production
2. Straight line
3. Declining balance
4. Sum-of-the-years digits (SOYD)

The IRS allows for one-time changes in the method used for calculating depreciation for certain types of assets such as real property. For additional information on this option, see IRS publication 534.

13.3.1 Production Depreciation

Production depreciation is based on the number of units of production (output) and the useful life of an asset in terms of production such as units, tons, feet, meters, cubic yards, cubic meters, hours, or mileage. Production depreciation is commonly used by construction or manufacturing firms, where the depreciable assets are heavy construction equipment or large processing equipment. Examples of the measurements used for determining the useful life of an asset when calculating production depreciation are the following:

- 10,000 hours for mobile equipment
- 200,000 hours for transport vehicles
- 1,000,000 yards (914,400 meters) of production for manufacturing or process plants

The formula for production depreciation per unit of production is Equation 13.1.

$$\text{Production depreciation} = \frac{\text{Number of units}}{\text{Life in units of production}} \times (\text{Cost} - \text{salvage value}) \tag{13.1}$$

In this equation, the salvage value is subtracted from the cost since an asset should not be depreciated below the salvage value. If an asset is depreciated below the salvage value and then sold for more than its depreciated book value, there will be recaptured depreciation on the amount realized on the sale above the depreciated value and this amount is subject to taxes.

Example 13.1 uses Equation 13.1 to calculate the production depreciation of an asset.

Example 13.1

A construction firm purchases a new bulldozer for $160,000.00 that will be operated for six hours per day, five days per week, and 52 weeks per year. The bulldozer will have a salvage value of $10,000.00 in 10 years. Determine the production depreciation.

Solution

Use Equation 13.1 to calculate the deprecation for the bulldozer:

$$\text{Yearly production} = \frac{6 \text{ hours}}{\text{Day}} \times \frac{5 \text{ days}}{\text{Week}} \times \frac{52 \text{ weeks}}{\text{Year}}$$

$$= 1,560 \text{ hours per year}$$

$$\text{Production depreciation} = \frac{\text{Number of hours}}{\text{Life in hours of production}} \times (\text{Cost} - \text{salvage value})$$

$$= \frac{1,560 \text{ hours}}{10,000 \text{ hours}} \times (\$160,000.00 - \$10,000.00)$$

$$= 0.156 \times \$150,000.00$$

$$= \$23,400.00$$

If the yearly use is not known for construction equipment, an average of 2,000 hours per year is used for the purpose of calculating yearly depreciation.

Another example calculating production depreciation is Example 13.2.

Example 13.2

A manufacturing plant purchased a new processing machine for \$100,000,000.00. The processing machine will have a salvage value of \$30,000,000.00 at the end of its useful life. The processing machine produces 50,000 yards (45,720 meters) per year and its useful life is 1,000,000 yards (914,400 meters). Using the yards of production for manufacturing plants, calculate the depreciation for the processing machine.

Solution

$$\text{Production depreciation (yards)} = \frac{\text{Number of yards}}{\text{Life in yards of production}} \times (\text{Cost} - \text{salvage value})$$

$$= \frac{50,000 \text{ yards}}{1,000,000 \text{ yards}} \times (\$100,000,000.00 - \$30,000,000.00)$$

$$= 0.05 \times \$70,000,000.00$$

$$= \$3,500,000.00$$

$$\text{Production depreciation (meters)} = \frac{\text{Number of meters}}{\text{Life in meters of production}} \times (\text{Cost} - \text{salvage value})$$

$$= \frac{45,720 \text{ meters}}{914,400 \text{ meters}} \times (\$100,000,000.00 - \$30,000,000.00)$$

$$= 0.05 \times \$70,000,000.00$$

$$= \$3,500,000.00$$

Example 13.3 also calculates production depreciation.

Example 13.3

The owner of a manufacturing plant bought a new conveying system. It cost \$10,000,000 and it processes 1,900,000 units per year. It has a salvage value of \$1,500,000.00 and its useful life is 12,000,000 units. Determine the production depreciation.

Solution

$$\text{Production depreciation} = \frac{\text{Number of units}}{\text{Life in units of production}} \times (\text{Cost} - \text{salvage value})$$

$$= \frac{1{,}900{,}000 \text{ units}}{12{,}000{,}000 \text{ units}} \times (\$10{,}000{,}000.00 - \$1{,}500{,}000.00)$$

$$= 0.1583 \times \$8{,}500{,}000.00 = \$1{,}345{,}550.00$$

13.3.2 Straight Line Depreciation

The most frequently used method for calculating yearly depreciation is *straight line depreciation*. When using straight line depreciation, the book value of an asset decreases linearly since the yearly depreciation is the same for every year the asset is being depreciated for tax purposes. The formula for straight line depreciation is Equation 13.2.

$$\text{Straight line depreciation} = \frac{P - F}{n} \tag{13.2}$$

where
- P is the present value (cost)
- F is the future value (salvage value)
- n is the useful life

Note: The IRS may set the useful life in this equation; therefore, the useful life is subject to change and it should be verified by checking with the IRS when the useful life is being used for tax purposes.

The present value, which is the initial cost of an asset, includes the cost of the asset, taxes, delivery charges, installation costs, and any other costs incurred in purchasing and installing the equipment or asset. The salvage value is the amount realized by the sell of the asset or equipment at the end of its useful life, or whenever the asset is sold, minus any costs associated with removing or dismantling the asset.

When using straight line depreciation to calculate book values, the annual depreciation is multiplied by (m) the number of years of service and this is subtracted from the initial cost. The formula for the book value of an asset when using straight line depreciation is Equation 13.3.

$$BV_m = P - (m \times D) \tag{13.3}$$

where
- BV_m is the book value after (m) years
- P is the present value (cost)
- m is the number of years of depreciation
- D is the annual depreciation

The following examples demonstrate calculating straight line depreciation and book value.

Example 13.4

A manager of an electrical engineering firm purchases a new device costing $160,000.00. The device will have a salvage value of $10,000.00 after five years. What are the yearly straight line depreciation and the book value at the end of three years?

Solution

$$\text{Straight line depreciation} = \frac{P-F}{n} = \frac{\$160{,}000.00 - \$10{,}000.00}{5} = \$30{,}000.00 \text{ per year}$$

$$BV_m = P - (m \times D_m) = \$160{,}000.00 - (3 \times \$30{,}000.00) = \$70{,}000.00$$

Example 13.5

The purchasing manager for a nuclear power company purchases a new turbine for one of its generators for $1,100,000,000.00. The turbine has a useful life of 10 years and a salvage value of $400,000,000.00. What is the yearly straight line depreciation?

Solution

$$\text{Straight line depreciation} = \frac{P-F}{n} = \frac{\$1{,}100{,}000{,}000.00 - \$400{,}000{,}000.00}{10}$$

$$= \$70{,}000{,}000.00$$

Example 13.6

A civil engineer working for a design firm purchases a new 3D printer for $9,000.00. The IRS allows depreciating a printer over five years. At the end of five years, the printer will have a salvage value of $700.00. Calculate the straight line depreciation and the book value for each of the five years over which the printer will be depreciated for tax purposes.

Solution

$$\text{Straight line depreciation} = \frac{P-F}{n} = \frac{\$9{,}000.00 - \$700.00}{5} = \$1{,}660.00 \text{ per year}$$

Table 13.2 includes the straight line depreciation and the book values calculated using Equations 13.2 and 13.3 for all five years of the useful life of the printer.

TABLE 13.2
Straight Line Depreciation and Book Values for Example 13.6

Year	Book Value before Depreciation	Straight Line Depreciation	Book Value after Depreciation
1	$9,000.00	$1,660.00	$7,340.00
2	$7,340.00	$1,660.00	$5,680.00
3	$5,680.00	$1,660.00	$4,020.00
4	$4,020.00	$1,660.00	$2,360.00
5	$2,360.00	$1,660.00	$700.00

Note: The book value for the last year after depreciation should be equal to the salvage value.

Example 13.7

A tool and die company owns several models of lathes that have a total cost of $15,000,000.00, a salvage value of $2,750,000.00, and a life of five years. Develop a table of depreciation and book values for the lathes.

Solution

$$\text{Straight line depreciation} = \frac{P - F}{n} = \frac{\$15{,}000{,}000.00 - \$2{,}750{,}000.00}{5} = \$2{,}450{,}000.00$$

Table 13.3 lists the depreciation and book values for the lathes.

TABLE 13.3
Straight Line Depreciation and Book Values for Example 13.7

Year	Book Value before Depreciation	Straight Line Depreciation	Book Value after Depreciation
1	$15,000,000.00	$2,450,000.00	$12,550,000.00
2	$12,550,000.00	$2,450,000.00	$10,100,000.00
3	$10,100,000.00	$2,450,000.00	$7,650,000.00
4	$7,650,000.00	$2,450,000.00	$5,200,000.00
5	$5,200,000.00	$2,450,000.00	$2,750,000.00

Note: The book value for the last year after depreciation should not be below the salvage value.

13.3.3 Declining Balance (Accelerated Cost Recovery System) Depreciation

In addition to the straight line depreciation, the IRS also allows *accelerated cost recovery depreciation* methods for depreciating assets and deducting them from taxable income since most assets actually lose more value during the first few years of service. In the *declining balance depreciation* method, the annual depreciation is 2.0, 1.5, or 1.25 times the current book value of the asset divided by the total economic life. If 2.0 times the current book value is used, then it is *double declining balance* depreciation and it doubles the amount of depreciation compared to using straight line depreciation. If 1.5 times the book value is used, then the depreciation would be 1.5 times the depreciation calculated using straight line depreciation.

As with straight line depreciation, when using declining balance depreciation, the book value is not allowed to be less than the salvage value. If additional depreciation is deducted from taxable income beyond this limit, then the IRS requires a firm to pay taxes on the amount of the salvage value minus the book value when the asset is sold and this is called recaptured depreciation. Since by depreciating the asset beyond its salvage value the company realizes a tax savings, then if the asset is sold at a value that has been recaptured, the business is required to pay taxes on the recaptured amount.

The IRS allows firms to use declining balance depreciation and then switch to straight line depreciation before the depreciation is less than the amount of depreciation available using straight line depreciation. A firm may only change from using declining balance depreciation to straight line depreciation once during the life of an asset. After the company changes depreciation methods to straight line, the amount of depreciation will be a set amount each year of its remaining life.

Equation 13.4 is the formula for calculating double declining balance depreciation:

$$\text{Double declining balance depreciation} = \frac{2}{n} BV \tag{13.4}$$

Equation 13.5 is the formula for calculating the book value when using declining balance depreciation and Equation 13.6 summarizes Equation 13.5:

$$BV_m = P - \text{Depreciation to date} \tag{13.5}$$

$$BV_m = BV_{m-1} - R\left(P - \sum D\right) \tag{13.6}$$

where

BV_m is the book value ($BV_m > F$)

R is the depreciation rate $\left(\frac{2}{n}, \frac{1.5}{n}, \text{ or } \frac{1.25}{n}\right)$

P is the present value

$\sum D$ is the sum of the depreciation to date

The types of declining balance depreciation the IRS allows are listed in Table 13.4, along with the category of assets for each type of depreciation.

TABLE 13.4
Internal Revenue Service Allowed Declining Balance Depreciation Rates

Type of Declining Balance Depreciation	Category of Asset
$\frac{2}{n} BV$	All new property except real estate
$\frac{1.5}{n} BV$	All used property and real estate
$\frac{1.25}{n} BV$	Used rental residential property

The following examples calculate double declining balance depreciation and book values.

Example 13.8

Calculate the depreciation and book values for the data provided in Example 13.6 using double declining balance depreciation.

Solution

$$\text{Double declining balance depreciation} = \frac{2}{n}(BV)$$

Table 13.5 shows the calculations for the depreciation and book values.

TABLE 13.5
Double Declining Balance Depreciation and Book Values for Example 13.8

Year	Book Value before Depreciation	Depreciation for Year	Book Value after Depreciation
1	\$9,000.00	$\frac{2}{5}(\$9,000.00-0)=\$3,600.00$	$\$9,000.00-\$3,600.00$ $=\$5,400.00$
2	\$5,400.00	$\frac{2}{5}(\$9,000.00-\$3,600.00)$ $=\$2,160.00$	$\$5,400.00-\$2,160.00$ $=\$3,240.00$
3	\$3,240.00	$\frac{2}{5}(\$9,000.00-\$3,600.00)$ $-\$2,160.00=\$1,296.00$	$\$3,240.00-\$1,296.00$ $=\$1,944.00$
4	\$1,944.00	$\frac{2}{5}(\$9,000.00-\$3,600.00$ $-\$2,160.00-\$1,296.00)=\$777.60$	$\$1,944.00-\$777.60$ $=\$1,166.40$
5	\$1,166.40	$\frac{2}{5}(\$9,000.00-\$3,600.00-\$2,160.00$ $-\$1,296.00-\$777.60)=\$466.56$	$\$1,166.40-\$466.40$ $=\$700.00$

The allowable depreciation needs to be calculated and compared to the total depreciation to ensure that an asset is only being depreciated down to the salvage value:

$$\text{Allowable depreciation} = P - F = \$9,000.00 - \$700.00 = \$8,300.00$$

$$\text{Total depreciation} = \$3,600.00 + \$2,160.00 + \$1,296.00 + \$777.60 + \$466.56$$
$$= \$8,300.00$$

Example 13.9

Calculate the depreciation and book values using 1.5 declining balance depreciation for an asset costing \$50,000.00, with a salvage value of \$5,000.00, and a life of four years.

Solution

$$1.5\ \text{Declining balance depreciation} = \frac{1.5}{n}(BV)$$

Table 13.6 shows the calculations for the depreciation and book values.

TABLE 13.6
1.5 Declining Balance Depreciation and Book Value for Example 13.9

Year	Book Value before Depreciation	Depreciation for Year	Book Value after Depreciation
1	\$50,000.00	$\frac{1.5}{4}(\$50{,}000.00 - 0) = \$18{,}750.00$	$\$50{,}000.00 - \$18{,}750.00 = \$31{,}250.00$
2	\$31,250.00	$\frac{1.5}{4}(\$50{,}000.00 - \$18{,}750.00) = \$11{,}718.75$	$\$31{,}250.00 - \$11{,}718.75 = \$19{,}531.25$
3	\$19,531.25	$\frac{1.5}{4}(\$50{,}000.00 - \$18{,}750.00 - \$11{,}718.75) = \$7{,}324.22$	$\$19{,}531.25 - \$7{,}324.22 = \$12{,}207.03$
4	\$12,207.03	$\frac{1.5}{4}(\$50{,}000.00 - \$18{,}750.00 - \$11{,}718.75 - \$7{,}324.22) = \$4{,}577.64$	$\$12{,}207.03 - \$4{,}557.64 = \$7{,}629.39$

Verify that the total depreciation does not exceed the allowable depreciation:

$$\text{Allowable depreciation} = P - F = \$50{,}000.00 - \$5{,}000.00 = \$45{,}000.00$$

$$\text{Total depreciation} = \$18{,}750.00 + \$11{,}718.75 + \$7{,}324.23 + \$4{,}577.64$$
$$= \$42{,}370.62$$

Example 13.10

Calculate the depreciation and book values for the data from Example 13.7 using double declining balance depreciation.

Solution

Table 13.7 lists the deprecation and book values for Example 13.10.

$$\text{Double declining balance depreciation} = \frac{2}{n}\,(BV)$$

TABLE 13.7
Double Declining Balance Depreciation and Book Values for Example 13.10

Year	Book Value before Depreciation	Depreciation for Year	Book Value after Depreciation
1	\$15,000,000.00	$\frac{2}{5}$(\$15,000,000.00 – 0) = \$6,000,000.00	\$15,000,000.00 – \$6,000,000.00 = \$9,000,000.00
2	\$9,000,000.00	$\frac{2}{5}$(\$15,000,000.00 – \$6,000,000.00) = \$3,600,000.00	\$9,000,000.00 – \$3,600,000.00 = \$5,400,000.00
3	\$5,400,000.00	$\frac{2}{5}$(\$15,000,000.00 – \$6,000,000.00 – \$3,600,000.00) = \$2,160,000.00	\$5,400,000.00 – \$2,160,000.00 = \$3,240,000.00
4	\$3,240,000.00	$\frac{2}{5}$ (\$15,000,000.00 – \$6,000,000.00 – \$3,600,000.00 – \$2,160,000.00) = \$1,296,000.00	\$3,240,000.00 – \$1,296,000.00 = \$1,944,000.00 This value is below the salvage value, only allowed \$490,000.00 in depreciation
5	\$2,750,000.00	\$0	\$2,750,000.00

13.3.4 Sum-of-the-Years Digits Depreciation

Sum-of-the-years digits (SOYD) depreciation is another accelerated cost recovery method for calculating depreciation that allows for more depreciation in the earlier years during the life of an asset. Sum-of-the-years digits depreciation uses the sum of the years one through (n) and the total number of years of the life of the asset. The depreciation is calculated by multiplying the initial cost of the asset minus the salvage value ($P-F$) by the remaining number of years divided by the sum-of-the-years digits. The formulas for SOYD depreciation are Equations 13.7 and 13.8.

$$\text{SOYD depreciation} = \frac{\text{Years remaining}}{\text{Sum-of-the-years digits (SOYD)}}(P-F) \tag{13.7}$$

$$= \frac{m}{\frac{n(n+1)}{2}}(P-F) \tag{13.8}$$

where

m is the remaining years
n is the total number of years
$\text{SOYD} = \frac{n(n+1)}{2}$
P is the present value (initial cost)
F is the future value (salvage value)

The book value when using sum-of-the-years digits depreciation for any given year is calculated using Equation 13.9.

$$BV_m = P - \left[\frac{m\left(n - \left(\frac{m}{2}\right) + 0.5\right)}{\text{SOYD}} \right](P - F) \tag{13.9}$$

Example 13.11 uses Equation 13.6 to calculate the sum-of-the-years digits depreciation for two different time frames.

Example 13.11

Calculate the sum-of-the-years digits for n = 5 years and n = 10 years.

Solution

For n = 5 years,

$$\text{SOYD} = \frac{n(n+1)}{2} = \frac{5(5+1)}{2} = \frac{5(6)}{2} = \frac{30}{2} = 15$$

or

$$\text{SOYD} = 5 + 4 + 3 + 2 + 1 = 15$$

For n = 10 years,

$$\text{SOYD} = \frac{n(n+1)}{2} = \frac{10(10+1)}{2} = \frac{10(11)}{2} = \frac{110}{2} = 55$$

or

$$\text{SOYD} = 10 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 55$$

Example 13.12 uses Equation 13.6 to calculate sum-of-the-years digits depreciation and book values.

Example 13.12

Using the data from Example 13.5, calculate the sum-of-the-years digits depreciation and book values for five years.

Solution

Table 13.8 shows the calculations for depreciation and the book values for Example 13.12:

$$\text{SOYD depreciation} = \frac{\text{Years remaining}}{\text{Sum-of-the-years digits (SOYD)}}(P - F)$$

TABLE 13.8
Sum-of-the-Years Digits Depreciation and Book Values for Example 13.12

Year	Book Value before Depreciation	SOYD Depreciation for Year	Book Value after Depreciation
1	\$9,000.00	$\frac{5}{15}(\$9{,}000.00 - \$700.00) = \$2{,}766.67$	\$9,000.00 – \$2,766.67 = \$6,233.33
2	\$6,233.33	$\frac{4}{15}(\$9{,}000.00 - \$700.00) = \$2{,}213.33$	\$6,233.33 – \$2,213.33 = \$4020.00
3	\$4,020.00	$\frac{3}{15}(\$9{,}000.00 - \$700.00) = \$1{,}660.00$	\$4,020.00 – \$1,660.00 = \$2,360.00
4	\$2,360.00	$\frac{2}{15}(\$9{,}000.00 - \$700.00) = \$1{,}106.67$	\$2,360.00 – \$1,106.67 = \$1,253.33
5	\$1,253.33	$\frac{1}{15}(\$9{,}000.00 - \$700.00) = \$553.33$	\$1,253.33 – \$553.33 = \$700.00

Note: The book value for the last year after depreciation is \$700.00 and it should be equal to the salvage value, which is \$700.00.

Example 13.13 provides another problem solving for depreciation using sum-of-the-years digits digits depreciation.

Example 13.13

Calculate the depreciation and book values using sum-of-the-years digits deprecation for the first three years of the life of an asset if it has an initial cost of \$250,000.00, a salvage value of \$40,000.00, and a life of eight years using Equation 13.8.

Solution

Calculate the sum-of-the-years digits and then the sum-of-the-years digits depreciation:

$$\text{SOYD} = \frac{n(n+1)}{2} = \frac{8(8+1)}{2} = \frac{8(9)}{2} = \frac{72}{2} = 36$$

or

$$\text{SOYD} = 8+7+6+5+4+3+2+1 = 36$$

Calculate the sum-of-the-years digits depreciation for years one through three using Equation 13.8

$$\text{SOYD depreciation} = \frac{m}{\frac{n(n+1)}{2}}(P-F)$$

$$\text{SOYD depreciation year 1} = \frac{8}{36}(\$250{,}000.00 - \$40{,}000.00)$$

$$= 0.22222 \times \$210{,}000.00 = \$46{,}666.20$$

$$\text{SOYD depreciation year 2} = \frac{7}{36}(\$250{,}000.00 - \$40{,}000.00)$$
$$= 0.19444 \times \$210{,}000.00 = \$40{,}832.40$$

$$\text{SOYD depreciation year 3} = \frac{6}{36}(\$250{,}000.00 - \$40{,}000.00)$$
$$= 0.16667 \times \$210{,}000.00 = \$35{,}000.70$$

Use Equation 13.9 to calculate the book value for years one through three:

$$BV_1 = BV - \frac{8}{36}(P - F)$$
$$= \$250{,}000.00 - \frac{8}{36}(\$250{,}000.00 - \$40{,}000.00)$$
$$= \$250{,}000.00 - (0.2222 \times \$210{,}000.00)$$
$$= \$250{,}000.00 - \$46{,}666.20$$
$$= \$203{,}333.80$$

$$BV_2 = BV_1 - \frac{7}{36}(P - F)$$
$$= \$203{,}333.80 - \frac{7}{36}(\$210{,}000.00)$$
$$= \$203{,}333.80 - (0.19444 \times \$210{,}000.00)$$
$$= \$203{,}333.80 - \$40{,}832.40$$
$$= \$162{,}501.40$$

$$BV_3 = BV_2 - \frac{6}{36}(P - F)$$
$$= \$162{,}501.40 - \frac{6}{36}(\$210{,}000.00)$$
$$= \$162{,}501.40 - (0.16667 \times \$210{,}000.00)$$
$$= \$162{,}501.40 - \$35{,}000.70$$
$$= \$127{,}500.70$$

Example 13.14

Calculate the sum-of-the-years digits depreciation and book values using the data in Problem 13.7.

Solution

Table 13.9 contains the depreciation and book values for Example 13.14:

$$\text{SOYD depreciation} = \frac{\text{Years remaining}}{\text{Sum-of-the-years digits (SOYD)}}(P - F)$$

TABLE 13.9
Sum-of-the-Years Digits Depreciation and Book Values for Example 13.14

Year	Book Value before Depreciation	SOYD Depreciation for Year	Book Value after Depreciation
1	\$15,000,000.00	$\frac{5}{15}$(\$15,000,000.00 – \$2,750,000.00) =4,083,333.33	\$15,000,000.00 – \$4,083,333.33 = \$10,916,666.67
2	\$10,916,666.67	$\frac{4}{15}$(\$15,000,000.00 – \$2,750,000.00) = \$3,266,667.67	\$10,916,666.67 – \$3,266,667.67 = \$7,650,000.00
3	\$7,650,000.00	$\frac{3}{15}$(\$15,000,000.00 – \$2,750,000.00) = \$2,450,000.00	\$7,650,000.00 – \$2,450,000.00 = \$5,200,000.00
4	\$5,200,000.00	$\frac{2}{15}$(\$15,000,000.00 – \$2,750,000.00) = \$1,633,333.33	\$5,200,000.00 – \$1,633,333.33 = \$3,566,666.67
5	\$3,566,666.67	$\frac{1}{15}$(\$15,000,000.00 – \$2,750,000.00) = \$816,666.67	\$3,566,666.67 – \$816,666.67 = \$2,750,000.00

Appendix C provides spreadsheet formulas for solving for all four types of depreciation.

Chapter 14 explains incorporating depreciation into taxes and after-tax rate of return analysis.

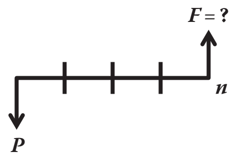
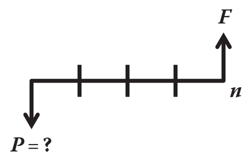
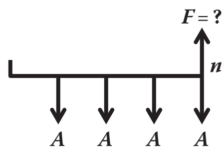
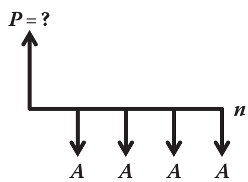
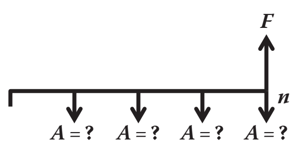
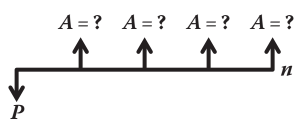
13.4 SUMMARY

This chapter introduced the concept of depreciation and four methods for calculating depreciation for tax purposes—production, straight line, declining balance, and sum-of-the-years digits. This chapter also provided definitions for depreciation and related terms and discussed the components considered when calculating deprecation. The last part of this chapter covered the methods for calculating the four different types of depreciation and provided examples demonstrating how to calculate each type of depreciation.

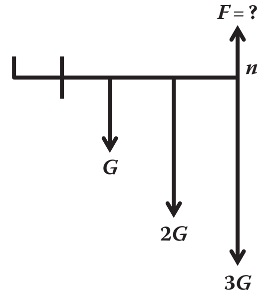
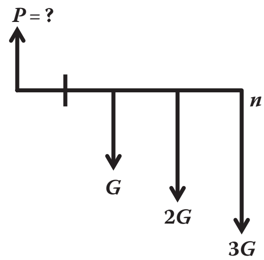
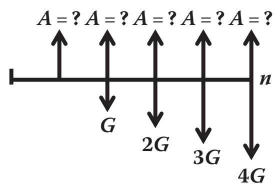
KEY TERMS

Accelerated cost recovery depreciation
Book value
Declining balance depreciation
Depreciation
Deterioration
Double declining balance depreciation
Market value

Appendix A: Basic Engineering Economic Equations and Cash Flow Diagrams

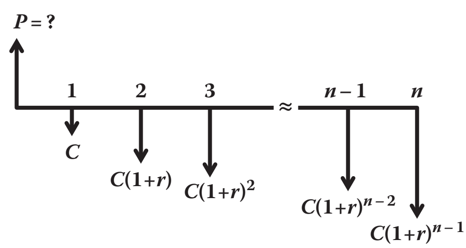
Factor	Equation	Cash Flow Diagram	As $n = \infty$
$F = P\ (F/P), i, n)$ Future worth (F) of a present value (P) n = number of time periods	$F = P_0(1 + i)^n$ $F = Pe^{i_n \times n}$ (continuous compounding)		${}^{F}\!/_{P} \Rightarrow \infty$
$P = F(P/F, i, n)$ Present worth (P) of a future value (F) n = number of time periods	$P = F\left[\frac{1}{(1+i)^n}\right]$		${}^{P}\!/_{F} \Rightarrow \infty$
$F = A(F/A, i, n)$ Future worth (F) of a periodic uniform series (A) n = number of time periods	$F = A\left[\frac{(1+i)^n - 1}{i}\right]$		${}^{F}\!/_{A} \Rightarrow \infty$
$P = A(P/A, i, n)$ Present worth (P) of a periodic uniform series (A) n = number of time periods	$P = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$		${}^{P}\!/_{A} \Rightarrow \frac{1}{i}$
$A = F(A/F, i, n)$ Periodic uniform series (A) of a future worth (F) n = number of time periods	$A = F\left[\frac{i}{(1+i)^n - 1}\right]$		
$A = P(A/P, i, n)$ Periodic uniform series (A) of a present worth (P) n = number of time periods	$A = P\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right]$		

(*Continued*)

Factor	**Equation**	**Cash Flow Diagram**	**As $n = \infty$**
$F = G(F/G, i, n)$ Future worth (F) of an arithmetic gradient series (G) n = number of payments plus one	$F = G\left[\left(\frac{1}{i}\right)\frac{(1+i)^n-1}{i} - n\right]$		$F/G \Rightarrow \infty$
$P = G(P/G, i, n)$ Present worth (P) of an arithmetic gradient series (G) n = number of payments plus one	$P = \frac{G}{i}\left[\frac{(1+i)^n-1}{i(1+i)^n} - \frac{n}{(1+i)^n}\right]$		$P/G \Rightarrow \frac{1}{i^2}$
$A = G(A/G, i, n)$ Periodic uniform series (A) equivalent of arithmetic gradient series (G)	$A = G\left[\frac{1}{i} - \frac{n}{(1+i)^n-1}\right]$		

Present worth (P) of a geometric gradient series
n = number of end of period payments

1. When $r > i$, then $w = \frac{1+r}{1+i} - 1$, $P = \frac{C}{1+i}\left(F/A, w, n\right) = \frac{C}{1+i}\left[\frac{(1+w)^n-1}{w}\right]$ $\qquad P/C \Rightarrow \infty$
2. When $r < i$, then $w = \frac{1+i}{1+r} - 1$ and $P = \frac{C}{1+r}\left(P/A, w, n\right) = \frac{C}{1+r}\left[\frac{(1+w)^n-1}{w(1+w)^n}\right]$ $\qquad P/C \Rightarrow \frac{1}{(1+r)w}$
3. When $r = i$, then $P = \frac{C \times n}{(1+r)} = \frac{C \times n}{(1+i)}$ $\qquad P/C \Rightarrow \infty$



Appendix B: Interest Factor Tables

	i = 0.5%			*i* = 0.5%			*i* = 0.5%			
	Present Sum (*P*)			**Uniform Series (*A*)**			**Future Sum (*F*)**			
n	***P/F***	***P/A***	***P/G***	***A/F***	***A/P***	***A/G***	***F/P***	***F/A***	***F/G***	***n***
1	0.99502	0.9950	0.0000	1.00000	1.00500	0.0000	1.0050	1.0000	0.0000	1
2	0.99007	1.9851	0.9900	0.49875	0.50375	0.4987	1.0100	2.0050	1.0000	2
3	0.98515	2.9702	2.9603	0.33167	0.33667	0.9966	1.0150	3.0150	3.0050	3
4	0.98025	3.9505	5.9011	0.24813	0.25313	1.4937	1.0201	4.0301	6.0200	4
5	0.97537	4.0258	9.8026	0.19801	0.20301	1.9900	1.0252	5.0502	10.050	5
6	0.97052	5.8963	14.655	0.16460	0.16960	2.4854	1.0303	6.0755	15.100	6
7	0.96569	6.8620	20.449	0.14073	0.14573	2.9800	1.0355	7.1058	21.175	7
8	0.96089	7.8229	27.175	0.12283	0.12783	3.4738	1.0407	8.1414	28.281	8
9	0.95610	8.7790	34.824	0.10891	0.11391	3.9667	1.0459	9.1821	36.423	9
10	0.95135	9.7304	43.386	0.09777	0.10277	4.4588	1.0511	10.228	45.605	10
11	0.94661	10.677	52.852	0.08866	0.09366	4.9501	1.0564	11.279	55.833	11
12	0.94191	11.618	63.213	0.08107	0.08607	5.4405	1.0616	12.335	67.112	12
13	0.93722	12.556	74.460	0.07464	0.07964	5.9301	1.0669	13.397	79.448	13
14	0.93256	13.488	86.583	0.06914	0.07414	6.4189	1.0723	14.464	92.845	14
15	0.92792	14.416	99.574	0.06436	0.06936	6.9069	1.0776	15.536	107.31	15
16	0.92330	15.339	113.42	0.06019	0.06519	7.3940	1.0830	16.614	133.84	16
17	0.91871	16.258	128.12	0.05651	0.06151	7.8803	1.0884	17.697	139.46	17
18	0.91414	17.172	143.66	0.05323	0.05823	8.3657	1.0939	18.785	157.15	18
19	0.90959	18.082	150.03	0.05030	0.05530	8.8504	1.0994	19.879	175.94	19
20	0.90506	18.987	177.23	0.04767	0.05267	9.3341	1.1049	20.979	195.82	20
21	0.90056	19.888	195.24	0.04528	0.05028	9.8171	1.1104	22.084	216.80	21
22	0.89608	20.784	214.06	0.04311	0.04811	10.299	1.1159	23.194	238.88	22
23	0.89162	21.675	233.67	0.04113	0.04613	10.780	1.1215	23.310	262.08	23
24	0.88719	22.562	254.08	0.03932	0.04432	11.261	1.1271	25.432	286.39	24
25	0.88277	23.445	275.26	0.03765	0.04265	11.740	1.1328	26.559	311.82	25
26	0.87838	24.324	297.22	0.03611	0.04111	12.219	1.1384	27.691	338.38	26
27	0.87401	25.198	319.95	0.03469	0.03969	12.697	1.1441	28.830	366.07	27
28	0.86966	26.067	343.43	0.03336	0.03836	13.174	1.1498	29.974	394.90	28
29	0.96533	26.933	367.66	0.03213	0.03713	13.651	1.1556	31.124	424.87	29
30	0.86103	27.794	392.63	0.03098	0.03598	14.126	1.1616	32.280	456.00	30
32	0.85248	29.503	444.76	0.02889	0.03389	15.075	1.1730	34.608	521.72	32
34	0.84402	31.195	499.75	0.02706	0.03206	16.020	1.1848	36.960	592.11	34
36	0.83564	32.871	557.56	0.02542	0.03042	16.962	1.1966	39.336	667.22	36
48	0.78710	42.580	959.91	0.01849	0.02349	22.543	1.2704	54.097	1219.5	48
60	0.74137	51.725	1448.6	0.01433	0.01933	28.006	1.3488	69.770	1954.0	60
120	0.54963	90.073	4823.5	0.00610	0.01110	53.550	1.8194	163.87	8775.8	120
180	0.40748	118.50	9031.3	0.00344	0.00844	76.211	2.4540	290.81	22163.	180
240	0.30210	139.58	13415.	0.00216	0.00716	96.113	3.3102	462.04	44408.	240
300	0.22397	155.20	17603.	0.00144	0.00644	113.41	4.4649	602.99	78598.	300
360	0.16604	166.79	21403.	0.00100	0.00600	128.32	6.0225	1004.5	128903	360
INF	0.00000	200.00	40000	0.00000	0.00500	200.00	INF	INF	INF	INF

	i = 0.75%			*i* = 0.75%			*i* = 0.75%			
	Present Sum (*P*)			**Uniform Series (*A*)**			**Future Sum (*F*)**			
n	***P/F***	***P/A***	***P/G***	***A/F***	***A/P***	***A/G***	***F/P***	***F/A***	***F/G***	***n***
1	0.99256	0.9925	0.0000	1.00000	1.00750	0.0000	1.0075	1.0000	0.0000	1
2	0.98517	1.9777	0.9851	0.49813	0.50563	0.4981	1.0150	2.0075	1.0000	2
3	0.97783	2.9555	2.9408	0.33085	0.33835	0.9950	1.0226	3.0225	3.0075	3
4	0.97055	3.9261	5.8525	0.24721	0.25471	1.4906	1.0303	4.0452	6.0300	4
5	0.96333	4.8894	9.7058	0.19702	0.20452	2.9850	1.0380	5.0755	10.075	5
6	0.95616	5.8456	14.486	0.16357	0.17107	2.4782	1.0458	6.1136	15.150	6
7	0.94904	6.7946	20.180	0.13967	0.14717	2.9701	1.0537	7.1594	21.264	7
8	0.94198	7.7366	26.774	0.12176	0.12926	3.4607	1.0616	8.2131	28.424	8
9	0.93496	8.6715	34.254	0.10782	0.11532	3.9501	1.0695	9.2747	36.637	9
10	0.92800	9.5995	42.606	0.09667	0.10417	4.4383	1.0775	10.344	45.911	10
11	0.92109	10.520	51.817	0.08755	0.09505	4.9252	1.0856	11.421	56.256	11
12	0.91424	11.434	61.874	0.07995	0.08745	5.4109	1.0938	12.507	67.678	12
13	0.90743	12.342	72.763	0.07352	0.08102	5.8954	1.1020	13.601	80.185	13
14	0.90068	13.243	84.472	0.06801	0.07551	6.3786	1.1102	14.703	93.787	14
15	0.89397	14.137	96.987	0.06324	0.07074	6.8605	1.1186	15.813	108.49	15
16	0.88732	15.024	110.29	0.05906	0.06656	7.3412	1.1269	16.932	124.30	16
17	0.88071	15.905	124.38	0.05537	0.06287	7.8207	1.1354	18.059	141.23	17
18	0.87416	16.779	139.24	0.05210	0.05960	8.2989	1.1439	19.194	159.29	18
19	0.86765	17.646	154.86	0.04917	0.05667	8.7759	1.1525	20.338	178.49	19
20	0.86119	18.508	171.23	0.04653	0.05403	9.2516	1.1611	21.491	198.82	20
21	0.85478	19.362	188.32	0.04415	0.05165	9.7621	1.1698	22.654	220.32	21
22	0.84842	20.211	206.14	0.04198	0.04938	10.199	1.1786	23.822	242.97	22
23	0.84210	21.053	224.66	0.04000	0.04750	10.671	1.1875	25.001	266.79	23
24	0.83583	21.889	243.89	0.03818	0.04568	11.142	1.1964	26.188	291.79	24
25	0.82961	22.718	263.80	0.03652	0.04402	11.611	1.2053	27.384	317.98	25
26	0.82343	23.542	284.38	0.03498	0.04248	12.080	1.2144	28.590	345.36	26
27	0.81730	24.359	305.63	0.03355	0.04105	12.547	1.2235	29.804	373.96	27
28	0.81122	25.170	327.54	0.03223	0.03973	13.012	1.2327	31.028	403.76	28
29	0.80518	25.975	350.08	0.03100	0.03850	13.447	1.2419	32.260	434.79	29
30	0.79919	26.755	373.26	0.02985	0.03735	13.940	1.2512	33.502	267.05	30
32	0.78733	28.355	421.46	0.02777	0.03527	14.863	1.2701	36.014	535.31	32
34	0.77565	29.912	472.07	0.02593	0.03343	15.781	1.2892	38.564	608.61	34
36	0.76415	31.446	524.99	0.02430	0.03180	16.694	1.3086	41.152	687.02	36
48	0.69861	40.184	886.84	0.01739	0.02489	22.069	1.4314	57.520	1269.4	48
60	0.63870	48.173	1313.5	0.01326	0.02076	27.266	1.5656	75.424	2056.5	60
120	0.40794	78.941	3995.5	0.00517	0.01267	50.652	2.4513	193.51	9801.9	120
180	0.26055	98.593	6892.6	0.00264	0.01014	69.909	3.8380	378.40	26454.	180
240	0.16641	111.14	9494.1	0.00150	0.00900	85.421	6.0091	667.88	57051.	240
300	0.10629	119.16	11636.	0.00089	0.00839	97.654	9.4084	1121.1	109483	300
360	0.06789	124.28	13312.	0.00055	0.00805	107.11	14.730	1830.7	196099	360
INF	0.00000	133.33	17777.	0.00000	0.00750	133.33	INF	INF	INF	INF

	i = 1%			i = 1%			i = 1%			
	Present Sum (P)			Uniform Series (A)			Future Sum (F)			
n	P/F	P/A	P/G	A/F	A/P	A/G	F/P	F/A	F/G	n
1	0.99010	0.9901	0.0000	1.00000	1.01000	0.0000	1.0100	1.0000	0.0000	1
2	0.98030	1.9704	0.9803	0.49751	0.50751	0.4975	1.0201	2.0100	1.0000	2
3	0.97059	2.9409	2.9214	0.33002	0.34002	0.9933	1.0303	3.0301	3.0100	3
4	0.96098	3.9019	5.8044	0.24628	0.25628	1.4875	1.0406	4.0604	6.0401	4
5	0.95147	4.8534	9.6102	0.19604	0.20604	1.9801	1.0510	5.1010	10.100	5
6	0.94205	5.7954	14.320	0.16225	0.17255	2.4709	1.0615	6.1520	15.201	6
7	0.93272	6.7821	19.916	0.13863	0.14863	2.9602	1.0721	7.2135	21.353	7
8	0.92348	7.6516	25.381	0.12069	0.13069	3.4477	1.0828	8.2856	28.567	8
9	0.91434	8.5660	33.695	0.10674	0.11674	3.9336	1.0936	9.3685	36.852	9
10	0.90529	9.4713	41.843	0.09558	0.10558	4.4179	1.1046	10.462	46.221	10
11	0.89632	10.367	50.806	0.08645	0.09645	4.9005	1.1156	11.566	56.683	11
12	0.88745	11.255	60.568	0.07885	0.08885	5.3814	1.1268	12.682	68.250	12
13	0.87866	12.133	71.112	0.07241	0.08241	5.8607	1.1380	13.809	80.932	13
14	0.86996	13.003	82.422	0.06690	0.07690	6.3383	1.1494	14.947	94.742	14
15	0.86135	13.865	94.481	0.06212	0.07212	6.8143	1.1609	16.096	109.69	15
16	0.85282	14.717	107.27	0.05794	0.06794	7.2886	1.1725	17.257	125.78	16
17	0.84438	15.562	120.78	0.05426	0.06426	7.7613	1.1843	18.430	143.04	17
28	0.83602	16.398	134.99	0.05098	0.06098	8.2323	1.1961	19.614	161.47	18
19	0.82774	17.226	149.89	0.04805	0.05805	8.7016	1.2081	20.810	181.09	19
20	0.81954	18.045	154.46	0.04542	0.05542	9.1693	1.2201	22.019	201.90	20
21	0.81143	18.857	181.69	0.04303	0.05303	9.6354	1.2323	23.239	223.91	21
22	0.80340	19.660	198.56	0.04086	0.05086	10.099	1.2447	24.471	247.15	22
23	0.79544	20.455	216.06	0.03889	0.04889	10.562	1.2571	25.716	271.63	23
24	0.87857	21.243	234.18	0.03737	0.04704	11.023	1.2697	26.973	297.34	24
25	0.77977	22.023	252.89	0.03541	0.04541	11.483	1.2824	28.243	324.32	25
26	0.77205	22.795	272.19	0.03387	0.04387	11.940	1.2952	28.525	352.56	26
27	0.76440	23.559	292.07	0.03245	0.04245	12.397	1.3082	30.820	382.08	27
28	0.75684	24.316	312.50	0.03112	0.04112	12.851	1.3212	32.129	412.91	28
29	0.74934	25.065	333.48	0.02990	0.03990	13.304	1.3345	33.450	445.03	29
30	0.74192	25.807	355.00	0.02875	0.03875	13.755	1.3478	34.784	478.48	30
32	0.72730	27.269	399.58	0.02667	0.03667	14.663	1.3749	37.493	549.40	32
34	0.71297	38.702	446.15	0.02484	0.03484	15.554	1.4025	40.257	625.77	34
36	0.69892	30.107	494.62	0.02321	0.03321	16.428	1.4307	43.076	707.68	36
48	0.62026	37.974	820.14	0.01633	0.02633	21.597	1.6122	61.222	1233.2	48
60	0.55045	44.955	1192.8	0.01224	0.02224	26.533	1.8167	81.669	2166.9	60
120	0.30299	69.700	3334.1	0.00435	0.01435	47.834	3.3003	230.03	11003.	120
180	0.16678	83.321	5330.0	0.00200	0.01200	63.969	5.9958	499.58	31958.	180
240	0.09181	90.819	6878.6	0.00101	0.01101	75.739	10.892	989.25	74925.	240
300	0.05053	94.946	7978.6	0.00053	0.01053	84.032	19.788	1878.8	157885	300
360	0.02782	97.218	8720.4	0.00029	0.01029	89.699	35.949	3494.9	313496	360
INF	0.00000	100.00	10000.	0.00000	0.01000	100.00	INF	INF	INF	INF

	i = 1.5%			i = 1.5%			i = 1.5%			
	Present Sum (P)			Uniform Series (A)			Future Sum (F)			
n	P/F	P/A	P/G	A/F	A/P	A/G	F/P	F/A	F/G	n
1	0.98522	0.9852	0.0000	1.00000	1.00500	0.0000	1.0150	1.0000	0.0000	1
2	0.97066	1.9558	0.9706	0.49628	0.51128	0.4962	1.0302	2.0150	1.0000	2
3	0.95632	2.9122	2.8833	0.32838	0.34338	0.9900	1.0456	3.0452	3.0150	3
4	0.94218	3.8543	5.7098	0.24444	0.25944	1.4813	1.0613	4.0909	6.0602	4
5	0.92826	4.7826	9.4228	0.19409	0.20909	1.9702	1.0772	5.1522	10.151	5
6	0.91454	5.6791	13.995	0.16053	0.17553	2.4565	1.0934	6.2295	15.303	6
7	0.90103	6.5982	19.401	0.13656	0.15156	2.9404	1.1098	7.3229	21.532	7
8	0.88771	7.4859	25.615	0.11858	0.13358	3.4218	1.1264	8.4328	28.855	8
9	0.97459	8.3605	32.612	0.10461	0.11961	3.9007	1.1433	9.5593	37.288	9
10	0.86167	9.2221	40.367	0.09343	0.10843	4.3772	1.1605	10.702	46.848	10
11	0.84893	10.071	48.856	0.08429	0.09929	4.8511	1.1779	11.863	57.550	11
12	0.83639	10.907	58.057	0.07668	0.09168	5.3226	1.1956	13.041	69.414	12
13	0.82403	11.731	67.945	0.07024	0.08524	5.7916	1.2135	14.236	82.455	13
14	0.81185	12.543	78.499	0.06472	0.07972	6.2582	1.2317	15.450	96.692	14
15	0.79985	13.343	89.697	0.05994	0.07494	6.7223	1.2502	16.682	112.14	15
16	0.78803	14.131	101.51	0.05577	0.07077	7.1839	1.2689	17.932	128.82	16
17	0.77639	14.907	113.94	0.05208	0.06708	7.6430	1.2880	19.201	146.75	17
18	0.76491	15.672	126.94	0.04881	0.06381	8.0997	1.3073	20.489	165.95	18
19	0.75361	16.426	140.50	0.04588	0.06088	8.5539	1.3268	21.796	186.44	19
20	0.74347	17.168	154.61	0.04325	0.05825	9.0056	1.3458	23.123	208.24	20
21	0.73150	17.900	169.24	0.04087	0.05587	9.4549	1.3670	24.470	231.36	21
22	0.72069	18.620	184.38	0.03870	0.05370	9.9018	1.3875	25.837	255.83	22
23	0.71004	19.330	200.00	0.03673	0.05173	10.346	1.4083	27.225	281.67	23
24	0.69954	20.030	216.09	0.03492	0.04992	10.788	1.4295	28.633	308.90	24
25	0.68921	20.719	232.63	0.03326	0.04826	11.227	1.4509	30.063	337.53	25
26	0.67902	21.398	249.60	0.03173	0.04673	11.664	1.4727	31.514	367.59	26
27	0.66899	22.067	267.00	0.03032	0.04532	12.099	1.4948	32.986	399.11	27
28	0.65910	22.726	284.79	0.02900	0.04400	12.531	1.5172	34.481	432.09	28
29	0.64936	23.376	302.97	0.02778	0.04278	12.961	1.5399	35.998	466.58	29
30	0.63976	24.015	321.53	0.02664	0.04164	13.388	1.5630	37.538	502.57	30
31	0.62099	25.267	359.69	0.02458	0.03958	14.235	1.6103	40.688	579.21	31
34	0.60277	26.481	399.16	0.02276	0.03776	15.073	1.6590	43.933	662.20	34
36	0.58509	27.660	429.83	0.02115	0.03615	15.900	1.7091	47.276	751.73	36
48	0.48936	34.042	703.54	0.01437	0.02937	20.666	2.0434	69.565	1437.6	48
60	0.40930	39.380	988.16	0.01039	0.02539	25.093	2.4432	96.214	2414.3	60
120	0.16752	55.498	2359.7	0.00302	0.01802	42.518	5.9693	331.29	14083.	120
180	0.06857	62.095	3316.9	0.00110	0.01610	53.416	14.584	905.62	48375.	180
240	0.02806	64.795	3870.6	0.00043	0.01543	59.736	35.632	2308.8	137924	240
300	0.01149	659.00	4163.6	0.00017	0.01517	63.180	87.058	5737.2	362474	300
360	0.00470	66.353	4310.7	0.00007	0.01507	64.966	212.70	14113.	916906	360
INF	0.00000	66.666	4444.4	0.00000	0.01500	66.666	INF	INF	INF	INF

	i = 2%			i = 2%			i = 2%			
	Present Sum (*P*)			Uniform Series (*A*)			Future Sum (*F*)			
n	***P/F***	***P/A***	***P/G***	***A/F***	***A/P***	***A/G***	***F/P***	***F/A***	***F/G***	***n***
1	0.98039	0.9803	0.0000	1.00000	1.02000	0.0000	1.0200	1.0000	0.0000	1
2	0.96117	1.9415	0.9611	0.49505	0.51505	0.4950	1.0404	2.0200	1.0000	2
3	0.94232	2.8838	2.8458	0.32675	0.34675	0.9868	1.0612	3.0604	3.0200	3
4	0.92385	3.8077	5.6173	0.24262	0.26262	1.4752	1.0824	4.1216	6.0804	4
5	0.90573	4.7134	9.2402	0.19216	0.21216	1.9604	1.1040	5.2040	10.202	5
6	0.88797	5.6014	13.680	0.15853	0.17853	2.4422	1.1261	6.3018	15.406	6
7	0.87056	6.4719	18.903	0.13451	0.15451	2.9208	1.1486	7.4342	21.714	7
8	0.85349	7.3254	24.877	0.11651	0.13651	3.3960	1.1716	8.5829	29.148	8
9	0.83676	8.1622	31.572	0.10252	0.12252	3.8680	1.1950	8.6546	37.731	9
10	0.82035	8.9825	38.955	0.09133	0.11133	4.3367	1.2189	10.949	47.486	10
11	0.80426	9.7868	46.997	0.08218	0.10218	4.8021	1.2433	12.168	58.435	11
12	0.78849	10.575	55.671	0.07456	0.09456	5.2642	1.2682	13.412	70.604	12
13	0.77303	11.348	64.947	0.06812	0.08812	5.7230	1.2936	14.680	84.016	13
14	0.75788	12.106	75.799	0.06260	0.08260	6.1786	1.3194	15.973	98.696	14
15	0.74301	12.849	85.202	0.05783	0.07783	6.6309	1.3458	17.293	114.67	15
16	0.72845	13.577	96.128	0.05365	0.07365	7.0799	1.3727	18.639	131.96	16
17	0.71416	14.291	107.55	0.04997	0.06997	7.5256	1.4004	20.012	150.60	17
18	0.70016	14.992	119.45	0.04670	0.06670	7.9681	1.4282	21.412	170.81	18
19	0.68643	15.678	131.81	0.04378	0.06378	8.4073	1.4568	22.840	192.01	19
20	0.67297	16.351	144.60	0.04116	0.06116	8.8432	1.4859	24.297	214.86	20
21	0.65978	17.011	157.79	0.03878	0.05878	9.2759	1.5156	25.783	239.16	21
22	0.65684	17.658	171.37	0.03663	0.05663	9.7054	1.5459	27.299	264.94	22
23	0.63416	18.292	185.33	0.03467	0.05467	10.131	1.5769	28.845	292.24	23
24	0.62172	18.913	199.63	0.03287	0.05287	10.554	1.6084	30.421	321.09	24
25	0.60953	19.523	214.25	0.03122	0.05122	10.974	1.6406	32.030	351.51	25
26	0.59758	20.131	229.19	0.02970	0.04970	11.391	1.6734	33.670	383.54	26
27	0.58586	20.706	244.43	0.02829	0.04829	11.804	1.7068	35.344	417.21	27
28	0.57437	21.281	259.93	0.02699	0.04699	12.214	1.7410	37.051	452.56	28
29	0.56311	21.844	275.70	0.02578	0.04578	12.621	1.7758	38.792	489.61	29
30	0.55207	22.396	291.71	0.02455	0.04465	13.025	1.8113	40.568	528.50	30
32	0.53063	23.468	324.40	0.02261	0.04261	13.823	1.8845	44.227	611.35	32
34	0.51003	24.498	357.88	0.02082	0.04082	14.608	1.9606	48.033	701.69	34
36	0.49002	25.488	392.04	0.01923	0.03923	15.380	2.0398	51.994	799.71	36
48	0.38654	30.673	605.96	0.01260	0.03260	19.755	2.5870	79.353	1567.6	48
60	0.30478	34.760	823.69	0.00877	0.02877	23.696	3.2810	114.05	2702.5	60
120	0.09289	45.355	1710.4	0.00205	0.02205	37.711	10.765	488.25	18412.	120
180	0.02831	48.584	2174.4	0.00058	0.02058	44.755	35.320	1716.0	76802.	180
240	0.00863	49.568	2374.8	0.00017	0.02017	47.911	115.88	5744.4	275222	240
300	0.00263	49.868	2453.9	0.00005	0.02005	49.208	380.23	18961.	930086	300
INF	0.00000	50.000	2500.0	0.00000	0.02000	50.000	INF	INF	INF	INF

	i = 2.5%			i = 2.5%			i = 2.5%			
	Present Sum (P)			Uniform Series (A)			Future Sum (F)			
n	P/F	P/A	P/G	A/F	A/P	A/G	F/P	F/A	F/G	n
1	0.97561	0.9856	0.0000	1.00000	1.02500	0.0000	1.0250	1.0000	0.0000	1
2	0.95181	1.9272	0.9518	0.49383	0.51883	0.4938	1.0506	2.0250	1.0000	2
3	0.92860	2.8560	2.8090	0.32514	0.35014	0.9835	1.0768	3.0756	3.0250	3
4	0.90595	3.7619	5.5268	0.24082	0.26582	1.4691	1.1039	4.1525	6.1006	4
5	0.88385	4.6458	9.0622	0.19025	0.21525	1.9506	1.1314	5.2563	10.253	5
6	0.86230	5.5081	13.373	0.15655	0.18155	2.4280	1.1596	6.3877	15.509	6
7	0.84127	6.3493	18.421	0.13250	0.15750	2.9012	1.1886	7.5474	21.897	7
8	0.82075	7.1701	23.166	0.11447	0.13947	3.3704	1.2184	8.7361	29.444	8
9	0.80073	7.9708	30.572	0.10046	0.12546	3.8355	1.2488	9.4545	38.180	9
10	0.78120	8.7520	38.603	0.08926	0.11426	4.2964	1.2800	11.203	48.135	10
11	0.76214	9.5142	45.224	0.08011	0.10511	4.7533	1.3120	12.483	59.338	11
12	0.74356	10.257	53.403	0.07249	0.09749	5.2061	1.3448	13.795	71.822	12
13	0.72542	10.983	62.108	0.06605	0.09105	5.6549	1.3785	15.140	85.617	13
14	0.70773	11.690	71.309	0.06054	0.08554	6.0995	1.4129	16.519	100.75	14
15	0.69047	12.381	80.975	0.05577	0.08077	6.5401	1.4483	17.931	117.27	15
16	0.67362	13.055	91.080	0.05160	0.07660	6.9766	1.4845	19.380	135.20	16
17	0.65720	13.712	101.59	0.04793	0.07293	7.4091	1.5216	20.864	154.58	17
18	0.64117	14.353	112.49	0.04467	0.06967	7.8375	1.5596	22.396	175.45	18
19	0.62553	14.979	123.75	0.04176	0.06676	8.2619	1.5986	23.946	197.84	19
20	0.61027	15.589	135.35	0.03915	0.06415	8.6823	1.6386	25.544	221.78	20
21	0.59539	16.184	147.25	0.03679	0.06179	9.0986	1.6795	27.183	247.33	21
22	0.58086	16.765	159.45	0.03465	0.05965	9.5109	1.7215	28.862	274.51	22
23	0.56670	17.332	171.92	0.03270	0.05770	9.9193	1.7646	30.584	303.37	23
24	0.55288	17.885	184.63	0.03091	0.05591	10.323	1.8087	32.349	333.96	24
25	0.53939	18.424	197.58	0.02928	0.05428	10.724	1.8539	34.157	366.31	25
30	0.47674	20.930	265.12	0.02278	0.04778	12.666	2.0975	43.902	556.10	30
35	0.42137	23.145	335.88	0.01821	0.04321	14.512	2.3732	54.928	797.12	35
36	0.41109	23.556	350.27	0.01745	0.04245	14.869	2.4325	57.301	852.05	36
40	0.37243	25.102	408.22	0.01484	0.03984	16.262	2.6850	67.402	1096.1	40
48	0.30567	27.773	524.03	0.01101	0.03601	18.868	3.2714	90.859	1714.3	48
50	0.29094	28.362	552.60	0.01026	0.03526	19.483	3.4371	97.484	1899.3	50
60	0.22728	30.908	690.86	0.00735	0.03235	22.351	4.3997	135.99	3039.6	60
100	0.08465	36.614	1125.9	0.00231	0.02731	30.752	11.813	432.54	13301.	100
120	0.05166	37.933	1269.3	0.00136	0.02636	33.463	19.358	734.32	24573.	120
180	0.01174	39.530	1496.6	0.00030	0.02530	37.861	85.171	3366.8	127475	180
240	0.00267	39.893	1570.1	0.00007	0.02507	39.357	374.73	14949.	588381	240
INF	0.00000	40.000	1600.0	0.00000	0.02500	40.000	INF	INF	INF	INF

	i = 3%			i = 3%			i = 3%			
	Present Sum (*P*)			Uniform Series (*A*)			Future Sum (*F*)			
n	***P/F***	***P/A***	***P/G***	***A/F***	***A/P***	***A/G***	***F/P***	***F/A***	***F/G***	***n***
1	0.97087	0.9708	0.0000	1.00000	1.03000	0.0000	1.0030	1.0000	0.0000	1
2	0.94260	1.9134	0.9426	0.49261	0.52261	0.4926	1.0609	2.0300	1.0000	2
3	0.91514	2.8286	2.7728	0.32353	0.35353	0.9803	1.0927	3.0909	3.0300	3
4	0.88849	3.7171	5.4283	0.23903	0.26903	1.4630	1.1255	4.1836	6.1209	4
5	0.86261	4.5797	8.8887	0.18835	0.21835	1.9409	1.1592	5.3091	10.304	5
6	0.83748	5.4171	13.076	0.15460	0.18460	2.4138	1.1940	6.4684	15.613	6
7	0.81309	6.2302	17.954	0.13051	0.16051	2.8818	1.2298	7.6624	22.082	7
8	0.78941	7.0196	23.480	0.11246	0.14246	3.3449	1.2667	8.8923	29.744	8
9	0.76642	7.7861	29.611	0.09843	0.12843	3.8031	1.3047	10.159	38.636	9
10	0.74409	8.5302	36.308	0.08723	0.11723	4.2565	1.3439	11.463	48.796	10
11	0.72242	9.2526	43.533	0.07808	0.10808	4.7049	1.3842	12.807	60.259	11
12	0.70138	9.9540	51.248	0.07046	0.10046	5.1485	1.4257	14.192	73.067	12
13	0.68095	10.635	59.419	0.06403	0.09403	5.5872	1.4685	15.617	87.259	13
14	0.66112	11.296	68.014	0.05853	0.08853	6.0210	1.5125	17.086	102.87	14
15	0.64186	11.937	77.000	0.05377	0.08377	6.4500	1.5579	18.598	119.96	15
16	0.62317	12.561	86.347	0.04961	0.07961	6.8742	1.6047	20.156	138.56	16
17	0.60502	13.166	96.028	0.04595	0.07595	7.2935	1.6528	21.761	158.72	17
18	0.58739	13.753	106.01	0.04271	0.07271	7.7081	1.7024	23.414	180.48	18
19	0.57029	14.323	116.27	0.03981	0.06981	8.1178	1.7535	25.116	203.89	19
20	0.55368	14.877	126.79	0.03722	0.06722	8.5228	1.8061	26.870	229.01	20
21	0.53755	15.415	137.55	0.03487	0.06487	8.9230	1.8602	28.676	255.88	21
22	0.52189	15.936	148.50	0.03275	0.06275	9.3185	1.9161	30.536	284.55	22
23	0.50669	16.443	159.65	0.03081	0.06081	9.7093	1.9735	32.452	315.09	23
24	0.49193	16.935	170.97	0.02905	0.05905	10.095	2.0327	34.426	347.54	24
25	0.47761	17.413	182.43	0.02743	0.05743	10.476	2.0937	36.459	381.97	25
26	0.46369	17.876	194.02	0.02594	0.05594	10.853	2.1565	38.553	418.43	26
28	0.43708	18.764	217.53	0.02329	0.05329	11.593	2.2879	42.930	497.69	28
30	0.41199	19.600	241.36	0.02102	0.05102	12.314	2.4272	47.575	585.84	30
35	0.35538	21.487	301.62	0.01654	0.04654	14.037	2.8138	60.462	848.73	35
36	0.34503	21.832	313.70	0.01580	0.04580	14.368	2.8982	63.275	909.19	36
40	0.30665	23.114	351.75	0.01326	0.04326	15.650	3.2620	75.401	1180.0	40
45	0.26444	24.518	420.63	0.01079	0.04079	17.155	3.7816	92.719	1590.6	45
48	0.24200	25.266	455.02	0.00958	0.03958	18.008	4.1322	104.40	1880.2	48
50	0.22811	25.729	477.48	0.00887	0.03887	18.557	4.3839	112.79	2093.2	50
60	0.16973	27.675	583.05	0.00613	0.03613	21.067	5.8916	163.05	3435.1	60
70	0.12630	29.123	676.08	0.00434	0.03434	23.214	7.9178	230.59	5353.1	70
80	0.09398	30.200	756.08	0.00311	0.03311	23.035	10.640	321.36	8045.4	80
90	0.06693	31.002	823.63	0.00226	0.03226	26.566	14.300	443.34	11778.	90
100	0.05203	31.598	879.85	0.00165	0.03165	27.844	19.218	607.28	16909.	100
120	0.02881	32.373	963.86	0.00089	0.03089	29.773	34.711	1123.7	33456.	120
180	0.00489	33.170	1076.3	0.00015	0.03015	32.448	204.50	6783.4	220115	180
INF	0.00000	33.333	1111.1	0.00000	0.03000	33.333	INF	INF	INF	INF

	i = 4%			*i* = 4%			*i* = 4%			
	Present Sum (*P*)			Uniform Series (*A*)			Future Sum (*F*)			
n	***P/F***	***P/A***	***P/G***	***A/F***	***A/P***	***A/G***	***F/P***	***F/A***	***F/G***	***n***
1	0.96154	0.9615	0.0000	1.00000	1.04000	0.0000	1.0400	1.0000	0.0000	1
2	0.92456	1.8860	0.9245	0.49020	0.53020	0.4902	1.0816	2.0400	1.0000	2
3	0.88900	2.7750	2.7025	0.32035	0.36035	0.9738	1.1248	3.1216	3.0400	3
4	0.85480	3.6299	5.2669	0.23549	0.27549	1.4510	1.1669	4.2464	6.1616	4
5	0.82193	4.4518	8.5546	0.18463	0.22463	1.9216	1.2166	5.4163	10.408	5
6	0.79031	5.2421	12.506	0.15076	0.19076	2.3857	1.2653	6.6329	15.824	6
7	0.75992	6.0020	17.065	0.12661	0.16661	2.8433	1.3159	7.8982	22.457	7
8	0.73069	6.7327	22.180	0.10853	0.14853	3.2944	1.3685	9.2142	30.355	8
9	0.70259	7.4353	27.801	0.09499	0.13499	3.7390	1.4233	10.582	39.569	9
10	0.67556	8.1109	33.881	0.08329	0.12329	4.1772	1.4802	12.006	50.152	10
11	0.65958	8.7604	40.377	0.07415	0.11415	4.6090	1.5394	13.486	62.158	11
12	0.62460	9.3850	47.247	0.06655	0.10655	5.0343	1.6010	15.025	75.645	12
13	0.60057	9.9856	54.454	0.06014	0.10014	5.4532	1.6650	16.626	90.670	13
14	0.57748	10.563	61.981	0.05467	0.09467	5.8658	1.7316	18.291	107.29	14
15	0.55526	11.118	69.735	0.04994	0.08994	6.2720	1.8009	20.023	125.59	15
16	0.53391	11.652	77.744	0.04582	0.08582	6.6720	1.8729	21.824	145.61	16
17	0.51337	12.165	85.958	0.04220	0.08220	7.0656	1.9479	23.697	167.43	17
18	0.49363	12.659	94.349	0.03899	0.07899	7.4530	2.0258	25.645	191.13	18
19	0.47464	13.133	102.89	0.03614	0.07614	7.8341	2.1068	27.671	216.78	19
20	0.45639	13.590	111.56	0.03358	0.07358	8.2091	2.1911	29.788	244.45	20
21	0.43883	14.029	120.34	0.03128	0.07128	8.5779	2.2787	31.969	274.23	21
22	0.42196	14.451	129.20	0.02920	0.06920	8.9406	2.3699	34.248	306.19	22
23	0.40573	14.856	138.12	0.02731	0.06731	9.2972	2.4647	36.617	340.44	23
24	0.39012	15.247	147.10	0.02559	0.06559	9.6479	2.5633	39.082	377.06	24
25	0.37512	15.622	156.10	0.02401	0.06401	9.9925	2.6658	41.645	416.14	25
26	0.36069	15.982	165.12	0.02257	0.06257	10.331	2.7724	44.311	457.79	26
28	0.33348	16.663	183.14	0.02001	0.06001	10.990	2.9987	49.967	549.19	28
30	0.30832	17.292	201.06	0.01783	0.05783	11.627	3.2434	56.084	652.12	30
35	0.25342	18.664	224.87	0.01358	0.05358	13.119	3.9460	73.652	966.30	35
36	0.24367	18.908	253.40	0.01289	0.05289	13.401	4.1039	77.598	1039.0	36
40	0.20829	19.792	286.51	0.01052	0.05052	14.476	4.8010	95.025	1375.6	40
45	0.17120	20.720	325.40	0.00826	0.04826	15.704	5.8411	121.02	1900.7	45
48	0.15219	21.195	347.24	0.00718	0.04718	16.383	6.5705	139.26	2281.5	48
50	0.14071	21.482	361.16	0.00655	0.04655	16.812	7.1066	152.66	2566.6	50
55	0.11566	22.108	383.68	0.00523	0.04523	17.807	8.6463	191.15	3403.9	55
60	0.09506	22.623	422.99	0.00420	0.04420	18.697	10.519	237.99	4449.7	60
70	0.06422	23.394	472.47	0.00275	0.04275	20.196	15.571	364.29	7357.2	70
80	0.04338	23.915	511.11	0.00181	0.04181	21.371	23.049	551.24	11781.	80
90	0.02931	24.267	540.73	0.00121	0.04121	22.282	34.119	827.98	18449.	90
100	0.01980	24.505	463.12	0.00081	0.04081	22.980	50.504	1237.6	28440.	100
120	0.00904	24.774	592.24	0.00036	0.04036	23.905	110.66	2741.5	65539.	120
180	0.00086	24.978	620.59	0.00003	0.04003	24.845	1164.1	29078.	724456	180
INF	0.00000	25.000	625.00	0.00000	0.04000	25.000	INF	INF	INF	INF

	i = 5%			i = 5%			i = 5%			
	Present Sum (P)			Uniform Series (A)			Future Sum (F)			
n	*P/F*	*P/A*	*P/G*	*A/F*	*A/P*	*A/G*	*F/P*	*F/A*	*F/G*	*n*
1	0.95238	0.9523	0.0000	1.00000	1.05000	0.0000	1.0500	1.0000	0.0000	1
2	0.90703	1.8594	0.9070	0.48780	0.53780	0.4878	1.1025	2.0500	1.0000	2
3	0.86384	2.7232	2.6347	0.31721	0.36721	0.9674	1.1576	3.1525	3.0500	3
4	0.82270	3.5459	5.1028	0.23201	0.28201	1.4309	1.2155	4.3101	6.2025	4
5	0.78353	4.3294	8.2369	0.18097	0.23097	1.9025	1.2762	5.5256	10.512	5
6	0.74622	5.0756	11.968	0.14702	0.19702	2.3579	1.3401	6.8019	16.038	6
7	0.71068	5.7863	16.232	0.12282	0.17282	2.8052	1.4071	8.1420	22.840	7
8	0.67684	6.4632	20.970	0.10472	0.15472	3.2445	1.4774	9.5491	30.982	8
9	0.64461	7.1078	26.126	0.09069	0.14069	3.6757	1.5513	11.026	40.531	9
10	0.61391	7.7217	31.652	0.07950	0.12950	4.0990	1.6288	12.577	51.557	10
11	0.58468	8.3054	37.498	0.07039	0.12039	4.5144	1.7103	14.206	64.135	11
12	0.55684	8.8632	43.624	0.06283	0.11283	4.9219	1.7958	15.917	78.342	12
13	0.53032	9.3935	49.987	0.05646	0.10646	5.3215	1.8856	17.713	94.259	13
14	0.50507	9.8986	56.553	0.05102	0.10102	5.7132	1.9799	19.598	111.97	14
15	0.48102	10.379	63.288	0.04634	0.09634	6.0973	2.0789	21.578	131.57	15
16	0.45811	10.837	70.159	0.04227	0.09227	6.4736	2.1828	23.657	153.15	16
17	0.43630	11.274	77.140	0.03870	0.08870	6.8422	2.2920	25.840	176.80	17
18	0.41552	11.689	84.204	0.03555	0.08555	7.2033	2.4066	28.132	202.64	18
19	0.39573	12.085	91.327	0.03275	0.08275	7.5569	2.5269	30.539	230.78	19
20	0.37689	12.462	98.488	0.03024	0.08024	7.9029	2.6533	33.066	261.31	20
21	0.35894	12.821	105.66	0.02800	0.07800	8.2416	2.7859	35.719	294.38	21
22	0.34185	13.163	112.84	0.02597	0.07597	8.5729	2.9252	38.505	330.10	22
23	0.32557	13.488	120.00	0.02414	0.07414	8.8970	3.0715	41.430	368.61	23
24	0.31007	13.798	127.14	0.02247	0.07247	9.2139	3.2251	44.502	410.04	24
25	0.29530	14.093	134.22	0.02095	0.07095	9.5237	3.3863	47.727	454.54	25
25	0.28124	14.375	141.25	0.01956	0.06956	9.8265	3.3556	51.113	502.26	26
28	0.25509	14.898	155.11	0.01712	0.06712	10.411	3.9201	58.402	608.05	28
30	0.23138	15.372	168.62	0.01505	0.06505	10.969	4.3219	66.438	728.77	30
35	0.18129	16.374	200.58	0.01107	0.06107	12.249	5.5160	90.320	1106.4	35
36	0.17266	16.546	206.62	0.01043	0.06043	12.487	5.7918	95.836	1196.7	36
40	0.14205	17.159	229.54	0.00828	0.05828	13.377	7.0399	120.80	1616.0	40
45	0.11130	17.774	255.31	0.00626	0.05626	14.364	8.9850	159.70	2294.0	45
48	0.09614	18.077	269.24	0.00532	0.05532	14.894	10.401	188.02	2800.5	48
50	0.08720	18.255	277.91	0.00478	0.05478	15.223	11.467	209.34	3186.9	50
55	0.06833	18.633	297.51	0.00367	0.05367	15.966	14.635	272.71	4354.2	55
60	0.05354	18.929	314.34	0.00283	0.05283	16.606	18.679	353.58	5871.6	60
70	0.03287	19.432	340.84	0.00170	0.05170	17.621	30.426	588.52	10370.	70
80	0.02018	19.596	359.64	0.00103	0.05203	18.352	49.561	971.22	17824.	80
90	0.01239	19.752	372.74	0.00063	0.05063	18.781	80.730	1594.6	30092.	90
100	0.00760	19.847	381.74	0.00038	0.05038	19.233	131.50	2610.0	50200.	100
120	0.00287	19.942	391.97	0.00014	0.05014	19.655	348.91	6958.2	136765.	120
INF	0.00000	20.000	40.000	0.00000	0.05000	20.000	INF	INF	INF	INF

	i = 6%			*i* = 6%			*i* = 6%			
	Present Sum (*P*)			**Uniform Series (*A*)**			**Future Sum (*F*)**			
n	***P/F***	***P/A***	***P/G***	***A/F***	***A/P***	***A/G***	***F/P***	***F/A***	***F/G***	***n***
1	0.94340	0.9434	0.0000	1.00000	1.05000	0.0000	1.0600	1.0000	0.0000	1
2	0.89000	1.8333	0.8900	0.48544	0.54544	0.4854	1.1236	2.0600	1.0000	2
3	0.83962	2.6730	2.5692	0.31411	0.37411	0.9611	1.1910	3.1836	3.0600	3
4	0.79209	3.4651	4.9455	0.22859	0.28859	1.4272	1.2624	4.3746	6.2436	4
5	0.74726	4.2123	7.9345	0.17740	0.23740	1.8836	1.3382	5.6370	10.618	5
6	0.70496	4.9173	11.459	0.14336	0.20336	2.3304	1.4185	6.9753	16.255	6
7	0.66506	5.5823	15.449	0.11914	0.17914	2.7675	1.5036	8.3938	23.230	7
8	0.62741	6.2097	19.841	0.10104	0.16104	3.1952	1.5938	9.8974	31.524	8
9	0.59190	6.8016	24.576	0.08702	0.14702	3.6133	1.6894	11.491	41.521	9
10	0.55839	7.3600	29.602	0.07587	0.13587	4.0220	1.7908	13.180	53.013	10
11	0.52679	7.8868	34.870	0.06679	0.12679	4.4212	1.8983	14.971	66.194	11
12	0.49697	8.3838	40.336	0.05928	0.11928	4.8112	2.0122	16.869	81.165	12
13	0.46884	8.8526	45.962	0.05296	0.11296	5.1919	2.1329	18.882	98.035	13
14	0.44230	9.2949	51.712	0.04758	0.10758	5.5635	2.2609	21.015	116.91	14
15	0.41727	9.7122	57.554	0.04296	0.10296	5.9259	2.3965	23.276	137.93	15
16	0.39365	10.105	63.459	0.03895	0.09895	6.2794	2.5403	25.672	161.20	16
17	0.37136	10.477	69.401	0.03544	0.09544	6.6239	2.6927	28.212	186.88	17
18	0.35034	10.827	75.356	0.03236	0.09236	6.9597	2.8543	30.905	215.09	18
19	0.33051	11.158	81.306	0.02962	0.08962	7.2867	3.0256	33.760	246.00	19
20	0.31180	11.469	97.230	0.02718	0.08718	7.6051	3.2071	36.785	279.76	20
21	0.29416	11.764	93.113	0.02500	0.08500	7.9150	3.3995	39.992	315.54	21
22	0.22751	12.041	98.941	0.02305	0.08305	8.2166	3.6035	42.392	356.53	22
23	0.26180	12.303	104.70	0.02128	0.08128	8.5099	3.8197	46.995	399.92	23
24	0.24698	12.550	110.38	0.01968	0.07968	8.7950	4.0489	50.815	446.92	24
25	0.23300	12.783	115.97	0.01823	0.07823	9.0722	4.2918	54.864	497.74	25
26	0.21981	13.003	121.46	0.01690	0.07690	8.3414	4.5493	59.156	552.60	26
27	0.20737	13.210	126.86	0.01570	0.07570	9.6029	4.8223	63.705	611.76	27
28	0.19563	13.406	132.14	0.01459	0.07459	9.8568	5.1116	68.528	675.46	28
29	0.18456	13.590	137.31	0.01358	0.07358	10.103	5.4183	73.639	743.99	29
30	0.17411	13.764	142.35	0.01265	0.07265	10.342	5.7434	79.058	817.63	30
35	0.13011	14.498	155.74	0.00897	0.06897	11.431	7.6860	111.43	1273.9	35
40	0.09722	15.046	185.95	0.00646	0.06646	12.359	10.285	154.76	1912.7	40
45	0.07265	15.455	203.11	0.00470	0.06470	13.141	13.764	212.74	2795.7	45
50	0.05429	15.761	217.45	0.00344	0.06344	13.786	18.420	290.33	4005.6	50
55	0.04057	15.990	229.32	0.00254	0.06254	14.341	24.650	394.17	5652.8	55
60	0.03031	16.161	239.04	0.00188	0.06188	14.790	32.987	533.12	7885.4	60
65	0.02265	16.289	246.94	0.00139	0.06139	15.160	44.145	719.08	10901.	65
70	0.01693	16.384	253.32	0.00103	0.06103	15.461	59.075	967.93	14965.	70
80	0.00945	16.509	262.54	0.00057	0.06057	15.903	105.79	1746.6	27776.	80
90	0.00528	16.578	268.39	0.00032	0.06032	16.189	189.46	3141.0	50851.	90
100	0.00295	16.617	272.04	0.00018	0.06018	16.371	339.30	5638.3	92306.	100
120	0.00092	16.651	275.68	0.00006	0.06006	16.556	1088.1	18119.	299997.	120
INF	0.00000	16.666	277.77	0.00000	0.06000	16.666	INF	INF	INF	INF

	i = 7%			i = 7%			i = 7%			
	Present Sum (*P*)			Uniform Series (*A*)			Future Sum (*F*)			
n	***P/F***	***P/A***	***P/G***	***A/F***	***A/P***	***A/G***	***F/P***	***F/A***	***F/G***	***n***
1	0.93458	0.9345	0.0000	1.00000	1.07000	0.0000	1.0700	1.0000	0.0000	1
2	0.87344	1.8080	0.8734	0.48309	0.55309	0.4830	1.1449	2.0700	1.0000	2
3	0.81630	2.6243	2.5060	0.31105	0.38105	0.9549	1.2250	3.2149	3.0700	3
4	0.76290	3.3872	4.7947	0.22523	0.29523	1.4155	1.3108	4.4399	6.2849	4
5	0.71299	4.1002	7.6466	0.17389	0.24389	1.8649	1.4025	5.7507	10.724	5
6	0.66634	4.7665	10.978	0.13980	0.20980	2.3032	1.5007	7.1532	16.475	6
7	0.62275	5.3893	14.714	0.11555	0.18555	2.7303	1.6057	8.6540	23.628	7
8	0.58201	5.9713	18.788	0.09747	0.16747	3.1465	1.7181	10.259	32.282	8
9	0.54393	6.5152	23.140	0.08349	0.15349	3.5517	1.8384	11.978	42.542	9
10	0.50835	7.0235	27.715	0.07238	0.14238	3.9460	1.9671	13.816	54.520	10
11	0.47509	7.4986	32.466	0.06336	0.13336	4.3296	2.1048	15.783	68.337	11
12	0.44401	7.9426	37.350	0.05590	0.12590	4.7025	2.2521	17.888	84.120	12
13	0.41496	8.3576	42.330	0.04965	0.11965	5.0648	2.4098	20.140	102.00	13
14	0.38782	8.7454	47.371	0.04434	0.11434	5.4167	2.5785	22.550	122.15	14
15	0.36245	0.1079	52.446	0.03979	0.10979	5.7582	2.7590	24.129	144.70	15
16	0.33873	9.4466	57.527	0.03586	0.10586	6.0896	2.9521	27.888	169.83	16
17	0.31657	9.7632	62.592	0.03243	0.10243	6.4110	3.1588	30.840	197.71	17
18	0.29586	10.059	67.621	0.02941	0.09941	6.7224	3.3799	33.999	228.55	18
19	0.27651	10.335	72.599	0.02675	0.09675	7.0241	3.6165	37.379	262.55	19
20	0.25842	10.594	77.509	0.02439	0.09439	7.3163	3.8696	40.995	299.93	20
21	0.24151	10.835	82.339	0.02229	0.09229	7.5990	4.1405	44.865	349.93	21
22	0.22571	11.061	87.079	0.02041	0.09041	7.8424	4.4304	49.005	385.79	22
23	0.21095	11.272	91.720	0.01871	0.08871	8.1368	4.7405	53.436	434.80	23
24	0.19715	11.469	06.254	0.01719	0.08719	8.3923	5.0723	58.176	477.23	24
25	0.18425	11.653	100.67	0.01581	0.08581	8.6391	5.4274	63.249	546.41	25
26	0.17220	11.825	104.98	0.01456	0.08456	8.8773	5.8073	68.676	609.66	26
27	0.16093	11.986	109.16	0.01343	0.08343	9.1072	6.2138	74.483	678.34	27
28	0.15040	12.137	113.22	0.01239	0.08239	9.3289	6.6488	80.697	752.82	28
29	0.14056	12.277	117.16	0.01145	0.08145	9.5427	7.1142	87.346	833.52	29
30	0.13137	12.409	120.97	0.01059	0.08059	9.7486	7.6122	94.460	920.86	30
35	0.09366	12.947	138.13	0.00723	0.07723	10.668	10.676	138.23	1474.8	35
40	0.06678	13.331	152.29	0.00501	0.07501	11.423	14.974	199.63	2280.5	40
45	0.04761	13.605	163.75	0.00350	0.07350	12.036	21.002	285.74	3439.2	45
50	0.03395	13.800	172.90	0.00246	0.07246	12.528	29.457	406.52	5093.2	50
55	0.02420	13.949	180.12	0.00174	0.07174	12.921	41.315	575.92	7441.8	55
60	0.01726	14.039	185.76	0.00123	0.07123	13.232	57.946	813.52	10764.	60
65	0.01230	14.109	190.14	0.00087	0.07087	13.476	81.272	1146.7	15453.	65
70	0.00877	15.160	193.51	0.00062	0.07062	13.666	113.93	1614.1	22059	70
80	0.00446	14.222	198.07	0.00031	0.07031	13.927	224.23	3189.0	44415.	80
90	0.00227	14.253	200.70	0.00016	0.07016	14.081	441.10	6287.1	88531.	90
100	0.00115	14.269	202.20	0.00008	0.07008	14.170	867.71	12381.	175452	100
120	0.00030	14.281	203.51	0.00002	0.07002	14.250	3357.7	47954.	683345	120
INF	0.00000	14.285	204.08	0.00000	0.07000	14.285	INF	INF	INF	INF

	i = 8%			*i* = 8%			*i* = 8%			
	Present Sum (*P*)			**Uniform Series (*A*)**			**Future Sum (*F*)**			
n	***P/F***	***P/A***	***P/G***	***A/F***	***A/P***	***A/G***	***F/P***	***F/A***	***F/G***	***n***
1	0.92593	0.9259	0.0000	1.00000	1.08000	0.0000	1.0800	1.0000	0.0000	1
2	0.85734	1.7832	0.8573	0.48077	0.56077	0.4807	1.1664	2.0800	1.0000	2
3	0.79383	2.5771	2.4450	0.30803	0.38803	0.9487	1.2597	3.2464	3.0800	3
4	0.73503	3.3121	4.6500	0.22192	0.30192	1.4039	1.3604	3.5061	6.3264	4
5	0.68058	3.9927	7.3724	0.17046	0.25046	1.8464	1.4693	5.8666	10.832	5
6	0.63107	4.6228	10.523	0.13632	0.21632	2.2763	1.5868	7.3359	16.699	6
7	0.58349	5.2063	14.024	0.11207	0.19207	2.6936	1.7138	8.9228	24.035	7
8	0.54027	5.7466	17.806	0.09401	0.17401	3.0985	1.8509	10.636	32.957	8
9	0.50025	6.2468	21.808	0.08008	0.16008	3.4910	1.9990	12.487	43.594	9
10	0.46319	6.7100	25.976	0.06903	0.14903	3.8713	2.1589	14.486	56.082	10
11	0.42888	5.1389	30.265	0.06008	0.14003	4.2395	2.3316	16.645	70.568	11
12	0.39711	7.5360	34.633	0.05270	0.13270	4.5957	2.5181	18.977	87.214	12
13	0.36770	4.9037	39.046	0.04652	0.12652	4.9402	2.7196	21.495	106.19	13
14	0.34046	8.2442	43.472	0.04130	0.12130	5.2730	2.9371	24.214	127.68	14
15	0.31524	8.5594	47.885	0.03683	0.11683	5.5944	3.1721	27.152	151.90	15
16	0.29189	8.8513	52.264	0.03298	0.11298	5.9046	3.4259	30.324	179.05	16
17	0.27027	9.1216	56.588	0.02963	0.10963	6.2037	3.7000	33.750	209.37	17
18	0.25025	9.3718	60.842	0.02670	0.10670	6.4920	3.9960	37.450	243.12	18
19	0.23171	9.6036	65.013	0.02413	0.10413	6.7696	4.3157	41.446	280.57	19
20	0.21455	9.8181	69.089	0.02185	0.10185	7.0369	4.6609	45.762	322.02	20
21	0.19866	10.016	73.062	0.01981	0.09983	7.2940	5.0338	50.422	367.78	21
22	0.18394	10.200	76.925	0.01803	0.09803	7.5411	5.4365	55.456	418.20	22
23	0.17032	10.371	80.672	0.01642	0.09642	7.7786	5.8714	60.893	473.66	23
24	0.15770	10.528	84.299	0.01498	0.09498	8.0066	6.3411	66.764	534.55	24
25	0.14602	10.674	87.804	0.01368	0.09368	8.2253	6.8484	7.3105	601.32	25
26	0.13520	10.810	91.184	0.01251	0.09251	8.4351	7.3963	79.954	674.43	26
27	0.12519	10.935	94.439	0.01145	0.09145	8.6362	7.9880	87.350	754.38	27
28	0.11591	11.051	97.568	0.01049	0.09049	8.8288	8.6271	95.338	841.73	28
29	0.10733	11.158	100.57	0.00962	0.08962	9.0132	9.3172	103.96	937.07	29
30	0.09938	11.257	103.45	0.00883	0.08883	9.1897	10.062	113.28	1041.0	30
35	0.06763	11.654	116.09	0.00580	0.08580	9.9610	14.785	172.31	1716.4	35
40	0.04603	11.924	126.04	0.00386	0.08386	10.569	21.724	259.05	2738.2	40
45	0.03133	12.108	133.73	0.00259	0.08259	11.044	31.920	386.50	4268.8	45
50	0.02132	12.233	139.59	0.00174	0.08174	11.410	46.901	573.77	6547.1	50
55	0.01451	12.318	144.00	0.00118	0.08118	11.690	68.913	848.92	9924.0	55
60	0.00988	12.376	147.30	0.00080	0.08080	11.901	101.25	1253.2	14915.	60
65	0.00672	12.416	149.73	0.00054	0.08054	12.060	148.78	1847.2	22278.	65
70	0.00457	12.442	151.53	0.00037	0.08037	12.178	218.60	2720.0	33126	70
80	0.00212	12.473	153.80	0.00017	0.08017	12.330	471.95	5866.9	72586.	80
90	0.00098	12.487	154.99	0.00008	0.08008	12.411	1018.9	12723.	157924.	90
100	0.00045	12.494	155.61	0.00004	0.08004	12.454	2199.7	27484.	342306.	100
INF	0.00000	12.500	156.25	0.00000	0.08000	12.500	INF	INF	INF	INF

	i = 9%			i = 9%			i = 9%			
	Present Sum (P)			Uniform Series (A)			Future Sum (F)			
n	P/F	P/A	P/G	A/F	A/P	A/G	F/P	F/A	F/G	n
1	0.91743	0.9174	0.0000	1.00000	1.09000	0.0000	1.0900	1.0000	0.0000	1
2	0.84168	1.7591	0.8416	0.47847	0.56847	0.4783	1.1881	2.0900	1.0000	2
3	0.77218	2.5312	2.3860	0.30505	0.39505	0.9426	1.2950	3.2781	3.0900	3
4	0.70843	3.2397	4.5113	0.21867	0.30867	1.3925	1.4115	4.5731	6.3681	4
5	0.64993	3.8896	7.1110	0.16709	0.25709	1.8282	1.5486	5.9847	10.941	5
6	0.59627	4.4859	10.092	0.13292	0.22292	2.2497	1.6771	7.5233	16.925	6
7	0.54703	5.0329	13.374	0.10869	0.19869	2.6574	1.8280	9.2004	24.449	7
8	0.50187	5.5348	16.887	0.09067	0.18067	3.0511	1.9925	11.028	33.649	8
9	0.46043	5.9952	20.571	0.07680	0.16680	3.4312	2.1718	13.021	44.678	9
10	0.42241	6.4176	24.372	0.06582	0.15582	3.7977	2.3673	15.192	57.699	10
11	0.38753	6.8051	28.248	0.05695	0.14695	4.1509	2.5804	17.560	72.892	11
12	0.35553	7.1607	32.159	0.04965	0.13965	4.4910	2.8126	20.140	90.452	12
13	0.32618	7.4869	36.073	0.04357	0.13357	4.8181	3.0658	22.953	110.59	13
14	0.29925	7.7861	39.963	0.03843	0.12843	5.1326	3.3417	26.019	133.54	14
15	0.27454	8.0606	43.806	0.03406	0.12406	5.4346	3.6424	29.360	159.56	15
16	0.25187	8.3125	47.584	0.03030	0.12030	5.7244	3.9703	33.003	188.92	16
17	0.23107	8.5436	51.282	0.02705	0.11705	6.0023	4.3276	36.973	221.93	17
18	0.21199	8.7556	54.886	0.02421	0.11421	6.2686	4.7171	41.301	258.90	18
19	0.19449	8.9501	58.386	0.02173	0.11173	6.5235	5.1416	46.018	300.20	19
20	0.17843	9.1285	61.777	0.01955	0.10955	6.7674	5.6044	51.160	346.22	20
21	0.16370	9.2922	65.050	0.01762	0.10762	7.0005	6.1088	56.764	397.38	21
22	0.15018	9.4424	68.204	0.01590	0.10590	7.2232	6.6586	62.873	454.14	22
23	0.13778	9.5802	71.235	0.01438	0.10438	7.4357	7.2578	69.531	517.02	23
24	0.12640	9.7066	74.143	0.01302	0.10302	7.6384	7.9110	76.789	586.55	24
25	0.15597	9.8225	76.926	0.01181	0.10181	7.8316	8.6230	84.700	663.34	25
26	0.10639	9.9289	79.586	0.01072	0.10072	8.0155	9.3991	93.324	748.04	26
27	0.09761	10.026	82.124	0.00973	0.09973	8.1906	10.245	102.72	841.36	27
28	0.08955	10.116	84.541	0.00885	0.09885	8.3571	11.167	112.96	944.09	28
29	0.08215	10.198	86.842	0.00806	0.09806	8.5153	12.172	124.13	1057.0	29
30	0.07537	10.273	89.028	0.00734	0.09734	8.6656	13.267	136.30	1181.1	30
35	0.04899	10.566	98.359	0.00464	0.09464	9.3082	20.414	215.71	2007.9	35
40	0.03184	10.757	105.37	0.00296	0.09296	9.7957	31.409	337.88	3309.8	40
45	0.02069	10.881	110.55	0.00190	0.09190	10.160	48.327	525.85	5342.8	45
50	0.01345	10.961	114.32	0.00123	0.09123	10.429	74.357	815.08	8500.9	50
55	0.00874	11.014	117.03	0.00079	0.09079	10.626	114.40	1260.0	13389.	55
60	0.00568	11.048	118.96	0.00051	0.09051	10.768	176.03	1944.7	20942.	60
65	0.00369	11.070	120.33	0.00033	0.09033	10.870	270.84	2998.2	32592.	65
70	0.00240	11.084	121.29	0.00022	0.09022	10.942	416.73	4619.2	50546.	70
80	0.00101	11.099	122.43	0.00009	0.09009	11.029	986.55	10950.	120784.	80
90	0.00043	11.106	122.97	0.00004	0.09004	11.072	2335.5	25939	287213.	90
100	0.00018	11.109	123.23	0.00002	0.09002	11.093	5529.0	61422.	681363.	100
INF	0.00000	11.111	123.45	0.00000	0.09000	11.111	INF	INF	INF	INF

	i = 10%			i = 10%			i = 10%			
	Present Sum (P)			Uniform Series (A)			Future Sum (F)			
n	P/F	P/A	P/G	A/F	A/P	A/G	F/P	F/A	F/G	n
1	0.90909	0.9090	0.0000	1.00000	1.10000	0.0000	1.1000	1.0000	0.0000	1
2	0.82645	1.7355	0.8264	0.47619	0.57619	0.4761	1.2100	2.1000	1.0000	2
3	0.75131	2.4868	2.3290	0.30211	0.40211	0.9365	1.3310	3.3100	3.1000	3
4	0.68301	3.1698	4.3781	0.21547	0.31547	1.3811	1.4641	4.6410	6.4100	4
5	0.62092	3.7907	6.8618	0.16380	0.26380	1.8101	1.6105	6.1051	11.051	5
6	0.56447	4.3552	9.6842	0.12961	0.22961	2.2235	1.7715	7.1756	17.156	6
7	0.51316	4.8684	12.763	0.10541	0.20541	2.6216	1.9487	9.4871	24.871	7
8	0.46651	5.3349	16.028	0.08744	0.18744	3.0044	2.1435	11.435	34.358	8
9	0.42410	5.7590	19.421	0.07364	0.17364	3.3723	2.3579	13.579	45.794	9
10	0.38554	6.1445	22.891	0.06275	0.16275	3.7254	2.5937	15.937	59.374	10
11	0.35049	6.4950	26.396	0.05396	0.15396	4.0640	2.8531	18.531	75.311	11
12	0.31863	6.8136	29.901	0.04676	0.14676	4.3884	3.1384	21.384	93.842	12
13	0.28966	7.1033	33.377	0.04708	0.14078	4.6987	3.4522	24.522	115.22	13
14	0.26333	7.3666	36.800	0.03575	0.13575	4.9955	3.7975	27.975	139.75	14
15	0.23939	7.6060	40.152	0.03147	0.13147	5.2789	4.1772	31.772	167.72	15
16	0.21763	7.8237	43.416	0.02782	0.12782	5.5493	4.5949	35.949	199.49	16
17	0.19784	8.0215	46.581	0.02466	0.12466	5.8071	5.0544	40.544	235.44	17
18	0.17986	8.2014	40.639	0.02193	0.12193	6.0525	5.5599	45.599	275.99	18
19	0.16351	8.3649	52.582	0.01955	0.11955	6.2681	6.1159	51.159	321.59	19
20	0.14864	8.5136	55.406	0.01746	0.11746	6.5080	6.7275	57.275	372.75	20
21	0.13513	8.6486	58.109	0.01562	0.11562	6.7188	7.4002	64.002	430.02	21
22	0.12285	8.7715	60.689	0.01401	0.11401	6.9188	8.1402	71.402	494.02	22
23	0.11168	8.8832	63.146	0.01257	0.11257	7.1084	8.9543	59.543	565.43	23
24	0.10153	8.9847	65.481	0.01130	0.11130	7.2880	9.8497	88.497	644.97	24
25	0.09230	9.0770	67.696	0.01017	0.11017	7.4579	10.834	98.347	733.47	25
26	0.08391	9.1609	69.794	0.00916	0.10916	6.4186	11.918	109.18	831.81	26
27	0.07628	9.3272	71.777	0.00826	0.10826	7.7704	13.110	121.10	940.99	27
28	0.06934	9.3065	73.659	0.00745	0.10745	7.9137	14.421	134.21	1062.1	28
29	0.06304	9.3696	75.414	0.00673	0.10673	8.0488	15.863	148.63	1196.3	29
30	0.05731	9.4269	77.076	0.00608	0.10608	8.1762	17.449	164.49	1344.9	30
35	0.03558	9.6441	83.987	0.00369	0.10369	8.7086	28.102	271.01	2360.2	35
40	0.02209	9.7790	88.952	0.00226	0.10226	9.0962	45.259	442.59	4025.9	40
45	0.01372	9.8628	92.454	0.00139	0.10139	9.3740	72.890	718.90	6739.0	45
50	0.00852	9.9148	94.888	0.00086	0.10086	9.5704	117.39	1163.0	11139.	50
55	0.00529	9.9471	96.561	0.00053	0.10053	9.7075	189.05	1880.5	18255.	55
60	0.00328	9.9671	97.701	0.00033	0.10033	9.8022	304.48	3034.8	29748.	60
65	0.00204	9.9796	98.470	0.00020	0.10020	9.8671	490.37	4893.7	48287.	65
70	0.00127	9.9873	98.987	0.00013	0.10013	9.9112	789.74	7887.4	78174.	70
80	0.00049	9.9951	99.560	0.00005	0.10005	9.9609	2048.4	20474.	203940.	80
90	0.00019	9.9981	99.811	0.00002	0.10002	9.9830	5313.0	53120.	530302.	90
INF	0.00000	10.000	100.00	0.00000	0.10000	10.000	INF	INF	INF	INF

	i = 11%			i = 11%			i = 11%			
	Present Sum (P)			Uniform Series (A)			Future Sum (F)			
n	P/F	P/A	P/G	A/F	A/P	A/G	F/P	F/A	F/G	n
1	0.90090	0.9009	0.0000	1.00000	1.11000	0.0000	1.1100	1.0000	0.0000	1
2	0.81162	1.7125	0.8116	0.47393	0.58393	0.4739	1.2321	2.1100	1.0000	2
3	0.73119	2.4437	2.2740	0.29921	0.40921	0.9305	1.3676	3.3421	3.1100	3
4	0.65873	3.1024	4.2502	0.21233	0.32233	1.3699	1.5180	4.7097	6.4521	4
5	0.59345	3.6959	6.6240	0.16057	0.27057	1.7922	1.6850	6.2278	11.161	5
6	0.53464	4.2305	9.2972	0.12638	0.23638	2.1976	1.8704	7.9128	17.389	6
7	0.48166	4.7122	12.187	0.10222	0.21222	2.5863	2.0761	9.7832	25.302	7
8	0.43393	5.1461	15.224	0.08432	0.19432	2.9584	2.3045	11.859	35.085	8
9	0.39092	5.5370	18.352	0.07060	0.18060	3.3144	2.5580	14.164	46.945	9
10	0.35218	5.8892	21.521	0.05980	0.16980	3.6544	2.8394	16.722	61.109	10
11	0.31728	6.2065	24.694	0.05112	0.16112	3.9788	3.1517	19.561	77.831	11
12	0.28584	6.4923	27.838	0.04403	0.15403	4.2879	3.4984	22.713	97.392	12
13	0.25751	6.7498	30.929	0.03815	0.14815	4.5821	3.8832	26.211	120.10	13
14	0.23199	6.9818	33.944	0.03323	0.14323	4.8618	4.3104	30.094	146.31	14
15	0.20900	7.1908	36.870	0.02907	0.13907	5.1274	4.7845	34.405	176.41	15
16	0.18829	7.3791	39.695	0.02552	0.13552	5.3793	5.3108	39.189	210.81	16
17	0.16963	7.5487	42.409	0.02247	0.13247	5.6180	5.8950	44.500	250.00	17
18	0.15282	7.7016	45.007	0.01984	0.12984	5.8438	6.5435	50.395	294.50	18
19	0.13768	7.8392	47.485	0.01756	0.12756	6.0573	7.2633	56.939	344.90	19
20	0.12403	7.9633	49.842	0.01558	0.12558	6.2589	8.0623	64.202	401.84	20
21	0.11174	8.0750	52.077	0.01384	0.12384	6.4491	8.9491	72.265	466.04	21
22	0.10067	8.1757	54.191	0.01231	0.12231	6.6282	9.9335	81.214	538.31	22
23	0.09069	8.2664	56.186	0.01097	0.12097	6.7969	11.026	91.147	619.52	23
24	0.08170	8.3481	58.065	0.00979	0.11979	6.9555	12.239	102.17	710.67	24
25	0.07361	8.4217	59.832	0.00874	0.11874	7.1044	13.585	114.41	812.84	25
26	0.06631	8.4880	61.490	0.00781	0.11781	7.2443	15.079	127.99	927.26	26
27	0.05974	8.5478	63.043	0.00699	0.11699	7.3753	16.738	143.07	1055.2	27
28	0.05382	8.6016	64.496	0.00626	0.11626	7.4981	18.579	159.81	1198.3	28
29	0.04849	8.6501	65.854	0.00561	0.11561	7.6131	20.623	178.39	1358.1	29
30	0.04368	8.6937	67.121	0.00502	0.11502	7.7205	22.892	199.02	1536.5	30
35	0.02592	8.8552	72.253	0.00293	0.11293	8.1594	38.574	341.59	2787.1	35
40	0.01538	8.9510	75.778	0.00172	0.11172	8.4659	65.000	581.82	4925.6	40
45	0.00913	9.0079	78.155	0.00101	0.11101	8.6762	109.53	986.63	8560.3	45
50	0.00542	9.0416	79.734	0.00060	0.11060	8.8185	184.56	1668.7	14715.	50
55	0.00322	9.0616	70.771	0.00035	0.11035	8.9134	311.00	2818.2	25120.	55
60	0.00191	9.0735	81.446	0.00021	0.11021	8.9762	524.05	4755.0	42682.	60
65	0.00113	9.0806	81.881	0.00012	0.11012	9.0172	883.06	8018.7	72307.	65
70	0.00067	9.0848	82.161	0.00007	0.11007	9.0438	1488.0	13518.	122258.	70
INF	0.00000	9.0909	82.644	0.00000	0.11000	9.0909	INF	INF	INF	INF

	i = 12%			i = 12%			i = 12%			
	Present Sum (P)			Uniform Series (A)			Future Sum (F)			
n	P/F	P/A	P/G	A/F	A/P	A/G	F/P	F/A	F/G	n
1	0.89286	0.8928	0.0000	1.00000	1.12000	0.0000	1.1200	1.0000	0.0000	1
2	0.79719	1.6900	0.7971	0.47170	0.59170	0.4717	1.2544	2.1200	1.1000	2
3	0.71178	2.4018	2.2207	0.29635	0.41635	0.9246	1.4049	3.3744	3.1200	3
4	0.63552	3.0373	4.1273	0.20923	0.32923	1.3588	1.5735	4.7793	6.4944	4
5	0.56743	3.6047	6.3970	0.15741	0.27741	1.7745	1.7623	6.3528	11.273	5
6	0.50663	4.1114	8.9301	0.12323	0.24323	2.1720	1.9738	8.1151	17.626	6
7	0.45235	4.5637	11.644	0.09912	0.21912	2.5514	2.2106	10.089	25.741	7
8	0.40388	4.9676	14.471	0.08130	0.20130	2.9131	2.4759	12.299	35.830	8
9	0.36061	5.3282	17.356	0.06768	0.18768	3.2574	2.7730	14.775	48.130	9
10	0.32197	5.6502	20.254	0.05698	0.17698	3.5846	3.1058	17.548	62.906	10
11	0.28748	5.9377	23.128	0.04842	0.16842	3.8952	3.4785	20.654	80.454	11
12	0.25668	6.1943	25.952	0.04144	0.16144	4.1896	3.8959	24.133	101.10	12
13	0.22917	6.4235	28.702	0.03568	0.15568	4.4683	4.3634	28.029	125.24	13
14	0.20462	6.6281	31.362	0.03087	0.15087	4.7316	4.8871	32.392	153.27	14
15	0.18270	6.8108	33.920	0.02682	0.14682	4.9803	5.4735	37.279	185.66	15
16	0.16312	6.9739	36.367	0.02339	0.14339	5.2146	6.1303	42.753	222.94	16
17	0.14564	7.1196	38.697	0.02046	0.14046	5.4353	6.8660	48.883	265.69	17
18	0.13004	7.2496	40.908	0.01794	0.13794	5.6427	7.6899	55.749	314.58	18
19	0.11611	7.3657	42.997	0.01576	0.13576	5.8375	8.6127	63.439	370.33	19
20	0.10367	7.4694	44.967	0.01388	0.13388	6.0202	9.6462	72.052	433.77	20
21	0.09256	7.5620	46.818	0.01224	0.13224	6.1913	10.803	81.698	505.82	21
22	0.08264	7.6446	48.554	0.01081	0.13081	6.3514	12.100	92.502	587.52	22
23	0.07379	7.7184	50.177	0.00956	0.12956	6.5010	13.552	104.60	680.01	23
24	0.06588	7.7843	51.692	0.00846	0.12846	6.6406	15.178	118.15	784.62	24
25	0.05882	7.8431	53.104	0.00750	0.12750	6.7708	17.000	133.33	902.78	25
26	0.05252	7.8956	54.417	0.00665	0.12665	6.8921	19.040	150.33	1036.1	26
27	0.04689	7.9425	55.636	0.00590	0.12590	7.0049	21.324	169.37	1186.4	27
38	0.04187	7.9844	56.767	0.00524	0.12524	7.1097	23.883	190.69	1355.8	28
29	0.03738	8.0218	57.814	0.00466	0.12466	7.2071	26.749	214.58	1546.5	29
30	0.03338	8.0551	58.782	0.00414	0.12414	7.2974	29.959	241.33	1761.1	30
35	0.01894	8.1755	62.605	0.00232	0.12232	7.6576	52.799	431.66	3305.5	35
40	0.01075	8.2437	65.113	0.00130	0.12130	7.8987	93.051	767.09	6059.1	40
45	0.00610	8.2525	66.734	0.00074	0.12074	8.0572	163.98	1358.2	10943.	45
50	0.00346	8.3045	67.762	0.00042	0.12042	8.1597	289.00	2400.0	19583.	50
55	0.00196	8.3169	68.408	0.00024	0.12024	8.2251	509.32	4236.0	34841.	55
60	0.00111	8.3240	68.810	0.00013	0.12013	8.2664	897.59	7471.6	61763.	60
65	0.00063	8.3280	69.058	0.00008	0.12008	8.2922	1581.8	13173.	109241.	65
70	0.00036	8.3303	69.210	0.00004	0.12004	8.3082	2787.8	23223.	192944.	70
INF	0.00000	8.3333	69.4444	0.00000	0.12000	8.3333	INF	INF	INF	INF

	i = 13%			i = 13%			i = 13%			
	Present Sum (P)			Uniform Series (A)			Future Sum (F)			
n	P/F	P/A	P/G	A/F	A/P	A/G	F/P	F/A	F/G	n
1	0.88496	0.8849	0.0000	1.00000	1.13000	0.0000	1.1300	1.0000	0.0000	1
2	0.78315	1.6681	0.7831	0.46948	0.59948	0.4694	1.2769	2.1300	1.0000	2
3	0.69305	2.3611	2.1692	0.29352	0.42352	0.9187	1.4429	3.4069	3.1300	3
4	0.61332	2.9744	4.0092	0.20619	0.33619	1.3478	1.6304	4.8498	6.5369	4
5	0.54276	3.5172	6.1802	0.15431	0.28431	1.7571	1.8424	6.4802	11.386	5
6	0.48032	3.9975	8.5818	0.12015	0.25015	2.1467	2.0819	8.3227	17.857	6
7	0.42506	4.4226	11.132	0.09611	0.22611	2.5171	2.3526	10.404	26.189	7
8	0.37616	4.7987	13.765	0.07839	0.20839	2.8685	2.6584	12.757	36.594	8
9	0.33288	5.1316	16.428	0.06487	0.19487	3.2013	3.0040	15.415	49.351	9
10	0.29459	5.4262	19.079	0.05429	0.18429	3.5161	3.3945	18.419	64.767	10
11	0.26070	5.6869	21.686	0.04584	0.17584	3.8134	3.8358	21.814	83.187	11
12	0.23071	5.9176	24.224	0.03899	0.16899	4.0935	4.3345	25.650	105.00	12
13	0.20416	6.1218	26.674	0.03335	0.16335	4.3572	4.8980	29.984	130.65	13
14	0.18068	6.3024	29.023	0.02867	0.15867	4.6050	5.5347	34.882	160.63	14
15	0.15989	6.4623	31.261	0.02474	0.15474	4.8374	6.2542	40.417	195.51	15
16	0.14150	6.6038	33.384	0.02143	0.15143	5.0552	5.0673	46.671	235.93	16
17	0.12522	6.7290	35.387	0.01861	0.14861	5.2589	7.9860	53.739	282.60	17
18	0.11081	6.8399	37.271	0.01620	0.14620	5.4491	9.0242	61.725	336.34	18
19	0.09806	6.9379	39.036	0.01413	0.14413	5.6265	10.197	70.749	398.07	19
20	0.08678	7.0247	40.685	0.01235	0.14235	5.7917	11.523	80.946	468.82	20
21	0.07680	7.1015	42.221	0.01081	0.14081	5.9453	13.021	92.469	549.76	21
22	0.06796	7.1695	43.648	0.00948	0.13948	6.0880	14.713	105.49	642.23	22
23	0.06014	7.2296	44.971	0.00832	0.13832	6.2204	16.626	120.20	747.73	23
24	0.05323	7.2828	45.196	0.00731	0.13731	6.3430	18.788	136.83	867.93	24
25	0.04710	7.3299	47.326	0.00643	0.13643	6.4565	21.230	155.62	1004.7	25
26	0.04168	7.3716	48.368	0.00565	0.13565	6.5614	23.990	176.85	1160.3	26
27	0.03689	7.4085	49.327	0.00498	0.13498	6.6581	27.109	200.84	1337.2	27
28	0.03264	7.4412	50.209	0.00439	0.13439	6.7474	30.633	227.95	1538.0	28
29	0.02880	7.4700	51.017	0.00387	0.13387	6.8296	34.615	358.58	1766.0	29
30	0.02557	7.4956	51.759	0.00341	0.13341	6.9052	39.115	293.19	2024.6	30
35	0.01388	7.5855	54.614	0.00183	0.13183	7.1998	72.068	546.68	3936.0	35
40	0.00753	7.6343	56.408	0.00099	0.13099	7.3887	132.78	1013.7	7490.0	40
45	0.00409	7.6608	57.514	0.00053	0.13053	7.5076	244.64	1874.1	14070.	45
50	0.00222	7.6752	58.187	0.00029	0.13029	7.5811	450.73	3459.5	26227.	50
55	0.00120	7.6830	58.590	0.00016	0.13016	7.6260	830.45	6380.4	48656.	55
60	0.00065	7.6872	58.831	0.00009	0.13009	7.6530	1530.0	11761.	90015.	60
65	0.00035	7.6895	58.973	0.00005	0.13005	7.6692	2819.0	21677.	166247	65
70	0.00019	7.6908	59.056	0.00003	0.13003	7.6788	5193.8	39945.	306732.	70
INF	0.00000	7.6923	59.171	0.00000	0.13000	7.6923	INF	INF	INF	INF

	i = 14%			i = 14%			i = 14%			
	Present Sum (*P*)			Uniform Series (*A*)			Future Sum (*F*)			
n	***P/F***	***P/A***	***P/G***	***A/F***	***A/P***	***A/G***	***F/P***	***F/A***	***F/G***	***n***
1	0.87719	0.8771	0.0000	1.00000	1.14000	0.0000	1.1400	1.0000	0.0000	1
2	0.76947	1.6466	0.7694	0.46729	0.60729	0.4672	1.2996	2.1400	1.0000	2
3	0.67497	2.3216	2.1194	0.29073	0.43073	0.9129	1.4815	3.4396	3.1400	3
4	0.59208	2.9137	3.8956	0.20320	0.34320	1.3370	1.6889	4.9211	6.5796	4
5	0.51937	3.4330	5.9731	0.15128	0.29128	1.7398	1.9254	6.6101	11.500	5
6	0.45559	3.8886	8.2510	0.11716	0.25716	2.1218	2.1949	8.5355	18.110	6
7	0.39964	4.2883	10.648	0.09319	0.23319	2.4832	2.5022	10.730	26.646	7
8	0.35056	4.6388	13.102	0.07557	0.21557	2.8245	2.8525	13.232	37.276	8
9	0.30751	4.9463	15.562	0.06217	0.20217	3.1463	3.2519	16.085	50.609	9
10	0.26974	5.2161	17.990	0.05171	0.19171	3.4490	3.7072	19.337	66.695	10
11	0.23662	5.4527	20.356	0.04339	0.18339	3.7333	4.2262	23.044	86.032	11
12	0.20756	5.6601	22.639	0.03667	0.17667	3.9997	4.8179	27.270	109.07	12
13	0.18207	5.8423	24.824	0.03116	0.17116	4.2490	5.4924	32.088	136.24	13
14	0.15971	6.0020	26.900	0.02661	0.16661	4.4819	6.2613	37.581	168.43	14
15	0.14010	6.1421	28.862	0.02281	0.16281	4.6990	7.1379	43.842	206.01	15
16	0.12289	6.2650	30.705	0.01962	0.15692	4.9011	8.1372	50.980	249.86	16
17	0.10780	6.3728	32.430	0.01692	0.15692	5.0888	8.2764	59.117	300.84	17
18	0.09456	6.4674	34.038	0.01462	0.15462	5.2629	10.575	68.394	359.95	18
19	0.08295	6.5503	35.531	0.01266	0.15266	5.4242	12.055	78.969	428.35	19
20	0.07276	6.6231	36.913	0.01099	0.15099	5.5734	13.743	91.024	507.32	20
21	0.06383	6.8969	38.190	0.00954	0.14954	5.7111	15.667	104.76	598.34	21
22	0.05599	6.7429	39.365	0.00830	0.14830	5.8380	17.861	120.42	703.11	22
23	0.04911	6.7920	40.446	0.00723	0.14723	5.9549	20.361	138.29	823.55	23
24	0.04308	6.8351	41.437	0.00630	0.14630	6.0623	23.212	158.65	961.84	24
25	0.03779	6.8729	42.344	0.00550	0.14550	6.1610	26.461	181.87	1120.5	25
26	0.03315	6.9060	43.172	0.00480	0.14480	6.2514	30.166	208.33	1302.3	26
27	0.02908	6.9351	43.928	0.00419	0.14419	6.3342	34.389	238.49	1510.7	27
28	0.02551	6.9606	44.617	0.00366	0.14366	6.4099	39.204	272.88	1749.2	28
29	0.02237	6.9830	45.244	0.00320	0.14320	6.4791	44.693	312.09	2022.1	29
30	0.01963	7.0026	45.813	0.00280	0.14280	6.5422	50.950	356.78	2334.1	30
35	0.01019	7.0700	47.951	0.00144	0.14144	6.7824	98.100	693.57	4704.0	35
40	0.00529	7.1050	49.237	0.00075	0.14075	6.9299	188.88	1342.0	9300.1	40
45	0.00275	7.1232	49.996	0.00039	0.14039	7.0187	363.67	2590.5	18182.	45
50	0.00143	7.1326	50.437	0.00020	0.14020	7.0713	700.23	4494.5	35318.	50
55	0.00074	7.1375	50.691	0.00010	0.14010	7.1020	1348.2	9623.1	68343.	55
60	0.00039	4.1401	50.835	0.00005	0.14005	7.1197	2595.9	18535.	131965	60
65	0.00020	7.1414	50.917	0.00003	0.14003	7.1298	4998.2	35693.	254496.	65
70	0.00010	7.1421	50.963	0.00001	0.14001	7.1355	9623.6	68733.	490351.	70
INF	0.00000	7.1428	51.020	0.00000	0.14000	7.1428	INF	INF	INF	INF

	i = 15%			i = 15%			i = 15%			
	Present Sum (P)			Uniform Series (A)			Future Sum (F)			
n	P/F	P/A	P/G	A/F	A/P	A/G	F/P	F/A	F/G	n
1	0.86957	0.8695	0.0000	1.00000	1.15000	0.0000	1.1500	1.0000	0.0000	1
2	0.75614	1.6257	0.7561	0.46512	0.61512	0.4651	1.3225	2.1500	1.0000	2
3	0.65752	2.2832	2.0711	0.28798	0.43798	0.9071	1.5208	3.4725	3.1500	3
4	0.57175	2.8549	3.7864	0.20027	0.35027	1.3262	1.7490	4.9933	6.6225	4
5	0.49718	3.3521	5.7751	0.14832	0.29832	1.7228	2.0113	6.7423	11.615	5
6	0.43233	3.7844	7.9367	0.14424	0.26424	2.0971	2.3130	8.7537	18.358	6
7	0.37594	4.1604	10.192	0.09036	0.24036	2.4498	2.6600	11.066	27.122	7
8	0.32690	4.4873	12.480	0.07285	0.22285	2.7813	3.0590	13.726	38.178	8
9	0.28426	4.7715	14.754	0.05957	0.20957	3.0922	3.5178	16.785	51.905	9
10	0.24718	5.0187	16.979	0.04925	0.19925	3.3832	4.0455	20.303	68.691	10
11	0.21491	5.2337	19.128	0.04107	0.19107	3.6549	4.6523	24.349	88.995	11
12	0.18691	5.4206	21.184	0.03448	0.18448	3.9082	5.3502	29.001	113.34	12
13	0.16253	5.5831	23.135	0.02911	0.17911	4.1437	6.1527	34.351	142.34	13
14	0.14133	5.7244	24.972	0.02469	0.17469	4.3624	7.0757	40.504	176.69	14
15	0.12289	5.8473	26.693	0.02102	0.17102	4.5649	8.1370	47.580	217.20	15
16	0.10686	5.9542	28.296	0.01795	0.16795	4.7522	9.3576	55.717	264.78	16
17	0.09293	6.0471	29.782	0.01537	0.16537	4.9250	10.761	65.075	320.50	17
18	0.08081	6.1279	31.156	0.01319	0.16319	5.0843	12.375	75.836	385.57	18
19	0.07027	6.1982	32.421	0.01134	0.16134	5.2307	14.231	88.211	461.41	19
20	0.06110	6.2593	33.582	0.00976	0.15976	5.3651	16.366	102.44	549.62	20
21	0.05313	6.3124	34.644	0.00842	0.15842	5.4883	18.821	118.81	652.06	21
22	0.04620	6.3586	35.615	0.00727	0.15727	5.6010	21.644	137.63	770.87	22
23	0.04017	6.3988	36.498	0.00628	0.15628	5.7039	24.891	159.27	908.50	23
24	0.03493	6.4337	37.302	0.00543	0.15543	5.7978	28.625	184.16	1067.7	24
25	0.03038	6.4641	38.031	0.00470	0.15470	5.8834	32.919	212.79	1251.9	25
26	0.02642	6.4905	38.691	0.00407	0.15407	5.9612	37.856	245.71	1464.7	26
27	0.02297	6.5135	39.289	0.00353	0.15353	6.0319	43.535	283.56	1710.4	27
28	0.01997	6.5335	39.828	0.00306	0.15306	6.9060	50.065	327.10	1994.0	28
29	0.01737	6.5508	40.314	0.00265	0.15265	6.1540	57.575	377.17	2321.1	29
30	0.01510	6.5659	40.752	0.00230	0.15230	6.2066	66.211	434.74	2698.3	30
35	0.00751	6.6166	42.358	0.00113	0.15113	6.4018	133.17	881.17	5641.1	35
40	0.00373	6.6417	43.283	0.00056	0.15056	6.5167	267.86	1779.0	11593.	40
45	0.00186	6.6542	43.805	0.00028	0.15028	6.5829	538.76	3585.1	23600.	45
50	0.00092	6.6605	44.095	0.00014	0.15014	6.6204	1083.6	7217.7	47784.	50
55	0.00046	6.6636	44.255	0.00007	0.15007	6.6414	2179.6	14524.	95461.	55
60	0.00023	6.6651	44.343	0.00003	0.15003	6.6529	4384.0	29220.	184400.	60
65	0.00011	6.6659	44.390	0.00002	0.15002	6.6592	8817.7	5877.8	391424.	65
INF	0.00000	6.6666	44.444	0.00000	0.15000	6.6666	INF	INF	INF	INF

	i = 20%			i = 20%			i = 20%			
	Present Sum (*P*)			Uniform Series (*A*)			Future Sum (*F*)			
n	***P/F***	***P/A***	***P/G***	***A/F***	***A/P***	***A/G***	***F/P***	***F/A***	***F/G***	***n***
1	0.83333	0.8333	0.0000	1.00000	1.20000	0.0000	1.2000	1.0000	0.0000	1
2	0.69444	1.5277	0.6944	0.45455	0.65455	0.4545	1.4400	2.2000	1.0000	2
3	0.57870	2.1064	1.8518	0.27473	0.47473	0.8791	1.7280	3.6400	3.2000	3
4	0.48225	2.5887	3.2986	0.18629	0.38629	1.2742	2.0736	5.3680	6.8400	4
5	0.40188	2.9906	4.9061	0.13438	0.33438	1.6405	2.4883	7.4416	12.208	5
6	0.33490	3.3255	6.5806	0.10071	0.30071	1.9788	2.9859	9.9299	19.649	6
7	0.27908	3.6045	8.2551	0.07742	0.27742	2.2901	3.5831	12.915	29.579	7
8	0.23257	3.8371	9.8830	0.06061	0.26061	2.5756	4.2998	16.499	42.495	8
9	0.19381	4.0309	11.433	0.04808	0.24808	2.8364	5.1597	20.798	58.994	9
10	0.16151	4.1924	12.887	0.03852	0.23852	3.0738	6.1917	25.958	79.793	10
11	0.13459	4.3270	14.233	0.03110	0.23110	3.2892	7.4300	32.150	105.75	11
12	0.11216	4.4392	15.466	0.02526	0.22526	3.4841	8.9161	39.580	137.90	12
13	0.09346	4.5326	16.588	0.02062	0.22062	3.6597	10.699	48.496	177.48	13
14	0.07789	4.6105	17.600	0.01689	0.21689	3.8174	12.839	59.195	225.98	14
15	0.06491	4.6754	18.509	0.01388	0.21388	3.9588	15.407	72.035	285.17	15
16	0.05409	4.7295	19.320	0.01144	0.21144	4.0861	18.488	87.442	357.21	16
17	0.04507	4.7746	20.041	0.00944	0.20944	4.1975	22.186	105.93	444.65	17
18	0.03756	4.8121	20.680	0.00781	0.20781	4.2975	26.623	128.22	550.58	18
19	0.03130	4.8435	21.243	0.00646	0.20646	4.3860	31.948	154.74	678.70	19
20	0.02608	4.8695	21.739	0.00536	0.20536	4.4643	38.337	186.68	833.44	20
21	0.02174	4.8913	22.174	0.00444	0.20444	4.5333	46.005	225.02	1020.1	21
22	0.01811	4.9094	22.554	0.00369	0.20369	4.5941	55.206	271.03	1245.1	22
23	0.01509	4.9245	22.886	0.00307	0.20307	4.6475	66.247	326.23	1516.1	23
24	0.01258	4.9371	23.176	0.00255	0.20255	4.6942	79.496	392.48	1842.4	24
25	0.01048	4.9475	23.427	0.00212	0.20212	4.7351	95.396	471.98	2334.9	25
26	0.00874	4.9563	23.646	0.00176	0.20176	4.7708	114.47	567.37	2706.8	26
27	0.00728	4.9636	23.835	0.00147	0.20147	4.8020	137.37	681.85	3274.2	27
28	0.00607	4.9696	23.999	0.00122	0.20122	4.8291	164.84	819.22	3956.1	28
29	0.00506	4.9747	24.140	0.00102	0.20102	4.8526	197.81	984.06	4775.3	29
30	0.00421	4.9789	24.262	0.00085	0.20085	4.8730	237.37	1181.8	5759.4	30
35	0.00169	4.9915	24.661	0.00034	0.20034	4.9407	590.66	2948.3	14566.	35
40	0.00068	4.9966	24.846	0.00014	0.20014	4.9727	1469.7	7343.8	36519.	40
45	0.00027	4.9986	24.931	0.00005	0.20005	4.9876	3657.2	18281.	91181.	45
50	0.00011	4.9994	24.969	0.00002	0.20002	4.9945	9200.4	45497.	227236.	50
55	0.00004	4.9997	24.986	0.00001	0.20001	4.9975	22644.	113219.	565820.	55
INF	0.00000	5.0000	25.000	0.00000	0.20000	5.0000	INF	INF	INF	INF

	i = 25%			i = 25%			i = 25%			
	Present Sum (P)			Uniform Series (A)			Future Sum (F)			
n	P/F	P/A	P/G	A/F	A/P	A/G	F/P	F/A	F/G	n
1	0.80000	0.8000	0.0000	1.00000	1.25000	0.0000	1.2500	1.0000	0.0000	1
2	0.64000	1.4400	0.6400	0.44444	0.69444	0.4444	1.5625	2.2500	1.0000	2
3	0.51200	1.9520	1.6640	0.26230	0.51230	0.8524	1.9531	3.8125	3.2500	3
4	0.40960	2.3616	2.8928	0.17344	0.42344	1.2249	2.4414	5.7656	7.0625	4
5	0.32768	2.6892	4.2035	0.12185	0.37185	1.5630	3.0517	8.2070	12.828	5
6	0.26214	2.9514	5.5142	0.08882	0.33882	1.8683	3.8147	11.258	21.035	6
7	0.20972	3.1611	6.7725	0.06634	0.31634	2.1424	4.7683	15.073	32.293	7
8	0.16777	3.3289	7.9469	0.05040	0.30040	2.3872	5.9604	19.841	47.367	8
9	0.13422	3.4631	9.0206	0.03876	0.28876	2.6047	7.4505	25.802	67.209	9
10	0.10737	3.5705	9.9870	0.03007	0.28007	2.7971	9.3132	33.252	93.011	10
11	0.08590	3.6564	10.846	0.02349	0.27349	2.9663	11.641	42.566	126.26	11
12	0.06872	3.7251	11.602	0.01845	0.26845	3.1145	14.551	54.207	168.83	12
13	0.05498	3.7801	12.261	0.01454	0.26454	3.2437	18.189	68.759	223.03	13
14	0.04398	3.8240	12.833	0.01150	0.26150	3.3559	22.737	86.949	291.79	14
15	0.03518	3.8592	13.326	0.00912	0.25912	3.4529	28.421	109.68	378.74	15
16	0.02815	3.8874	13.748	0.00724	0.25724	3.5366	35.527	138.10	488.43	16
17	0.02252	3.9099	14.108	0.00576	0.25576	3.6083	44.408	173.63	626.54	17
18	0.01801	3.9279	14.414	0.00459	0.25459	3.6697	55.511	218.04	800.17	18
19	0.01441	3.9423	15.674	0.00366	0.25366	3.7221	69.388	273.55	1018.2	19
20	0.01553	3.9538	14.893	0.00292	0.25292	3.7667	86.736	342.94	1291.7	20
21	0.00922	3.9631	15.077	0.00233	0.25233	3.8045	108.42	429.68	1634.7	21
22	0.00738	3.9704	15.232	0.00186	0.25186	3.8364	135.52	538.10	2064.4	22
23	0.00590	3.9763	15.362	0.00148	0.25148	3.8634	169.40	673.62	2602.5	23
24	0.00472	3.9811	15.471	0.00119	0.25119	3.8861	211.75	843.03	3276.1	24
25	0.00378	3.9848	15.561	0.00095	0.25095	3.9051	264.69	1054.7	4119.1	25
26	0.00302	3.9879	15.637	0.00076	0.25076	3.9211	330.87	1319.4	5173.9	26
27	0.00242	3.9903	15.700	0.00061	0.25061	3.9345	413.59	1650.3	6493.4	27
28	0.00193	3.9922	15.752	0.00048	0.25048	3.9457	516.98	2063.9	8143.8	28
29	0.00155	3.9938	15.795	0.00039	0.25039	3.9550	646.23	2580.9	10207.	29
30	0.00124	3.9950	15.831	0.00031	0.25031	3.9628	807.79	3227.1	12788.	30
35	0.00041	3.9983	15.936	0.00010	0.25010	3.9858	2465.1	9856.7	39287.	35
40	0.00013	3.9994	15.976	0.00003	0.25003	3.9946	7523.1	30088.	120195.	40
45	0.00004	3.9998	15.991	0.00001	0.25001	3.9980	22958.	91831.	357146.	45
INF	0.00000	4.0000	16.000	0.00000	0.25000	4.0000	INF	INF	INF	INF

	i = 30%			*i* = 30%			*i* = 30%			
	Present Sum (*P*)			Uniform Series (*A*)			Future Sum (*F*)			
n	*P/F*	*P/A*	*P/G*	*A/F*	*A/P*	*A/G*	*F/P*	*F/A*	*F/G*	*n*
1	0.76923	0.7692	0.0000	1.00000	1.30000	0.0000	1.3000	1.0000	0.0000	1
2	0.59172	1.3609	0.5917	0.43478	0.73478	0.4347	1.6900	2.3000	1.0000	2
3	0.45517	1.8161	1.5020	0.25063	0.55063	0.8270	2.1970	3.9900	3.3000	3
4	0.35013	2.1662	2.5524	0.16163	0.46163	1.1782	2.8561	6.1870	7.2900	4
5	0.26933	2.4355	3.6297	0.11058	0.41058	1.4903	3.7129	9.0431	13.477	5
6	0.20718	2.6427	4.6656	0.07839	0.37839	1.7654	4.8268	12.756	22.520	6
7	0.15937	2.8021	5.6218	0.05687	0.35687	2.0062	6.2748	17.582	35.276	7
8	0.12259	2.9247	6.4799	0.04192	0.34192	2.2155	8.1573	23.857	52.850	8
9	0.09430	3.0190	7.2343	0.03124	0.33124	2.3962	10.604	32.015	76.716	9
10	0.07254	3.0915	7.8871	0.02346	0.32346	2.5512	13.785	42.619	108.73	10
11	0.05580	3.1473	8.4452	0.01773	0.31773	2.6822	17.921	56.405	151.35	11
12	0.04392	3.1902	8.9173	0.01345	0.31345	2.7951	23.298	74.327	207.75	12
13	0.03302	3.2232	9.3135	0.01024	0.32024	2.8894	30.287	97.625	282.08	13
14	0.02540	3.2486	9.6436	0.00782	0.30782	2.9685	39.373	127.91	379.70	14
15	0.01954	3.2682	9.9172	0.00598	0.30598	3.0344	51.185	167.28	507.62	15
16	0.01503	3.2832	10.142	0.00458	0.30458	3.0892	66.541	218.47	674.90	16
17	0.01156	3.2948	10.327	0.00351	0.30351	3.1345	86.504	285.01	893.38	17
18	0.00889	3.3036	10.478	0.00269	0.30269	3.1718	112.45	371.51	1178.3	18
19	0.00684	3.3105	10.601	0.00207	0.30207	3.2024	146.19	483.97	1549.9	19
20	0.00526	3.3157	10.701	0.00159	0.30159	3.2275	190.05	630.16	2033.8	20
21	0.00405	3.3198	10.782	0.00122	0.30122	3.2479	247.06	820.21	2664.0	21
22	0.00311	3.3229	10.848	0.00094	0.30094	3.2646	321.18	1067.2	3484.2	22
23	0.00239	3.3253	10.900	0.00072	0.30072	3.2781	417.53	1388.4	4551.5	23
24	0.00184	3.3271	10.943	0.00055	0.30055	3.2890	542.80	1806.0	5940.0	24
25	0.00142	3.3286	10.977	0.00043	0.30043	3.2978	705.64	2348.8	7746.0	25
26	0.00109	3.3297	11.004	0.00033	0.39933	3.3049	917.33	3054.4	10094.	26
27	0.00084	3.3305	11.026	0.00025	0.30025	3.3106	1192.5	3971.7	13149.	27
28	0.00065	3.3311	11.043	0.00019	0.30019	3.3152	1550.2	5164.3	17121.	28
29	0.00050	3.3316	11.057	0.00015	0.30015	3.3189	2015.3	6714.6	22285.	29
30	0.00038	3.3320	11.068	0.00011	0.30011	3.3218	2620.0	8729.9	29000.	30
35	0.00010	3.3329	11.098	0.00003	0.30003	3.3297	9727.8	32422.	107960.	35
INF	0.00000	3.3333	11.111	0.00000	0.30000	3.3333	INF	INF	INF	INF

	i = 40%			i = 40%			i = 40%			
	Present Sum (*P*)			Uniform Series (*A*)			Future Sum (*F*)			
n	***P/F***	***P/A***	***P/G***	***A/F***	***A/P***	***A/G***	***F/P***	***F/A***	***F/G***	***n***
1	0.71429	0.7142	0.0000	1.00000	1.40000	0.0000	1.4000	1.0000	0.0000	1
2	0.51020	1.2244	0.5102	0.41667	0.81667	0.4166	1.9600	2.4000	1.0000	2
3	0.36443	1.5889	1.2390	0.22936	0.62936	0.7798	2.7440	4.3600	3.4000	3
4	0.26031	1.8492	2.0199	0.14077	0.54077	1.0923	3.8416	7.1040	7.7600	4
5	0.18593	2.0351	2.7637	0.09136	0.49136	1.3579	5.3782	10.945	14.864	5
6	0.13281	2.1679	3.4277	0.06126	0.46126	1.5811	7.5295	16.323	25.809	6
7	0.09586	2.2628	3.9969	0.04192	0.44192	1.7663	10.541	23.853	42.133	7
8	0.06776	2.3306	4.4712	0.02907	0.42907	1.9185	14.757	34.394	65.986	8
9	0.04840	2.3790	4.8584	0.02034	0.42034	2.0422	20.661	48.152	100.38	9
10	0.03457	2.4135	5.1696	0.01432	0.41432	2.1419	28.925	69.813	149.53	10
11	0.02469	2.4382	5.4165	0.01013	0.41013	2.2214	40.495	98.739	219.34	11
12	0.01764	2.4559	5.6106	0.00718	0.40718	2.2845	56.693	139.23	318.08	12
13	0.01260	2.4685	5.7617	0.00510	0.40510	2.3341	79.371	195.92	457.32	13
14	0.00900	2.4775	5.8787	0.00363	0.40363	2.3728	111.12	275.30	653.25	14
15	0.00643	2.4839	5.9687	0.00259	0.40259	2.4029	156.56	386.42	928.55	15
16	0.00459	2.4885	6.0376	0.00185	0.40185	2.4262	217.79	541.98	1314.9	16
17	0.00328	2.4918	6.0901	0.00132	0.40132	2.4440	304.91	759.78	1856.9	17
18	0.00234	2.4941	6.1299	0.00094	0.40094	2.4577	426.87	1064.7	2616.7	18
29	00.0167	2.4958	6.1600	0.00067	0.40067	2.4681	597.63	1491.5	3681.4	19
20	0.00120	2.4970	6.1827	0.00048	0.40048	2.4760	836.68	2089.2	5173.0	20
21	0.00085	2.4978	6.1998	0.00034	0.40034	2.4820	1171.3	2925.8	7262.2	21
22	0.00061	2.4984	6.2126	0.00024	0.40024	2.4865	1639.9	4097.2	10188.	22
23	0.00044	2.4989	6.2222	0.00017	0.40017	2.4899	2295.8	5737.1	14285.	23
24	0.00031	2.4992	6.2293	0.00012	0.40012	2.4935	3214.2	8033.0	20022.	24
25	0.00022	2.4994	6.2347	0.00009	0.40009	2.4944	4499.8	11247.	28055.	25
INF	0.00000	2.5000	6.2500	0.00000	0.40000	2.5000	INF	INF	INF	INF

	i = 50%			*i* = 50%			*i* = 50%			
	Present Sum (*P*)			Uniform Series (*A*)			Future Sum (*F*)			
n	*P/F*	*P/A*	*P/G*	*A/F*	*A/P*	*A/G*	*F/P*	*F/A*	*F/G*	*n*
1	0.66667	0.6666	0.0000	1.00000	1.50000	0.0000	1.5000	1.0000	0.0000	1
2	0.44444	1.1111	0.4444	0.40000	0.90000	0.4000	2.2500	2.5000	1.0000	2
3	0.29630	1.4074	1.0370	0.21053	0.71053	0.7368	3.3750	4.7500	3.5000	3
4	0.19753	1.6049	1.6296	0.12308	0.62308	1.1053	5.0625	8.1250	8.2500	4
5	0.13169	1.7366	2.1563	0.07583	0.57583	1.2417	7.5937	13.187	16.375	5
6	0.08779	1.8244	2.5953	0.04812	0.43812	1.4225	11.390	20.781	29.562	6
7	0.05853	1.8829	2.9465	0.03108	0.53108	1.5648	17.085	32.171	50.343	7
8	0.03902	1.9219	3.1296	0.02030	0.52030	1.6751	25.628	49.257	82.515	8
9	0.02601	1.9479	3.4277	0.01335	0.51335	1.7596	38.443	74.886	131.77	9
10	0.01734	1.9653	3.5838	0.00882	0.50882	1.8235	57.665	113.33	206.66	10
11	0.01156	1.9768	3.6994	0.00585	0.50585	1.8713	86.497	170.99	319.99	11
12	0.00771	1.9845	3.7841	0.00388	0.50388	1.9067	129.74	257.49	490.98	12
13	0.00514	1.9897	3.8458	0.00258	0.50258	1.9328	194.62	387.23	748.47	13
14	0.00343	1.9931	3.8903	0.00172	0.50172	1.9518	291.92	581.85	1135.7	14
15	0.00228	1.9954	3.9223	0.00114	0.50114	1.9656	437.89	873.78	1717.5	15
16	0.00152	1.9969	3.9451	0.00076	0.50076	1.9756	656.83	1311.6	2591.3	16
17	0.00101	1.9979	3.9614	0.00051	0.50051	1.9827	985.26	1968.5	3903.0	17
18	0.00068	1.9986	3.9729	0.00034	0.50034	1.9878	1477.8	2953.7	5871.5	18
19	0.00045	1.9991	3.9810	0.00023	0.50023	1.9914	2216.8	4431.6	8825.3	19
20	0.00030	1.9994	3.9867	0.00015	0.50015	1.9939	3325.2	6648.5	13257.	20
21	0.00020	1.9996	3.9907	0.00010	0.50010	1.9957	4987.8	9973.7	19905.	21
22	0.00013	1.9997	3.9935	0.00007	0.50007	1.9970	7481.8	14961.	29879.	22
23	0.00009	1.9998	3.9955	0.00004	0.50004	1.9979	11222.	22443.	44841.	23
24	0.00006	1.9998	3.9969	0.00003	0.50003	1.9985	16834.	33666.	67284.	24
25	0.00004	1.9999	3.9978	0.00002	0.50002	1.9990	25251.	50500.	100951.	25
INF	0.00000	2.0000	4.0000	0.00000	0.50000	2.0000	INF	INF	INF	INF

Appendix C: Using Spreadsheets to Solve Engineering Economic Problems

The formulas used in spreadsheets to calculate the engineering economic solutions for the factors covered in this book are the following:

Present worth = PV
Future value = FV
Annuity = PMT
Net present worth = NPV
Internal rate of return = IRR
Effective interest rate= EFFECT
Straight line depreciation = SLN
Sum-of-the-years digits depreciation = SYD
Double declining balance depreciation = DDB

To calculate the different factors, the appropriate factor is listed as (=factor) and it is entered along with all of the different variables in parenthesis according to their location in the spreadsheet.

For example, to calculate present worth, first, enter all of the relevant variables in the following order:

A1) Interest rate
A2) Periods
A3) Uniform series
A4) Future value
A5) Present value

If a formula does not use one of the variables, enter a zero instead of a value.

For the present worth, enter the following after all of the variables are entered into the spreadsheet:

=PV(A1,A2,A3,A4)

where A1 through A4 are the cells where the values are listed for the variables. The present value will be calculated by the spreadsheet and displayed on the spreadsheet rather than the formula.

The following are examples that demonstrate using spreadsheets to calculate the answers to engineering economic problems. The first set of numbers demonstrates entering the data into the spreadsheet and the second set of numbers is the resulting spreadsheet with the answers.

Note: Values in parenthesis represent negative numbers.

Converting a present value into a future worth	
Interest rate	B5) 0.1
Periods	B6) 5
Uniform series	B7) 0
Present value	B8) $1,000.00
Future value	=FV(B5,B6,B7,B8)
Interest rate	10%
Periods	5
Uniform series	0
Present value	$1,000.00
Future value	($1,610.51)

Converting a future value into a present worth	
Interest rate	B13) 0.10
Periods	B14) 5
Uniform series	B15) 0
Future value	B16) $1,000.00
	=PV(B13,B14,B15,B16)
Interest rate	10%
Periods	5
Uniform series	0
Future value	$1,000.00
Present value	($620.92)

Converting a uniform series into a future worth	
Interest rate	B21) 0.09
Periods	B22) 10
Uniform series	B23) $500.00
Present value	B24) 0
Future value	=FV(B21,B22,B23,B24)
Interest rate	9%
Periods	10
Uniform series	$500.00
Present value	0
Future value	($7,596.46)

Converting a future value into a uniform series	
Interest rate	B29) 0.1
Periods	B30) 20
Present value	B32) 0
Future value	B32) $100,000.00
Uniform series	=PMT(B29,B30,B31,B32)

(*Continued*)

Converting a future value into a uniform series (*Continued*)	
Interest rate	10%
Periods	20
Present value	0
Future value	$100,000.00
Uniform series	($1,745.96)

Converting a uniform series into a present worth	
Interest	B37) 0.06
Periods	B38) 5
Uniform series	B39) $500.00
Future value	B40) 0
Present value	=PV(B37,B38,B39, B40)
Interest rate	6%
Periods	5
Uniform series	$500.00
Present value	($2,106.18)

Converting a nonuniform series into a present worth		
Interest rate	B44) 0.12	
Period	Amount	PV
A46) 0	B46) 0	C46) =PV(B44,A46,0,B46)
A47) 1	B47) $1,000.00	C47) =PV(B44,A47,0,B47)
A48) 2	B48) $1,000.00	C48) =PV(B44,A48,0,B48)
A49) 3	B49) $2,000.00	C49) =PV(B44,A49,0,B49)
A50) 4	B50) $1,000.00	C50) =PV(B44,A50,0,B50)
A51) 5	B51) $1,000.00	C51) =PV(B44,A51,0,B51)
	Total	=SUM(C46:C51)
Interest rate	12%	
Period	Amount	PV
0	0	$0.00
1	$1,000.00	($892.86)
2	$1,000.00	($797.19)
3	$2,000.00	($1,423.56)
4	$1,000.00	($635.52)
5	$1,000.00	($567.43)
	Total	($4,316.56)

Calculating net present worth

Year	Cash Flow	MARR
0	B57) ($120,000.00)	C57) 0.2
1	B58) $48,000.00	
2	B59) $48,000.00	
3	B60) $48,000.00	
4	B61) $48,000.00	
5	B62) $60,000.00	
	=NPV(C57,B58,B59,B60,B61,B62)+B57)	

Year	Cash Flow	MARR
0	($120,000.00)	20%
1	$48,000.00	
2	$48,000.00	
3	$48,000.00	
4	$48,000.00	
5	$60,000.00	
	$28,371.91	

Calculating net future worth

MARR	B68) 0.2	
Year	**Cash Flow**	
0	B70) ($120,000.00)	=FV(B68,5,0,B70)
1	B71) $48,000.00	=FV(B68,4,0,B71)
2	B72) $48,000.00	=FV(B68,3,0,B72)
3	B73) $48,000.00	=FV(B68,2,0,B73)
4	B74) $48,000.00	=FV(B68,1,0,B74)
5	B75) $60,000.00	=B75
	Future worth	=SUM(B70:B75)

MARR	20%	
Year	**Cash Flow**	
0	(120,000)	$298,598.40
1	$48,000.00	$99,532.80
2	$48,000.00	$82,944.00
3	$48,000.00	$69,120.00
4	$48,000.00	$57,600.00
5	$60,000.00	$60,000.00
	Future worth	$70,598.00

Calculating annual equivalent worth of present and future values	
MARR	B80) 0.18
Purchase price	B81) ($65,000.00)
Salvage value	B82) $15,000.00
Life years	B83) 7
Annual profit (or cost)	B84) $17,100.00
Annual equivalent	=PMT(B80,B81,B82,B83,B84)
MARR	18%
Purchase price	($65,000.00)
Salvage value	$15,000.00
Life years	7
Annual profit (or cost)	$17,100.00
Annual equivalent	$1,181.90

Calculating rate of return	
Year	**Cash Flow**
0	B91) ($120,000.00)
1	B92) $48,000.00
2	B93) $48,000.00
3	B94) $48,000.00
4	B95) $48,000.00
5	B96) $60,000.00
Rate of return	=IRR(B91:B96,0.2)
Year	**Cash Flow**
0	($120,000.00)
1	$48,000.00
2	$48,000.00
3	$48,000.00
4	$48,000.00
5	$60,000.00
Rate of return	30%

Determining monthly payments for a loan	
Annual interest rate	B103) 0.09
Monthly interest rate	B104) 0.0075
Years	B105) 30
Months	B106) 360

(*Continued*)

Determining monthly payments for a loan (Continued)	
Loan principal	B107) $150,000.00
Balloon payment	B108) 0
Payment	=PMT(B104,B106,B107,0)
Annual interest rate	9.00%
Monthly interest rate	0.75%
Years	30
Months	360
Loan principal	$150,000.00
Balloon payment	0
Payment	($1,206.93)

Converting annual percentage rate to annual effective rate	
Nominal interest rate	B113) 0.12
Compounding periods	B114) 4
Effective interest rate	=EFFECT(B113,B114)
Nominal interest rate	12%
Compounding periods	4
Effective interest rate	0.12550881 or 12.55%

Calculating the interest rate when replacing a loan	
Months	B119) 60
Existing loan amount	B120) $55,000.00
Closing costs	B121) $1,590.00
New loan principal	B122) =B120+B121
Monthly payment	=PMT(B122)
Monthly interest rate	0.0052
Nominal interest rate	0.0623
Months	60
Existing loan amount	$55,000.00
Closing costs	$1,590.00
New loan principal	$56,590.00
Monthly payment	($1,100.00)
Monthly interest rate	0.0052
Nominal interest rate	6.23%

Calculating monthly interest, principal reduction, and balance for a loan	
Annual interest rate	B130) 0.09
Monthly interest rate	B131) =B130/12
Monthly payment	B132) $1,206.93
Balance from last month	B133) $150,000.00
Interest for this month	B134) =(B131 * B133)
Principal reduction	B135) =(B132 – B134)
Balance from this month	B136 =B133 – B135
Annual interest rate	9%
Monthly interest rate	0.0075
Monthly payment	$1,206.93
Balance from last month	$150,000.0
Interest for this month	$1,125.00
Principal reduction	$81.93
Balance from this month	$149,918.07

Calculating straight line depreciation		
Purchase price	B140) $110,000.00	
Salvage value	B141) $10,000.00	
Recovery period in years	B142) 5	
n	Depreciation	BVm($)
0		C144) =B140
1	B145) =SLN(B140,B141,B142)	C145) =(C144 – B145)
2	B146) =SLN(B140,B141,B142)	C146) =(C145 – B146)
3	B147) =SLN(B140,B141,B142)	C147) =(C146 – B147)
4	B148) =SLN(B140,B141,B142)	C148) =(C147 – B148)
5	B149) =SLN(B140,B141,B142)	C149) =(C148 – B149)
Purchase price	$110,000.00	
Salvage value	$10,000.00	
Recovery period in years	5	
n	Depreciation	BVm($)
0		$110,000.00
1	$20,000.00	$90,000.00
2	$20,000.00	$70,000.00
3	$20,000.00	$50,000.00
4	$20,000.00	$30,000.00
5	$20,000.00	$10,000.00

Calculating sum-of-the-years digits depreciation		
Purchase price	B153) $110,000.00	
Salvage value	B154) $10,000.00	
Recovery period in years	B156) 5	
n	Depreciation	BVm
0		C157) =B153
1	B158) =SYD(B153,B154,B155,A158)	C158) =C157 − B158
2	B159) =SYD(B153,B154,B155,A159)	C159) =C158 − B159
3	B160) =SYD(B153,B154,B155,A160)	C160) =C159 − B160
4	B161) =SYD(B153,B154,B155,A161)	C161) =C160 − B161
5	B162)=SYD(B153,B154,B155,A162)	C162) =C161 − B162
Purchase price	$110,000.00	
Salvage value	$10,000.00	
Recovery period in years	5	
n	Depreciation	BVm
0		$110,000.00
1	$33,333.33	$76,666.67
2	$26,666.67	$50,000.00
3	$20,000.00	$30,000.00
4	$13,333.33	$16,666.67
5	$6,666.67	$10,000.00

Calculating declining balance depreciation		
Purchase price	B166) $110,000.00	
Salvage value	B167) $10,000.00	
Recovery period in years	B168) 5	
Rate (%)	B169) 200	
n	Depreciation	BVm
0		C171) =B166
1	B172) =DDB(B166,B167,B168,A171,A172)	C172) =C171 − B172
2	B173) =DDB(B166,B167,B168,A172,A173)	C173) =C172 − B173
3	B174) =DDB(B166,B167,B168,A173,A174)	C174) =C173 − B174
4	B175) =DDB(B166,B167,B168,A174,A175)	C175) =C174 − B175
5	B176) =DDB(B166,B167,B168,A175,A176)	C176) =C175 − B176
Purchase price	$110,000.00	
Salvage value	$10,000.00	
Recovery period in years	5	
Rate (%)	200	
n	Depreciation	BVm
0		$110,000.00
1	$44,000.00	$66,000.00
2	$26,400.00	$39,600.00
3	$15,840.00	$23,760.00
4	$9,504.00	$14,256.00
5	$4,256.00	$10,000.00

Environmental and Natural Resource Economics

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Chapter 2

The Economic Approach

Property Rights, Externalities, and Environmental Problems

> The charming landscape which I saw this morning, is indubitably made up of some twenty or thirty farms. Miller owns this field, Locke that, and Manning the woodland beyond. But none of them owns the landscape. There is a property in the horizon which no man has but he whose eye can integrate all the parts, that is, the poet. This is the best part of these men's farms, yet to this their land deeds give them no title.
>
> —Ralph Waldo Emerson, *Nature* (1836)

Introduction

Before examining specific environmental problems and the policy responses to them, it is important that we develop and clarify the economic approach, so that we have some sense of the forest before examining each of the trees. By having a feel for the conceptual framework, it becomes easier not only to deal with individual cases but also, perhaps more importantly, to see how they fit into a comprehensive approach.

In this chapter, we develop the general conceptual framework used in economics to approach environmental problems. We begin by examining the relationship between human actions, as manifested through the economic system, and the environmental consequences of those actions. We can then establish criteria for judging the desirability of the outcomes of this relationship. These criteria provide a basis for identifying the nature and severity of environmental problems, and a foundation for designing effective policies to deal with them.

Throughout this chapter, the economic point of view is contrasted with alternative points of view. These contrasts bring the economic approach into sharper focus and stimulate deeper and more critical thinking about all possible approaches.

The Human–Environment Relationship

The Environment as an Asset

In economics, the environment is viewed as a composite asset that provides a variety of services. It is a very special asset, to be sure, because it provides the life-support systems that sustain our very existence, but it is an asset nonetheless. As with other assets, we wish to enhance, or at least prevent undue depreciation of, the value of this asset so that it may continue to provide aesthetic and life-sustaining services.

The environment provides the economy with raw materials, which are transformed into consumer products by the production process, and energy, which fuels this transformation. Ultimately, these raw materials and energy return to the environment as waste products (see Figure 2.1).

The environment also provides goods and services directly to consumers. The air we breathe, the nourishment we receive from food and drink, and the protection we derive from shelter and clothing are all benefits we receive, either directly or indirectly, from the environment. One significant subclass of these, *ecosystem goods and services*, incorporates the benefits obtained directly from ecosystems, including biodiversity, breathable air, wetlands, water quality, carbon sequestration, and recreation. Anyone who has experienced the exhilaration of white-water rafting, the total serenity of a wilderness trek, or the breathtaking beauty of a

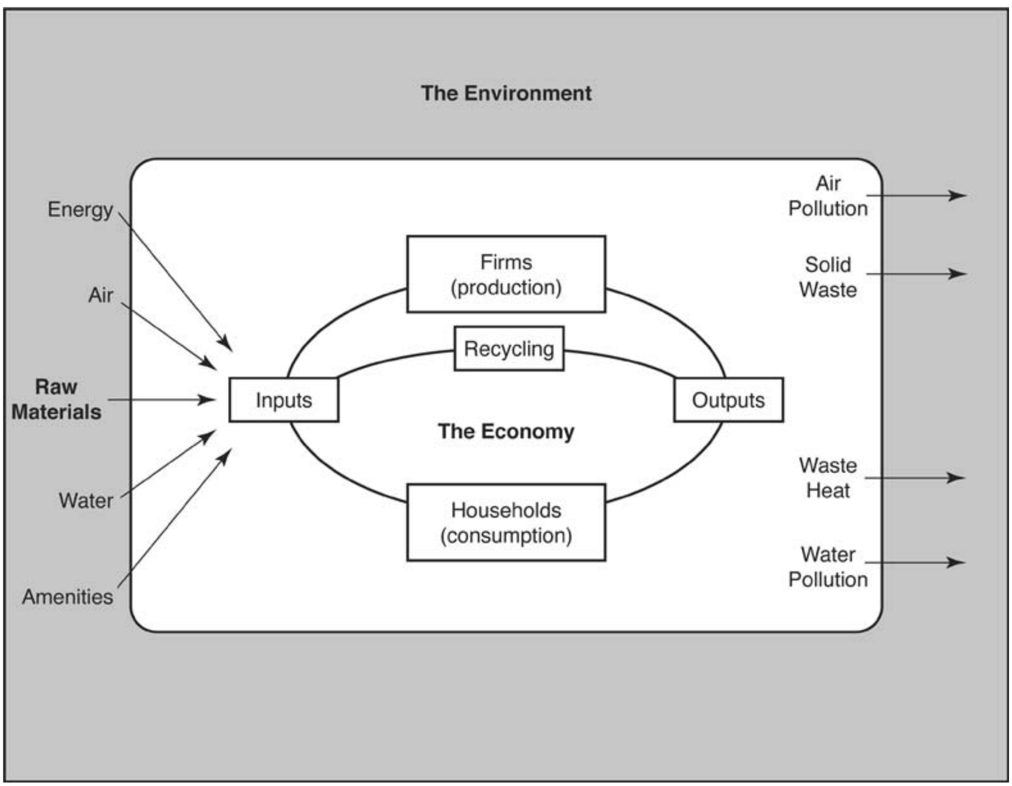

Figure 2.1 The Economic System and the Environment

sunset will readily recognize that ecosystems provide us with a variety of amenities for which no substitute exists. (Chapter 13 provides a closer look at the role economics plays in maintaining and protecting these very special goods and services.)

If the environment is defined broadly enough, the relationship between the environment and the economic system can be considered a *closed system*. For our purposes, a closed system is one in which no inputs (energy or matter) are received from outside the system and no outputs are transferred outside the system. An *open system*, by contrast, is one in which the system imports or exports matter or energy.

If we restrict our conception of the relationship in Figure 2.1 to our planet and the atmosphere around it, then clearly we do not have a closed system. We derive most of our energy from the sun, either directly or indirectly. We have also sent spaceships well beyond the boundaries of our atmosphere. Nonetheless, historically speaking, for *material* inputs and outputs (not including energy), this system can be treated as a closed system because the amount of exports (such as abandoned space vehicles) and imports (e.g., moon rocks) are negligible. Whether the system remains closed depends on the degree to which space exploration opens up the rest of our solar system as a source of raw materials.

The treatment of our planet and its immediate environs as a closed system has an important implication that is summed up in the *first law of thermodynamics*—energy and matter can neither be created nor destroyed.[1] The law implies that the mass of materials flowing into the economic system from the environment has either to accumulate in the economic system or return to the environment as waste. When accumulation stops, the mass of materials flowing into the economic system is equal in magnitude to the mass of waste flowing into the environment.

Excessive wastes can, of course, depreciate the asset; when they exceed the absorptive capacity of nature, wastes reduce the services that the asset provides. Examples are easy to find: air pollution can cause respiratory problems, polluted drinking water can cause cancer, smog obliterates scenic vistas, climate change can lead to flooding of coastal areas.

The relationship of people to the environment is also conditioned by another physical law, the *second law of thermodynamics*. Known popularly as the *entropy law*, this law states that "entropy increases." *Entropy* is the amount of energy unavailable for work. Applied to energy processes, this law implies that no conversion from one form of energy to another is completely efficient and that the consumption of energy is an irreversible process. Some energy is always lost during conversion, and the rest, once used, is no longer available for further work. The second law also implies that, in the absence of new energy inputs, any closed system must eventually use up its available energy. Since energy is necessary for life, life ceases when useful energy flows cease.

We should remember that our planet is not even approximately a closed system with respect to energy; we gain energy from the sun. The entropy law does remind us, however, that the flow of solar energy establishes an upper limit on the flow of available energy that can be sustained. Once the stocks of stored energy (such as fossil fuels and nuclear energy) are gone, the amount of energy available for useful work will be determined solely by flow resources such as solar, wind, and hydro, and by the amount that can be stored (through dams, trees, and so on). Thus, in the very long run, the growth process will be limited by the availability of these flow resources and our ability to put them to work.

The Economic Approach

Two different types of economic analysis can be applied to increase our understanding of the relationship between the economic system and the environment: *Positive* economics attempts to describe *what is*, *what was*, or *what will be*. *Normative* economics, by contrast, deals with

what *ought to be*. Disagreements within positive economics can usually be resolved by an appeal to the facts. Normative disagreements, however, involve value judgments.

Both branches are useful. Suppose, for example, we want to investigate the relationship between trade and the environment. Positive economics could be used to describe the kinds of impacts trade would have on the economy and the environment. It could not, however, provide any guidance on the question of whether trade was desirable. That judgment would have to come from normative economics, a topic we explore in the next section.

The fact that positive analysis does not, by itself, determine the desirability of some policy action does not mean that it is not useful in the policy process. Example 2.1 provides one example of the kinds of economic impact analyses that are used in the policy process.

EXAMPLE 2.1

Economic Impacts of Reducing Hazardous Pollutant Emissions from Iron and Steel Foundries

The U.S. Environmental Protection Agency (EPA) was tasked with developing a "maximum achievable control technology standard" to reduce emissions of hazardous air pollutants from iron and steel foundries. As part of the rule-making process, the EPA conducted an *ex ante* economic impact analysis to assess the potential economic impacts of the proposed rule.

If implemented, the rule would require some iron and steel foundries to implement pollution control methods that would increase the production costs at affected facilities. The interesting question addressed by the analysis is how large those impacts would be.

The impact analysis estimated annual costs for existing sources to be $21.73 million. These cost increases were projected to result in small increases in output prices. Specifically, prices were projected to increase by only 0.1 percent for iron castings and 0.05 percent for steel castings. The impacts of these price increases were expected to be experienced largely by iron foundries using cupola furnaces as well as consumers of iron foundry products. Unaffected domestic foundries and foreign producers of coke were actually projected to earn slightly higher profits as a result of the rule.

This analysis helped in two ways. First, by showing that the impacts fell under the $100 million threshold that mandates review by the Office of Management and Budget, the analysis eliminated the need for a much more time- and resource-consuming analysis. Second, by showing how small the expected impacts would be, it served to lower the opposition that might have arisen from unfounded fears of much more severe impacts.

Source: Office of Air Quality Planning and Standards, United States Environmental Protection Agency. (November 2002). *Economic Impact Analysis of Proposed Iron and Steel Foundries*. NESHAP Final Report; National Emissions Standards for Hazardous Air Pollutants for Iron and Steel Foundries. (April 17, 2007). Proposed Rule. *Federal Register*, 72(73), 19150–19164.

A rather different context for normative economics can arise when the possibilities are more open-ended. For example, we might ask, how much should we control emissions of greenhouse gases (which contribute to climate change) and how should we achieve that degree of control? Or we might ask, how much forest of various types should be preserved? Answering these questions requires us to consider the entire range of possible outcomes and to select the best or optimal one. Although that is a much more difficult question to answer than one that asks us only to compare two predefined alternatives, the basic normative analysis framework is the same in both cases.

Environmental Problems and Economic Efficiency

Static Efficiency

The chief normative economic criterion for choosing among various outcomes occurring at the same point in time is called *static efficiency*, or merely *efficiency*. An allocation of resources is said to satisfy the static efficiency criterion if the economic surplus derived from those resources is maximized by that allocation. Economic surplus, in turn, is the sum of consumer's surplus and producer's surplus.

Consumer surplus is the value that consumers receive from an allocation minus what it costs them to obtain it. Consumer surplus is measured as the area under the demand curve minus the consumer's cost. This is the shaded triangle in Figure 2.2. The cost to the consumer is the area under the price line, bounded from the left by the vertical axis and the right by the quantity of the good. This rectangle, which captures price times quantity, represents consumer expenditure on this quantity of the good.

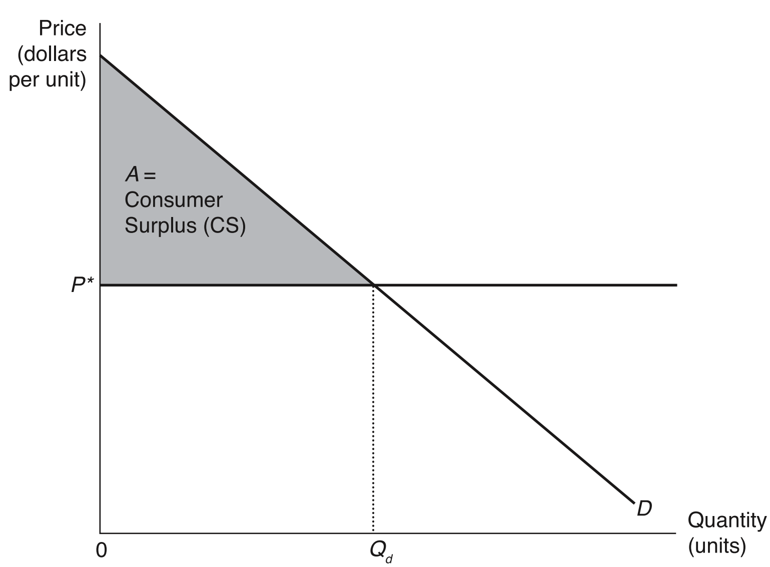

Figure 2.2 The Consumer's Choice

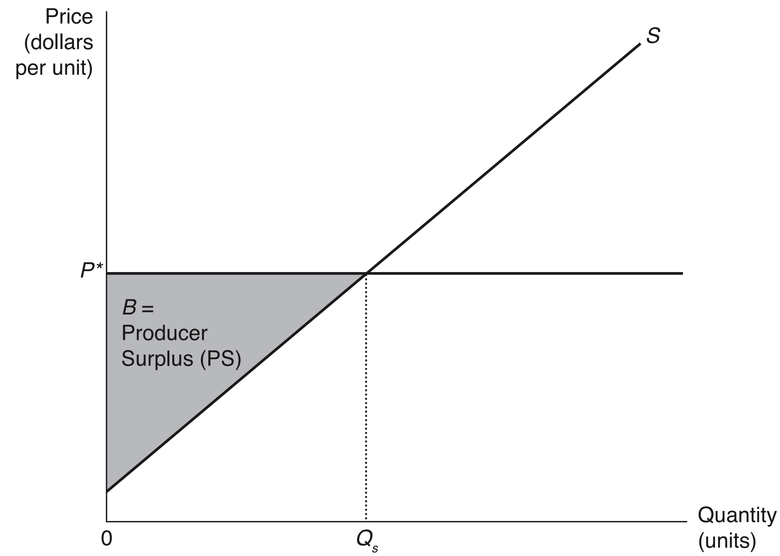

Figure 2.3 The Producer's Choice

Why is this area above the price line thought of as a surplus? For each quantity purchased, the corresponding point on the market demand curve represents the amount of money some person would have been willing to pay for the last unit of the good. The *total willingness to pay* for some quantity of this good—say, three units—is the sum of the willingness to pay for each of the three units. Thus, the total willingness to pay for three units would be measured by the sum of the willingness to pay for the first, second, and third units, respectively. It is now a simple extension to note that the total willingness to pay is the area under the continuous market demand curve to the left of the allocation in question. For example, in Figure 2.2 the total willingness to pay for Q_D units of the commodity is the total area under the demand curve up to Q_D. Thus it is the shaded triangle of consumer surplus plus the rectangle of cost. Total willingness to pay is the concept we shall use to define the total value a consumer would receive from the amount of the good they take delivery of. Thus, total value the consumer would receive is equal to the area under the market demand curve from the origin to the allocation of interest. Consumer surplus is thus the excess of total willingness to pay over the (lower) actual expenditure.

Meanwhile, sellers face a similar choice (see Figure 2.3). Given price P^*, the seller maximizes his or her own producer surplus by choosing to sell Q_s units. The *producer surplus* is designated by the shaded area *B*, the area under the price line that lies above the marginal cost curve (supply Curve S), bounded from the left by the vertical axis and the right by the quantity of the good. To calculate producer or consumer surplus, notice that as long as the functions are linear (as they are in the Figures), each area is represented as a right triangle. Remember that the area of a right triangle is calculated as 1/2 × the base of the triangle × the height of the triangle. Using this formula, try calculating these areas in the first self-test exercise at the end of this chapter.

Property Rights

Property Rights and Efficient Market Allocations

The manner in which producers and consumers use environmental resources depends on the property rights governing those resources. In economics, *property rights* refer to a bundle of entitlements defining the owner's rights, privileges, and limitations for use of the resource. By examining such entitlements and how they affect human behavior, we will better understand how environmental problems arise from government and market allocations.

These property rights can be vested either with individuals, groups or with the state. How can we tell when the pursuit of profits is consistent with efficiency and when it is not?

Efficient Property Rights Structures

Let's begin by describing the structure of property rights that could produce efficient allocations in a well-functioning market economy. An efficient structure has three main characteristics:

1. *Exclusivity*—All benefits and costs accrued as a result of owning and using the resources should accrue to the owner, and only to the owner, either directly or indirectly by sale to others.
2. *Transferability*—All property rights should be transferable from one owner to another in a voluntary exchange.
3. *Enforceability*—Property rights should be secure from involuntary seizure or encroachment by others.

An owner of a resource with a well-defined property right (one exhibiting these three characteristics) has a powerful incentive to use that resource efficiently because a decline in the value of that resource represents a personal loss. Farmers who own the land have an incentive to fertilize and irrigate it because the resulting increased production raises income. Similarly, they have an incentive to rotate crops when that raises the productivity of their land.

When well-defined property rights are exchanged, as in a market economy, this exchange facilitates efficiency. We can illustrate this point by examining the incentives consumers and producers face when a well-defined system of property rights is in place. Because the seller has the right to prevent the consumer from consuming the product in the absence of payment, the consumer must pay to receive the product. Given a market price, the consumer decides how much to purchase by choosing the amount that maximizes his or her individual consumer surplus.

Is this allocation efficient? According to our definition of static efficiency, it is clear the answer is yes. The economic surplus is maximized by the market allocation and, as seen in Figure 2.4, it is equal to the sum of consumer and producer surpluses (areas $A + B$). Thus, we have not only established a procedure for measuring efficiency, but also a means of describing how the surplus is distributed between consumers and producers.

This distinction is crucially significant. Efficiency is *not* achieved because consumers and producers are seeking efficiency. They aren't! In a system with well-defined property rights and competitive markets in which to sell those rights, producers try to maximize their surplus and consumers try to maximize their surplus. The price system, then, induces those self-interested parties to make choices that also turn out to be efficient from the point of view of society as a whole. It channels the energy motivated by self-interest into socially productive paths.

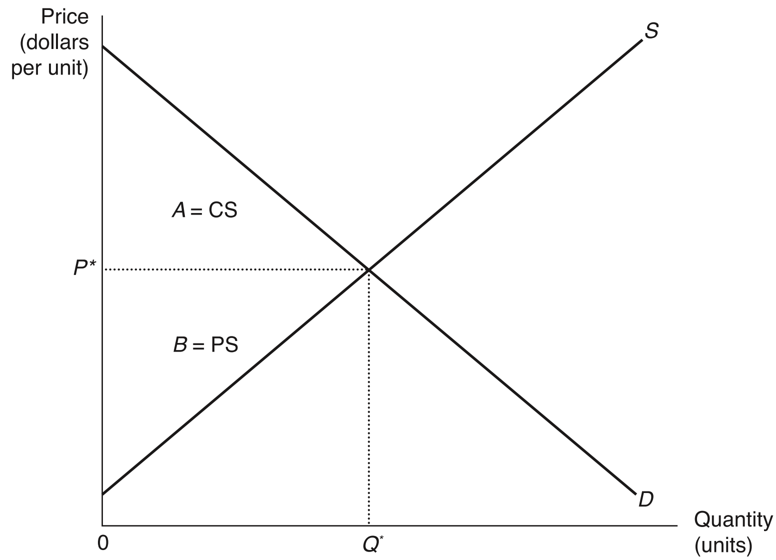

Figure 2.4 Market Equilibrium

Familiarity may have dulled our appreciation, but it is noteworthy that a system designed to produce a harmonious and congenial outcome could function effectively while allowing consumers and producers so much individual freedom in making choices. This is truly a remarkable accomplishment.

Producer's Surplus, Scarcity Rent, and Long-Run Competitive Equilibrium

Since the area under the price line is total revenue, and the area under the marginal cost (or supply) curve is total variable cost, producer's surplus is related to profits. In the short run when some costs are fixed, producer's surplus is equal to profits plus fixed cost. In the long run when all costs are variable, producer's surplus is equal to profits plus rent, the return to scarce inputs owned by the producer. As long as new firms can enter into profitable industries without raising the prices of purchased inputs, long-run profits and rent will equal zero.

Scarcity Rent. Most natural resource industries, however, do give rise to rent and, therefore, producer's surplus is not eliminated by competition, even with free entry. This producer's surplus, which persists in long-run competitive equilibrium, is called *scarcity rent*.

David Ricardo was the first economist to recognize the existence of scarcity rent. Ricardo suggested that the price of land was determined by the least fertile marginal unit of land. Since the price had to be sufficiently high to allow the poorer land to be brought into production, other, more fertile land could be farmed at an economic profit. Competition could not erode that profit because the amount of high-quality land was limited and lower prices would serve only to reduce the supply of land below demand. The only way to expand production would be to bring additional, less fertile land (more costly to farm) into production; consequently, additional production does not lower price, as it does in a constant-cost industry. As we shall see, other circumstances also give rise to scarcity rent for natural resources.

Externalities as a Source of Market Failure

The Concept Introduced

Exclusivity is one of the chief characteristics of an efficient property rights structure. This characteristic is frequently violated in practice. One broad class of violations occurs when an agent making a decision does not bear all of the consequences of his or her action.

Suppose two firms are located by a river. The first produces steel, while the second, somewhat downstream, operates a resort hotel. Both use the river, although in different ways. The steel firm uses it as a receptacle for its waste, while the hotel uses it to attract customers seeking water recreation. If these two facilities have different owners, an efficient use of the water is not likely to result. Because the steel plant does not bear the cost of reduced business at the resort resulting from waste being dumped into the river, it is not likely to be very sensitive to that cost in its decision making. As a result, it could be expected to dump too much waste into the river, and an efficient allocation of the river would not be attained.

This situation is called an externality. An *externality* exists whenever the welfare of some agent, either a firm or household, depends not only on his or her activities, but also on activities under the control of some other agent. In the example, the increased waste in the river imposed an external cost on the resort, a cost the steel firm could not be counted upon to consider appropriately in deciding the amount of waste to dump.

The effect of this external cost on the steel industry is illustrated in Figure 2.5, which shows the market for steel. Steel production inevitably involves producing pollution as well as steel. The demand for steel is shown by the demand curve *D*, and the private marginal cost of producing the steel (exclusive of pollution control and damage) is depicted as MC_p. Because society considers both the cost of pollution and the cost of producing the steel, the social marginal cost function (MC_s) includes both of these costs as well.

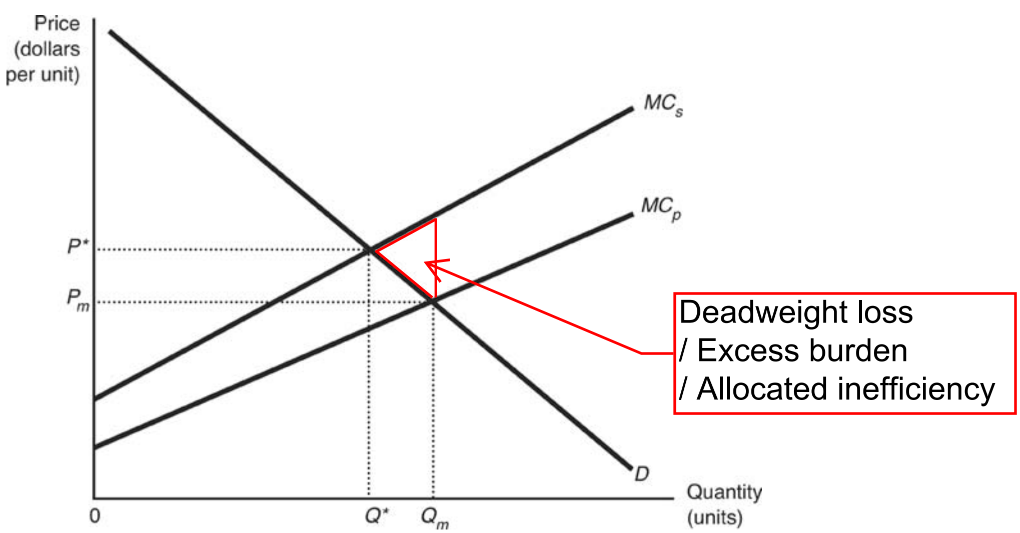

Figure 2.5 The Market for Steel

If the steel industry faced no outside control on its emission levels, it would seek to produce Q_m. That choice, in a competitive setting, would maximize its private producer surplus. But that is clearly not efficient, since the net benefit is maximized at Q^*, not Q_m. Can you see the deadweight loss? A deadweight loss arises whenever marginal social costs are not equal to marginal social benefits. In this case, at the market allocation, marginal social costs are higher than marginal benefits.

With the help of Figure 2.5, we can draw a number of conclusions about market allocations of commodities causing pollution externalities:

1. The output of the commodity is too large.
2. Too much pollution is produced.
3. The prices of products responsible for pollution are too low.
4. As long as the costs remain external, no incentives to search for ways to yield less pollution per unit of output are introduced by the market.
5. Recycling and reuse of the polluting substances are discouraged because release into the environment is so inefficiently cheap.

The effects of a market imperfection for one commodity end up affecting the demands for raw materials, labor, and so on. The ultimate effects are felt through the entire economy.

Types of Externalities

External effects, or externalities, can be positive or negative. Historically, the terms *external cost* (*external diseconomy*) and *external benefit* (*external economy*) have been used to refer, respectively, to circumstances in which the affected party is damaged by or benefits from the externality. Clearly, the water pollution example represents an external cost. External benefits are not hard to find, however. As noted in the opening quote to this chapter, private individuals who preserve a particularly scenic area provide an external benefit to all who pass. Generally, when external benefits are present, the market will undersupply the resources.

One other distinction is important. One class of externalities, known as *pecuniary externalities*, does not present the same kinds of problems as pollution does. Pecuniary externalities arise when the external effect is transmitted through altered prices. Suppose that a new firm moves into an area and drives up the rental price of land. That increase creates a negative effect on all those paying rent and, therefore, is an external diseconomy.

This pecuniary diseconomy, however, does not cause a market failure because the resulting higher rents are reflecting the true scarcity of land. The land market provides a mechanism by which the parties can bid for land; the resulting prices reflect the value of the land in its various uses. Without pecuniary externalities, the price signals would fail to sustain an efficient allocation.

The pollution example is *not* a pecuniary externality because the effect is not transmitted through prices. In this example, prices do not adjust to reflect the increasing waste load. The damage to the water resource is not reflected in the steel firm's costs. An essential feedback mechanism that is present for pecuniary externalities is not present for the pollution case.

The externalities concept is a broad one, covering a multitude of sources of market failure (Example 2.2 illustrates one). The next step is to investigate some specific circumstances that can give rise to externalities.

EXAMPLE 2.2

Shrimp Farming Externalities in Thailand

In the Tha Po village on the coast of Surat Thani Province in Thailand, more than half of the 1100 hectares of mangrove swamps have been cleared for commercial shrimp farms. Although harvesting shrimp is a lucrative undertaking, mangroves also serve as nurseries for fish and as barriers for storms and soil erosion. Following the destruction of the local mangroves, Tha Po villagers experienced a decline in fish catch and suffered storm damage and water pollution. Can market forces be trusted to strike the efficient balance between preservation and development for the remaining mangroves?

Calculations by economists Sathirathai and Barbier (2001) demonstrated that the value of the ecological services that would be lost from further destruction of the mangrove swamps exceeded the value of the shrimp farms that would take their place. Preservation of the remaining mangrove swamps would be the efficient choice.

Would a potential shrimp-farming entrepreneur make the efficient choice? Unfortunately, the answer is no. This study estimated the economic value of mangroves in terms of local use of forest resources, offshore fishery linkages, and coastal protection to be in the range of $27,264–$35,921 per hectare. In contrast, the economic returns to shrimp farming, once they are corrected for input subsidies and for the costs of water pollution, are only $194–$209 per hectare. However, as shrimp farmers are heavily subsidized and do not have to take into account the external costs of pollution, their financial returns are typically $7,706.95–$8,336.47 per hectare. In the absence of some sort of external control imposed by collective action, converting mangroves to shrimp farming would be the normal, if inefficient, result. The externalities associated with the ecological services provided by the mangroves support a biased decision that results in fewer social net benefits, but greater private net benefits.

Sources: Sathirathai, S., & Barbier, E. B. (April 2001). Valuing mangrove conservation in southern Thailand. *Contemporary Economic Policy*, *19*(2). 109–122; Barbier, E. B., & Cox, M. (2004). An economic analysis of shrimp farm expansion and mangrove conversion in Thailand. *Land Economics*, *80*(3), 389–407.

Perverse Incentives Arising from Some Property Right Structures

Private property is, of course, not the only possible way of defining entitlements to resource use. Other possibilities include:

- state-property regimes (the government owns and controls the property);
- common-property regimes (the property is jointly owned and managed by a specified group of co-owners); and
- *res nullius* or open-access regimes (in which no one owns or exercises control over the resources).

All of these create rather different incentives for resource use.

State-property regimes exist not only in former communist countries, but also to varying degrees in virtually all countries of the world. Parks and forests, for example, are frequently owned and managed by the government in capitalist as well as in socialist nations. Problems

with both efficiency and sustainability can arise in state-property regimes when the incentives of bureaucrats, who implement and/or make the rules for resource use, diverge from collective interests.

Common-property resources are those shared resources that are managed in common rather than privately. Entitlements to use common-property resources may be formal, protected by specific legal rules, or they may be informal, protected by tradition or custom. Common-property regimes exhibit varying degrees of efficiency and sustainability, depending on the rules that emerge from collective decision making. While some very successful examples of common-property regimes exist, unsuccessful examples are even more common.

One successful example of a common-property regime involves the system of allocating grazing rights in Switzerland. Although agricultural land is normally treated as private property, in Switzerland grazing rights on the Alpine meadows have been treated as common property for centuries. Overgrazing is protected by specific rules, enacted by an association of users, which limit the amount of livestock permitted on the meadow. The families included on the membership list of the association have been stable over time as rights and responsibilities have passed from generation to generation. This stability has apparently facilitated reciprocity and trust, thereby providing a foundation for continued compliance with the rules.

Unfortunately, that kind of stability may be the exception rather than the rule, particularly in the face of heavy population pressure. The more common situation can be illustrated by the experience of Mawelle, a small fishing village in Sri Lanka. Initially, a complicated but effective rotating system of fishing rights was devised by villagers to assure equitable access to the best spots and best times while protecting the fish stocks. Over time, population pressure and the infusion of outsiders raised demand and undermined the collective cohesion sufficiently that the traditional rules became unenforceable, producing overexploitation of the resource and lower incomes for all the participants.

Res nullius property resources, the main focus of this section, can be exploited on a first-come, first-served basis because no individual or group has the legal power to restrict access. *Open-access resources*, as we shall henceforth call them, have given rise to what has become known popularly as the "tragedy of the commons."

The problems created by open-access resources can be illustrated by recalling the fate of the American bison. Bison are an example of "common-pool" resources. *Common-pool resources* are shared resources characterized by nonexclusivity and divisibility. *Nonexclusivity* implies that resources can be exploited by anyone, while *divisibility* means that the capture of part of the resource by one group subtracts it from the amount available to the other groups. (Note the contrast between common-pool resources and public goods, the subject of the next section.) In the early history of the United States, bison were plentiful; unrestricted hunting access was not a problem. Frontier people who needed hides or meat could easily get whatever they needed; the aggressiveness of any one hunter did not affect the time and effort expended by other hunters. In the absence of scarcity, efficiency was not threatened by open access.

As the years slipped by, however, the demand for bison increased and scarcity became a factor. As the number of hunters increased, eventually every additional unit of hunting activity increased the amount of time and effort required to produce an additional yield of bison.

Consider graphically how various property rights structures (and the resulting level of harvest) affect the scarcity rent (in this case, equivalent to the economic surplus received by consumers and producers), where the amount of rent is measured as the difference between the revenues received from the harvest minus the costs associated with producing that harvest. Figure 2.6 compares the revenue and costs for various levels of harvest. In the top panel the

revenue is calculated by multiplying, for each level of hunting activity, the (assumed constant) price of bison by the amount harvested. The upward sloping total cost curve simply reflects that fact that increases in harvest effort result in higher total costs. (Marginal cost is assumed to be constant for this example.)

In terms of the top panel of Figure 2.6, the total surplus associated with any level of effort is measured as the vertical difference between the total revenue (benefits) curve and the total cost curve for that level of harvest.

In the bottom panel the marginal revenue curve is downward sloping (despite the constant price) because as the amount of hunting effort increases, the resulting bison population size decreases. Smaller populations support smaller harvests per unit of effort expended.

The efficient level of hunting activity in this model (E^*) maximizes the surplus. This can be seen graphically in two different ways. First, E^* maximizes the vertical difference between the total cost and total benefit (top panel). Second, in the bottom panel E^* is the level where the marginal revenue, which records the addition to the surplus from an additional unit of

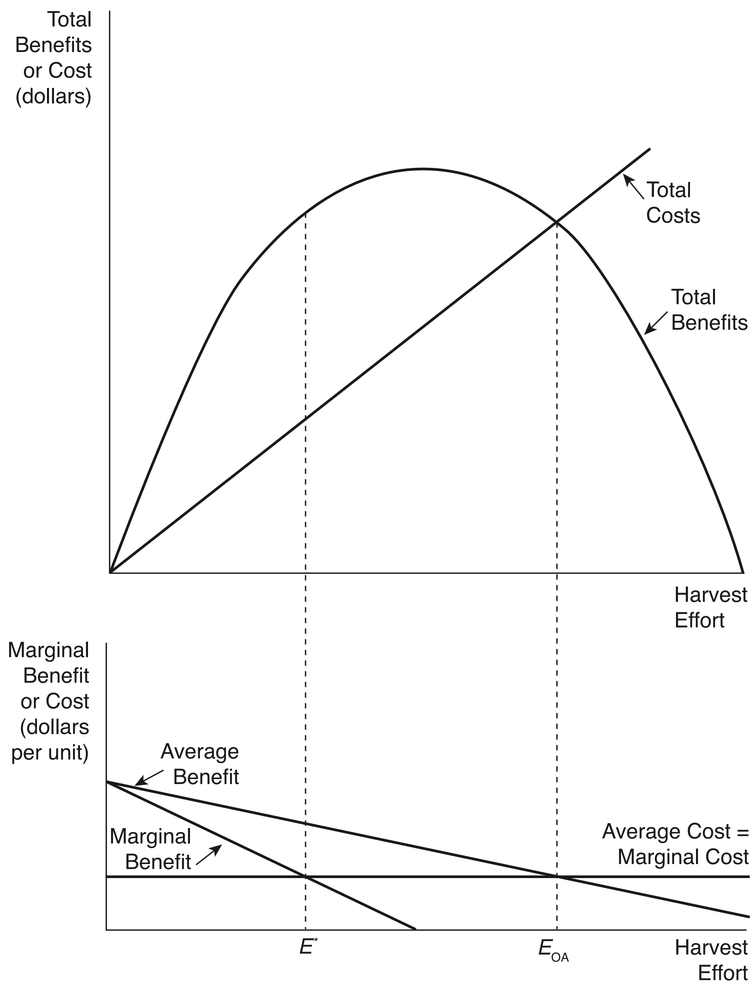

Figure 2.6 Bison Harvesting

effort, crosses the marginal cost curve, which measures the reduction in the surplus due to the additional cost of expending that last unit of effort. These two panels are simply two different (mathematically equivalent) ways to demonstrate the same outcome. (The curves in the bottom panel are derived from the curves in the top panel.)

With all hunters having completely unrestricted access to the bison, the resulting allocation would not be efficient. No individual hunter would have an incentive to protect scarcity rent by restricting hunting effort. Individual hunters, under this "open access" scenario (without exclusive rights), would exploit the resource until their total benefit equaled total cost, implying a level of effort equal to (E_{OA}). Excessive exploitation of the herd occurs because individual hunters cannot appropriate the scarcity rent; therefore, they ignore it. One of the losses from further exploitation that could be avoided by exclusive owners—the loss of scarcity rent due to overexploitation—is not part of the decision-making process of open-access hunters.

Two characteristics of this formulation of the open-access allocation are worth noting: (1) in the presence of sufficient demand, unrestricted access will cause resources to be overexploited; (2) the scarcity rent is dissipated—no one is able to appropriate the rent, so it is lost.

Why does this happen? Unlimited access destroys the incentive to conserve. A hunter who can preclude others from hunting his stock has an incentive to keep the herd at an efficient level. This restraint results in lower costs in the form of less time and effort expended to produce a given yield of bison. On the other hand, a hunter exploiting an open-access resource would not have an incentive to conserve because the potential additional economic surplus derived from self-restraint would, to some extent, be captured by other hunters who simply kept harvesting. Thus, unrestricted access to scarce resources promotes an inefficient allocation. As a result of excessive harvest and the loss of habitat as land was converted to farm and pasture, the Great Plains bison herds nearly became extinct (Lueck, 2002). Another example of open-access, fisheries, is the principal topic of Chapter 12.

Public Goods

Public goods, defined as those that exhibit both consumption indivisibilities and nonexcludability, present a particularly complex category of environmental resources. *Nonexcludability* refers to a circumstance where, once the resource is provided, even those who fail to pay for it cannot be excluded from enjoying the benefits it confers. Consumption is said to be *indivisible* when one person's consumption of a good does not diminish the amount available for others. Several common environmental resources are public goods, including not only the "charming landscape" referred to by Emerson, but also clean air, clean water, and biological diversity.[2]

Biological diversity includes two related concepts: (1) the amount of genetic variability among individuals within a single species, and (2) the number of species within a community of organisms. *Genetic diversity*, critical to species survival in the natural world, has also proved to be important in the development of new crops and livestock. It enhances the opportunities for crossbreeding and, thus, the development of superior strains. The availability of different strains was the key, for example, in developing new, disease-resistant barley.

Because of the interdependence of species within ecological communities, any particular species may have a value to the community far beyond its intrinsic value. Certain species contribute balance and stability to their ecological communities by providing food sources or holding the population of the species in check.

The richness of diversity within and among species has provided new sources of food, energy, industrial chemicals, raw materials, and medicines. Yet, considerable evidence suggests

that biological diversity is decreasing. Biodiversity is a valuable ecosystem service. Ecosystem services will be covered in detail in Chapter 13.

Can we rely solely on the private sector to produce the efficient amount of public goods, such as biological diversity? Unfortunately, the answer is no. Suppose that in response to diminishing ecological diversity we decide to take up a collection to provide some means of preserving endangered species. Would the collection yield sufficient revenue to pay for an efficient level of ecological diversity? The general answer is no. Let's see why.

In Figure 2.7, individual demand curves for preserving biodiversity have been presented for two consumers, A and B. The market demand curve is represented by the vertical summation of the two individual demand curves. A vertical summation is necessary because everyone can simultaneously consume the same amount of biological diversity. We are, therefore, able to determine the market demand by finding the sum of the amounts of money they would be willing to pay for that level of diversity.

What is the efficient level of diversity? It can be determined by a direct application of our definition of efficiency. The efficient allocation maximizes economic surplus, which is represented geometrically by the portion of the area under the market demand curve that lies above the constant marginal cost curve. The allocation that maximizes economic surplus is Q^*, the allocation where the demand curve crosses the marginal cost curve.

Why would a competitive market not be expected to supply the efficient level of this good? Since the two consumers have very different marginal willingness to pay from the efficient allocation of this good (*OA* versus *OB*), the efficient pricing system would require charging a different price to each consumer. Person *A* would pay *OA* and person *B* would pay *OB*. (Remember consumers tend to choose the level of the good that equates their marginal willingness to pay to the price they face.) Yet the producer would have no basis for figuring out how to differentiate the prices. In the absence of excludability, consumers are not likely to willingly reveal the strength of their preference for this commodity. All consumers have an incentive to understate the strength of their preferences to try to shift more of the cost burden to the other consumers.

Therefore, inefficiency results because each person is able to become a free rider on the other's contribution. A *free rider* is someone who derives the value from a commodity without paying an efficient amount for its supply. Because of the consumption indivisibility and nonexcludability properties of the public good, consumers receive the value of any diversity purchased by other people. When this happens it tends to diminish incentives to contribute, and the contributions are not sufficiently large to finance the efficient amount of the public good; it would be undersupplied. (In Chapter 17 we shall use the lens provided by game theory to show how the free rider effect helps to shape climate policy.)

The privately supplied amount may not be zero, however. Some diversity would be privately supplied. Indeed, as suggested by Example 2.3, the privately supplied amount may be considerable.

Imperfect Market Structures

Environmental problems also occur when one of the participants in an exchange of property rights is able to exercise an inordinate amount of power over the outcome. This can occur, for example, when a product is sold by a single seller, or *monopoly*.

It is easy to show that monopolies violate our definition of *efficiency* in the goods market (see Figure 2.8). According to our definition of *static efficiency*, the efficient allocation would result when *OB* is supplied. This would yield consumer surplus represented by triangle *IGC and* producer surplus denoted by triangle *GCH*. The monopoly, however, would produce and

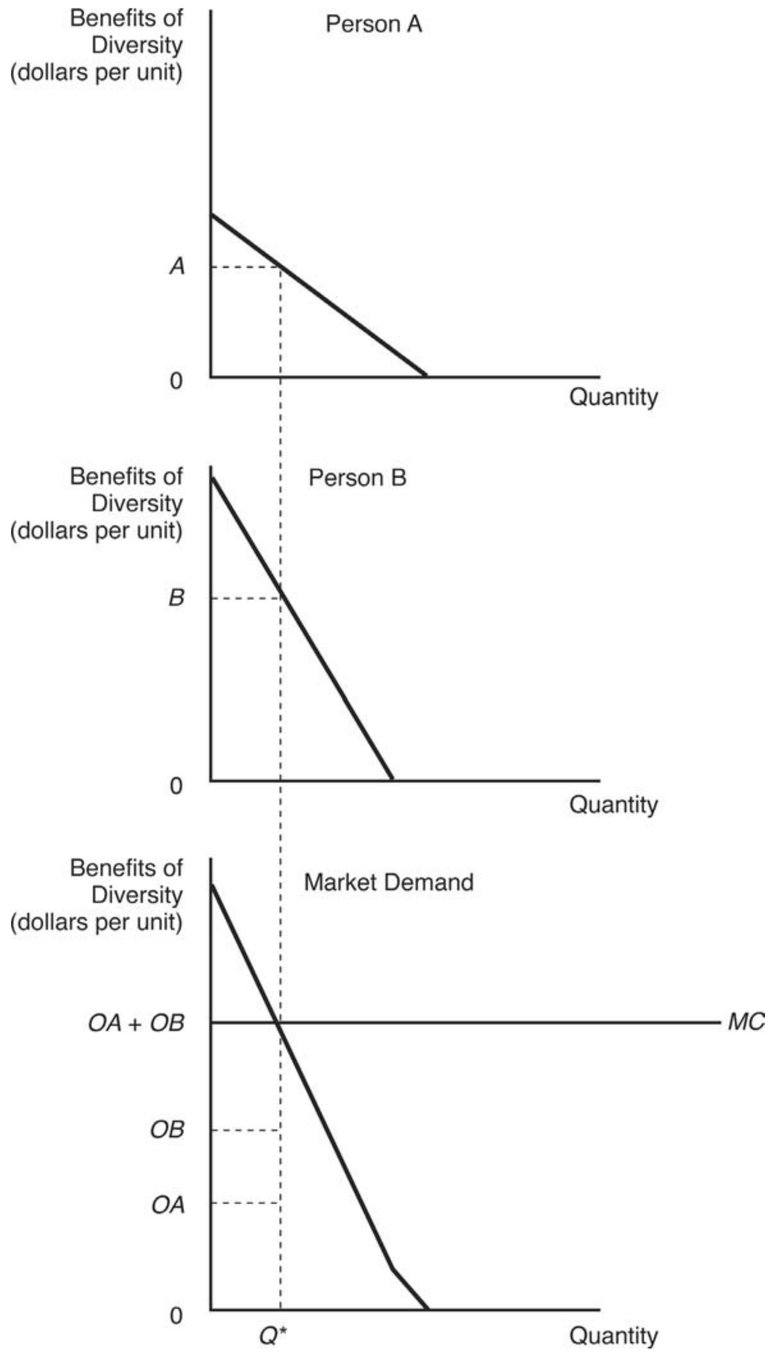

Figure 2.7 Efficient Provision of Public Goods

EXAMPLE 2.3

Public Goods Privately Provided: The Nature Conservancy

Can the demand for a public good such as biological diversity be observed in practice? Would the market respond to that demand? Apparently so, according to the existence of an organization called the Nature Conservancy.

The Nature Conservancy was born of an older organization called the Ecologist Union on September 11, 1950, for the purpose of establishing natural area reserves to aid in the preservation of areas, objects, and fauna and flora that have scientific, educational, or aesthetic significance. This organization purchases, or accepts as donations, land that has some unique ecological or aesthetic significance, to keep it from being used for other purposes. In so doing it preserves many species by preserving the habitat.

From humble beginnings, the Nature Conservancy has, as of 2017, been responsible for the preservation of 119 million acres of forests, marshes, prairies, mounds, and islands around the world. Additionally, the Nature Conservancy has protected 5000 miles of rivers and operates over 100 marine conservation projects. These areas serve as home to rare and endangered species of wildlife and plants. The Conservancy owns and manages the largest privately owned nature preserve system in the world.

This approach has considerable merit. A private organization can move more rapidly than the public sector. Because it has a limited budget, the Nature Conservancy sets priorities and concentrates on acquiring the most ecologically unique areas. Yet the theory of public goods reminds us that if this were to be the sole approach to the preservation of biological diversity, it would preserve a smaller-than-efficient amount.

Source: The Nature Conservancy, www.nature.org/about-us/vision-mission/index.htm?intc=nature.fnav.about

sell *OA*, where marginal revenue equals marginal cost, and would charge price *OF*. At this point, although the producer's surplus (*HFED*) is maximized, the sum of consumer and producer surplus is clearly not, because this choice causes society to lose economic surplus equal to triangle *EDC*.[3] Monopolies supply an inefficiently small amount of the good.

Imperfect markets clearly play some role in environmental problems. For example, the major oil-exporting countries have formed a cartel, resulting in higher-than-normal prices and lower-than-normal production. A *cartel* is a collusive agreement among producers to restrict production and raise prices. This collusive agreement allows the group to act as a monopolist. The inefficiency in the goods market would normally be offset to some degree by an associated reduction in social costs. The reduction in the combustion of oil would result in lower levels of pollution and, hence, the social cost associated with that pollution.

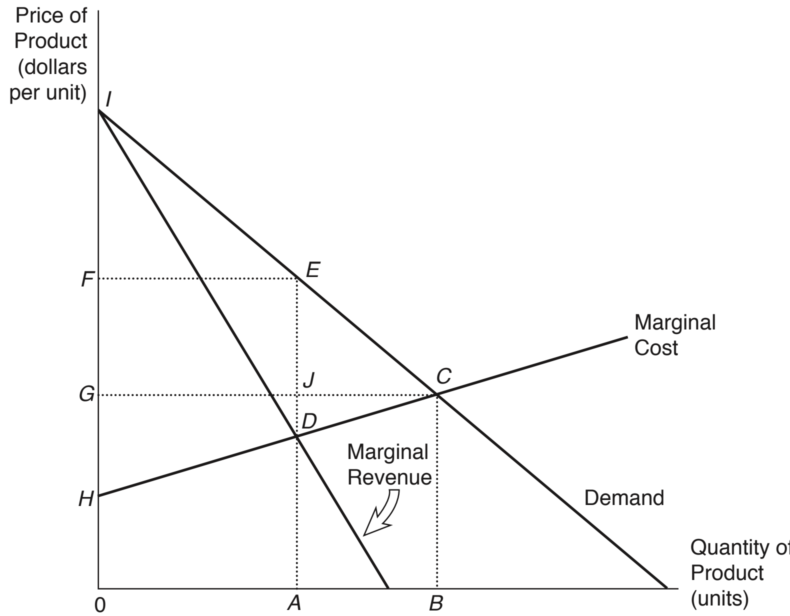

Figure 2.8 Monopoly and Inefficiency

Asymmetric Information

When all parties to a specific situation or transaction have access to the same amount of information about that situation, the information is said to be symmetrically distributed. If, however, one or more parties have more information than the others, the information distribution is said to be asymmetric.

Asymmetic information creates problems for the market when it results in a decision maker knowing too little to make an efficient choice. Suppose, for example, a consumer had a preference for organic food, but didn't know what food choices were truly organic. Since it would be relatively easy for producers to claim their produce was organically grown even if it were not, consumers who could not accurately distinguish truly organic produce from its fraudulent substitute would tend to be unwilling to pay a higher price for organic produce. As a result, both the profits and the output of organic farmers would be inefficiently low. If consumers do not have full information, negative externalities may result. (We shall encounter asymmetric information problems in several chapters, including energy, pollution control, toxic substances, and ecosystem services.)

Government Failure

Market processes are not the only sources of inefficiency. Political processes are fully as culpable. As will become clear in the chapters that follow, some environmental problems have arisen from a failure of political, rather than economic, institutions. To complete our study of the ability of institutions to allocate environmental resources, we must understand this source of inefficiency as well.

Government failure shares with market failure the characteristic that improper incentives are the root of the problem. Special interest groups use the political process to engage in what has become known as *rent seeking*. Rent seeking is the use of resources in lobbying and other activities directed at securing legislation that results in more profitable outcomes for those funding this activity. Successful rent-seeking activity will typically increase the net benefits going to the special interest group, but it will also frequently lower the surplus to society as a whole. In these instances, it is a classic case of the aggressive pursuit of a larger slice of the pie leading to a smaller pie.

Why don't the losers rise up to protect their interests? One main reason is voter ignorance. It is economically rational for voters to remain at least partially ignorant on many issues simply because of the high cost of keeping fully informed and the low probability that any single vote will be decisive. In addition, it is difficult for diffuse groups of individuals, each of whom is affected only to a small degree, to organize a coherent, unified opposition. Successful opposition is, in a sense, a public good, with its attendant tendency for free riding. Opposition to special interests would normally be underfunded, especially when the opposition is dispersed and the special interests are concentrated.

Rent seeking can take many forms. Producers can seek protection from competitive pressures brought by imports or can seek price floors to hold prices above their efficient levels. Consumer groups can seek price ceilings on goods or special subsidies to transfer part of their costs to the general body of taxpayers.

Rent seeking is not the only source of inefficient government policy. Sometimes governments act without full information and establish policies that are ultimately very inefficient. For example, some time ago, one technological strategy chosen by the government to control motor vehicle pollution involved adding a chemical substance (MTBE) to gasoline. Designed to promote cleaner combustion, this additive turned out to create a substantial water pollution problem.

Governments may also pursue social policy objectives that have the side effect of causing an environmental inefficiency. For example, looking back at Figure 2.5, suppose that the government, when pressured by lobbyists, decides to subsidize the production of steel. Figure 2.9 illustrates the outcome. The private marginal cost curve shifts down and to the right causing a further increase in production, lower prices, and even more pollution produced. Thus, the subsidy moves us even further away from where surplus is maximized at Q^*. The shaded triangle A shows the deadweight loss (inefficiency) without the subsidy. With the subsidy, the deadweight loss grows to areas $A + B + C$. This social policy has the side effect of increasing an environmental inefficiency.

In another example, in Chapter 7, we shall see how the desire to hold down natural gas prices for consumers led to subsequent shortages. These examples provide a direct challenge to the presumption that more direct intervention by the government automatically leads to either greater efficiency or greater sustainability.

These cases illustrate the general economic premise that environmental problems arise because of a divergence between individual and collective objectives. This is a powerful explanatory device because not only does it suggest why these problems arise, but it also suggests how they might be resolved—by realigning individual incentives to make them compatible with collective objectives. As self-evident as this approach may be, it is controversial when people disagree about whether the problem is our improper values or the improper translation of our quite proper values into action.

Economists have always been reluctant to argue that values of consumers are warped, because that would necessitate dictating the "correct" set of values. Both capitalism and democracy are based on the presumption that the majority knows what it is doing, whether it is casting ballots for representatives or dollar votes for goods and services.

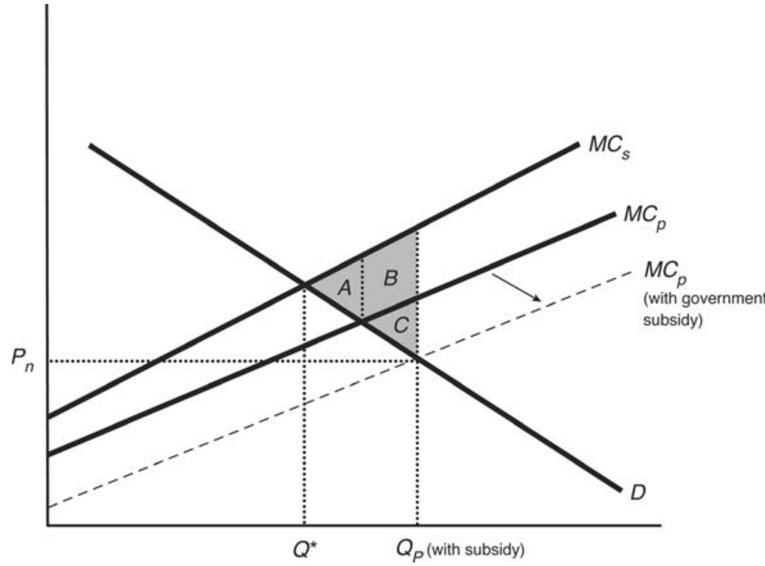

Figure 2.9 The Market for Steel Revisited

The Pursuit of Efficiency

We have seen that environmental problems can arise when property rights are ill defined, and when these rights are exchanged under something other than competitive conditions. We can now use our definition of efficiency to explore possible remedies, such as private negotiation, judicial remedies, and regulation by the legislative and executive branches of government.

Private Resolution through Negotiation—Property, Liability, and the Coase Theorem

The simplest means to restore efficiency occurs when the number of affected parties is small, making negotiation feasible. Suppose, for example, we return to the case used earlier in this chapter to illustrate an externality—the conflict between the polluting steel company and the downstream resort.

Figure 2.10 allows us to examine how this negotiation might take place. If the resort offers a payment of $C + D$ to the steel company, they would be better off if the steel firm responded by decreasing its production from Q_m to Q^*. Let's assume that the payment is equal to this amount. Would the steel company be willing to reduce production to the desired level? If they refused the compensation, their producer surplus would be $A + B + D$. If they accepted, their producer surplus would be $A + B$ plus the payment, so their total return would be $A + B + C + D$. Clearly, they are better off by C if they accept the payment. Society as a whole is better off by the amount C as well since the economic surplus from Q_m is $A - C$ and the economic surplus from Q^* is A.

Our discussion of individual negotiations raises two questions: (1) Should the property right always belong to the party who held it first (in this case the steel company)? (2) How can

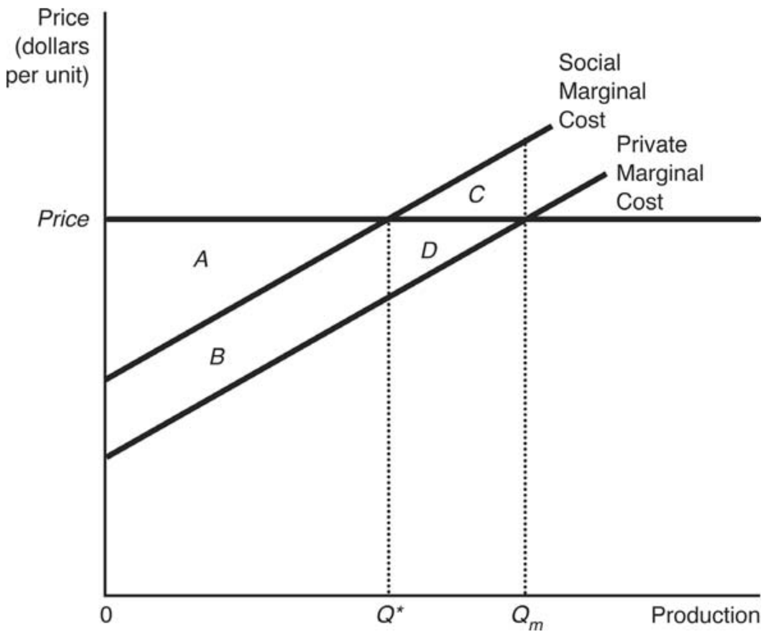

Figure 2.10 Efficient Output with Pollution Damage

environmental risks be handled when prior negotiation is clearly impractical? These questions are routinely answered by the court system.

The court system can respond to environmental conflicts by imposing either property rules or liability rules. Property rules specify the initial allocation of the entitlement or property right. The entitlements at conflict in our example are, on one hand, the right to add waste products to the river and, on the other, the right to an attractive river. In applying property rules, the court merely decides which right is preeminent and places an injunction against violating that right. The injunction is removed only upon obtaining the consent of the party whose right was violated. Consent is usually obtained in return for an out-of-court monetary settlement achieved via negotiation.

Note that, in the absence of a court decision, the entitlement is naturally allocated to the party that can most easily seize it. In our example, the natural allocation would give the entitlement to the steel company. The courts must decide whether to overturn this natural allocation.

How would they decide? And what difference would their decision make? The answers may surprise you.

In a classic article, economist Ronald Coase (1960) held that, as long as negotiation costs are negligible and affected consumers can negotiate freely with each other (when the number of affected parties is small), the court could allocate the entitlement to *either* party, and an efficient allocation would result. The only effect of the court's decision would be to change the distribution of surplus among the affected parties. This remarkable conclusion has come to be known as the *Coase theorem*.

Why is this so? In Figure 2.10, we showed that if the steel company has the property right, it is in the resort's interest to offer a payment that results in the desired level of output.

Suppose, now, that the resort has the property right instead. To pollute in this case, the steel company must pay the resort. Suppose it could pollute only if it compensated the resort for all damages. (In other words, it would agree to pay the difference between the two marginal cost curves up to the level of output actually chosen.) As long as this compensation was required, the steel company would choose to produce Q^* since that is the level at which its producer's surplus, given the compensation, is maximized. (Note that, due to the compensation, the curve the steel company uses to calculate its producer surplus is now the higher marginal cost curve.)

The difference between these different ways of allocating property rights lies in how the cost of obtaining the efficient level of output is shared between the parties. When the property right is assigned to the steel company, the cost is borne by the resort (part of the cost is the damage and part is the payment to reduce the level of damage). When the property right is assigned to the resort, the cost is borne by the steel company (it now must compensate for all damage). In either case, the efficient level of production results. The Coase theorem shows that the very existence of an inefficiency triggers pressures for improvements. Furthermore, the existence of this pressure does not depend on the assignment of property rights.

This is an important point, but the importance of this theorem should not be overstated. As we shall see in succeeding chapters, private efforts triggered by inefficiency can frequently prevent the worst excesses of environmental degradation. However, both theoretical and practical objections can be raised. The chief theoretical qualification concerns the assumption that wealth effects do not matter. The decision to confer the property right on a particular party results in a transfer of wealth to that party. This transfer might shift the demand curve for either steel or resorts out, as long as higher incomes result in greater demand. Whenever wealth effects are significant, the type of property rule issued by the court does affect the outcome.

Wealth effects normally are small, so the zero-wealth-effect assumption is probably not a fatal flaw. Some serious practical flaws, however, do mar the usefulness of the Coase theorem. The first involves the incentives for polluting that result when the property right is assigned to the polluter. Since pollution would become a profitable activity with this assignment, other polluters might be encouraged to increase production and pollution in order to earn the payments. That certainly would not be efficient.

Negotiation is also difficult to apply when the number of people affected by the pollution is large. You may have already noticed that in the presence of several affected parties, pollution reduction is a public good. The free-rider problem would make it difficult for the group to act cohesively and effectively for the restoration of efficiency.

When individual negotiation is not practical for one reason or another, the courts can turn to liability rules. These are rules that award monetary damages, after the fact, to the injured party. The amount of the award is designed to correspond to the amount of damage inflicted. Thus, returning to Figure 2.10, a liability rule would force the steel company to compensate the resort for all damages incurred. In this case, it could choose any production level it wanted, but it would have to pay the resort an amount of money equal to the area between the two marginal cost curves from the origin to the chosen level of output. In this case the steel plant would maximize its producer's surplus by choosing Q^*. (Why wouldn't the steel plant choose to produce more than that? Why wouldn't the steel plant choose to produce less than that?)

The moral of this story is that appropriately designed liability rules can also correct inefficiencies by forcing those who cause damage to bear the cost of that damage. Internalizing previously external costs causes profit-maximizing decisions to be compatible with efficiency. As we shall see in subsequent chapters, this "internalizing externalities" principle

plays a large role in the design of efficient policy in many areas of environmental and natural resource policy.

Liability rules are interesting from an economics point of view because early decisions create precedents for later ones. Imagine, for example, how the incentives to prevent oil spills facing an oil company are transformed once it has a legal obligation to clean up after an oil spill and to compensate fishermen for reduced catches. It quickly becomes evident that in this situation, accident prevention can become cheaper than retrospectively dealing with the damage once it has occurred.

This approach, however, also has its limitations. It relies on a case-by-case determination based on the unique circumstances for each case. Administratively, such a determination is very expensive. Expenses, such as court time, lawyers' fees, and so on, fall into a category called *transaction costs* by economists. In the present context, these are the administrative costs incurred in attempting to correct the inefficiency. The Coase theorem relies on an assumption that transaction costs are low. In reality though, this is rarely the case. Transaction costs in many cases can be quite high. When the number of parties involved in a dispute is large and the circumstances are common, we are tempted to correct the inefficiency by statutes or regulations rather than court decisions.

Legislative and Executive Regulation

These remedies can take several forms. The legislature could dictate that no one produce more steel or pollution than Q^*. This dictum might then be backed up with sufficiently large jail sentences or fines to deter potential violators. Alternatively, the legislature could impose a tax on steel or on pollution. A per-unit tax equal to the vertical distance between the two marginal cost curves would work (see Figure 2.10).

Legislatures could also establish rules to permit greater flexibility and yet reduce damage. For example, zoning laws would establish separate areas for steel plants and resorts. This approach assumes that the damage can be substantially reduced by keeping nonconforming uses apart.

They could also require the installation of particular pollution control equipment (as when catalytic converters were required on automobiles), or deny the use of a particular production ingredient (as when lead was removed from gasoline). In other words, they can regulate outputs, inputs, production processes, emissions, and even the location of production in their attempt to produce an efficient outcome. In subsequent chapters, we shall examine the various options policymakers have not only to show how they can modify environmentally destructive behavior, but also to establish the degree to which they can promote efficiency.

Payments are, of course, not the only means victims have at their disposal for lowering pollution. When the victims also consume the products produced by the polluters, consumer boycotts are possible. When the victims are employed by the polluter, strikes or other forms of labor resistance are also possible.

Legislation and/or regulation can also help to resolve the asymmetric information problem. Because the fundamental problem is that one or more of the parties do not have sufficient crucial, trustworthy information, the obvious solution involves providing that information. How should that information be provided?

Labeling is one attempt to provide more information to consumers. Examples of labeling for food products include notifying consumers about products containing genetically modified organisms, and identifying organically grown crops and fair trade products.

A recent source of encouragement for organic farms has been the demonstrated willingness of consumers to pay a premium for organically grown fruits and vegetables. To allow

consumers to discern which products are truly organic, growers need a reliable certification process. Additionally, fear of lost access to important foreign markets, such as the European Union, led to an industry-wide push in the United States for *mandatory* labeling standards that would provide the foundation for a national uniform seal. (*Voluntary* U.S. certification programs had proved insufficient to assure access to European markets, since they were highly variable by state.)

In response to these pressures, the Organic Foods Production Act (OFPA) was enacted in the 1990 Farm Bill.[4] Title 21 of that law states the following objectives:

> (1) to establish national standards governing the marketing of certain agricultural products as organically produced; (2) to assure consumers that organically produced products meet a consistent standard; and (3) to facilitate interstate commerce in fresh and processed food that is organically produced.[5]

The USDA National Organic Program, established as part of this Act, is responsible for a mandatory certification program for organic production. The Act also established the National Organic Standards Board (NOSB) and charged it with defining the "organic" standards. The new rules, which took effect in October 2002, require certification by the USDA for labeling. Foods labeled as "100 percent organic" must contain only organic ingredients. Foods labeled as "organic" must contain at least 95 percent organic agricultural ingredients, excluding water and salt. Products labeled as "Made with Organic Ingredients" must contain at least 70 percent organic agricultural ingredients.

Certification allows socially conscious consumers to make a difference. As Example 2.4 demonstrates, eco-certification for coffee seems to be one such case.

An Efficient Role for Government

While the economic approach suggests that government action could well be used to restore efficiency, it also suggests that inefficiency is not a sufficient condition to justify government intervention. Any corrective mechanism involves transaction costs. If these transaction costs are high enough, and the surplus to be derived from correcting the inefficiency is small enough, then it is best simply to live with the inefficiency.

Consider, for example, the pollution problem. Wood-burning stoves, which were widely used for cooking and heat in the late 1800s in the United States, were sources of pollution, but because of the enormous capacity of the air to absorb the emissions, no regulation resulted. More recently, however, the resurgence of demand for wood-burning stoves in cold climates with nearby forests, precipitated in part by high oil prices, has resulted in strict regulations for wood-burning stove emissions because the population density is so much higher.

Over time, the scale of economic activity and the resulting emissions have increased. Cities are experiencing severe problems from air and water pollutants because of the clustering of activities. Both the increase in the number of emitters and their clustering have increased the amount of emissions per unit volume of air or water. As a result, pollutant concentrations have caused perceptible problems with human health, vegetation growth, and aesthetics.

Historically, as incomes have risen, the demand for leisure activities has also risen. Many of these leisure activities, such as canoeing and backpacking, take place in unique, pristine environmental areas. With the number of these areas declining as a result of conversion to other uses, the value of remaining areas has increased. Thus, the values derived from protecting

EXAMPLE 2.4

Can Eco-Certification Make a Difference? Organic Costa Rican Coffee

Environmental problems associated with agricultural production for export in developing countries can be difficult to tackle using conventional regulation because producers are typically so numerous and dispersed, while regulatory agencies are commonly inadequately funded and staffed. In principle, eco-certification of production could circumvent these problems by providing a means for the socially conscious consumer to identify environmentally superior products, thereby providing a basis for paying a price premium for them. These premiums, in turn, would create financial incentives for producers to meet the certification standards.

Do socially conscious buyers care enough to actually pay a price premium that is high enough to motivate changes in the way the products are produced? Apparently, for Costa Rican coffee at least, they are.

One study examined this question for certified organic coffee grown in Turrialba, Costa Rica, an agricultural region in the country's central valley, about 40 miles east of San José, the capital city. This is an interesting case because Costa Rican farmers face significant pressure from the noncertified market to lower their costs, a strategy that can have severe environmental consequences. In contrast, organic production typically not only involves higher labor costs, but the conversion from chemically based production can also reduce yields. In addition, the costs of initial certification and subsequent annual monitoring and reporting are significant.

The authors found that organic certification did improve coffee growers' environmental performance. Specifically, they found that certification significantly reduced the use of pesticides, chemical fertilizers, and herbicides, and increased the use of organic fertilizer. In general, their results suggest that organic certification has a stronger causal effect on preventing negative practices than on encouraging positive ones. The study notes that this finding is consistent with anecdotal evidence that local inspectors tend to enforce the certification standards prohibiting negative practices more vigorously than the standards requiring positive ones.

Source: Blackman, A., & Naranjo, M. A. (2012). Does eco-certification have environmental benefits? Organic coffee in Costa Rica. *Ecological Economics*, *83*(November), 58–66.

some areas have risen over time until they have exceeded the transaction costs of protecting them from pollution and/or development.

The level and concentration of economic activity has increased pollution problems and driven up the demand for clean air and pristine areas. These changes have created the preconditions for government action. Can government respond efficiently or will rent seeking prevent efficient political solutions? We devote much of this book to pinning down the answer to that question.

Summary

How producers and consumers use the resources making up the environmental asset depends on the nature of the entitlements embodied in the property rights governing resource use. When property rights systems are exclusive, transferable, and enforceable, the owner of a resource has a powerful incentive to use that resource efficiently, since the failure to do so results in a personal loss.

The economic system will not always sustain efficient allocations, however. Specific circumstances that could lead to inefficient allocations include externalities, improperly defined property rights systems (such as open-access resources and public goods), imperfect markets for trading the property rights to the resources (monopoly), and asymmetric information. When these circumstances arise, market allocations typically do not maximize the surplus.

Due to rent-seeking behavior by special interest groups or the less-than-perfect implementation of efficient plans, the political system can produce inefficiencies as well. Voter ignorance on many issues, coupled with the public-good nature of any results of political activity, tends to create a situation in which maximizing an individual's private surplus (through lobbying, for example) can be at the expense of a lower economic surplus for all consumers and producers.

The efficiency criterion can be used to assist in the identification of circumstances in which our political and economic institutions lead us astray. It can also assist in the search for remedies by facilitating the design of regulatory, judicial, or legislative solutions.

Discussion Questions

1. In a well-known legal case, *Miller v. Schoene* (287 U.S. 272), a classic conflict of property rights was featured. Red cedar trees, used only for ornamental purposes, carried a disease that could destroy apple orchards within a radius of 2 miles. There was no known way of curing the disease except by destroying the cedar trees or by ensuring that apple orchards were at least 2 miles away from the cedar trees. Apply the Coase theorem to this situation. Does it make any difference to the outcome whether the cedar tree owners are entitled to retain their trees or the apple growers are entitled to be free of them? Why or why not?
2. In primitive societies, the entitlements to use land were frequently possessory rights rather than ownership rights. Those on the land could use it as they wished, but they could not transfer it to anyone else. One could acquire a new plot by simply occupying and using it, leaving the old plot available for someone else. Would this type of entitlement system cause more or less incentive to conserve the land than an ownership entitlement? Why? Would a possessory entitlement system be more efficient in a modern society or a primitive society? Why?
3. In this chapter we have discussed how markets work. Recently some new markets have emerged that focus on sharing of durable goods among a wider circle of users. Examples include Airbnb and Uber. The rise of these sharing markets may well have an impact on the relationship between the economy and the environment.
    a. What are the market niches these firms have found? How is Airbnb different from Hilton? How is Uber different from Hertz or Yellow Cab? Is this a matter mainly of a different type of supply or is the demand side affected as well?

b. Why now? Markets for personal transportation and temporary housing have been around for a long time. How can these new companies find profitable opportunities in markets that have existed for some time? Is it evidence that the markets are not competitive? Or have the new opportunities been created by some changes in market conditions?
c. Are these new sharing markets likely on balance to be good for or harmful to the environment? Why?

Self-Test Exercises

1. Suppose the state is trying to decide how many miles of a very scenic river it should preserve. There are 100 people in the community, each of whom has an identical inverse demand function given by $P = 10 - 1.0q$, where q is the number of miles preserved and P is the per-mile price he or she is willing to pay for q miles of preserved river. (a) If the marginal cost of preservation is $500 per mile, how many miles would be preserved in an efficient allocation? (b) How large is the economic surplus?
2. Suppose the market demand function (expressed in dollars) for a normal product is $P = 80 - q$, and the marginal cost (in dollars) of producing it is $MC = 1q$, where P is the price of the product and q is the quantity demanded and/or supplied.

   a. How much would be supplied by a competitive market?
   b. Compute the consumer surplus and producer surplus. Show that their sum is maximized.
   c. Compute the consumer surplus and the producer surplus assuming this same product was supplied by a monopoly. (*Hint*: The marginal revenue curve has twice the slope of the demand curve.)
   d. Show that, when this market is controlled by a monopoly, producer surplus is larger, consumer surplus is smaller, and the sum of the two surpluses is smaller than when the market is controlled by competitive industry.

3. Suppose you were asked to comment on a proposed policy to control oil spills. Since the average cost of an oil spill has been computed as $\$X$, the proposed policy would require any firm responsible for a spill immediately to pay the government $\$X$. Is this likely to result in the efficient amount of precaution against oil spills? Why or why not?
4. "In environmental liability cases, courts have some discretion regarding the magnitude of compensation polluters should be forced to pay for the environmental incidents they cause. In general, however, the larger the required payments the better." Discuss.
5. Label each of the following propositions as descriptive or normative and defend your choice:

   a. Energy efficiency programs have created jobs.
   b. Money spent on protecting endangered species is wasted.
   c. Fisheries must be privatized to survive.
   d. Raising transport costs lower suburban land values.
   e. Birth control programs are counterproductive.

6. Identify whether each of the following resource categories is a public good, a common-pool resource, or neither and defend your answer:

   a. A pod of whales in the ocean to whale hunters.
   b. A pod of whales in the ocean to whale watchers.

c. The benefits from reductions of greenhouse gas emissions.
d. Water from a town well that excludes nonresidents.
e. Bottled water.

Notes

1 We know, however, from Einstein's famous equation ($E = mc^2$) that matter can be transformed into energy. This transformation is the source of energy in nuclear power.
2 Notice that public "bads," such as dirty air and dirty water, are also possible.
3 Producers would lose area *JDC* compared to the efficient allocation, but they would gain area *FEJG*, which is much larger. Meanwhile, consumers would be worse off, because they lose area *FECJG*. Of these, *FEJG* is merely a transfer to the monopoly, whereas *EJC* is a pure loss to society. The total pure loss (*EDC*) is called a *deadweight loss*.
4 The European Union has followed a similar, but not identical, policy.
5 Golan et al. (2001).

Further Reading

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Mason, C. F. (2012). The economics of eco-labeling: Theory and empirical implications. *International Review of Environmental and Resource Economics*, *6*, 341–372. A survey of the growing literature on eco-labeling.

Ostrom, E. (1992). *Crafting Institutions for Self-Governing Irrigation Systems*. San Francisco, CA: ICS Press. A classic book by a Nobel Prize laureate that demonstrates that in favorable circumstances common-pool problems can sometimes be solved by voluntary organizations, rather than by a coercive state; among the cases considered are communal tenure in meadows and forests, irrigation communities, and fisheries.

Ostrom, E., Dietz, T., Dolsak, N., Stern, P., Stonich, S., & Weber, E. U. (Eds.). (2002). *The Drama of the Commons*. Washington, D.C.: National Academy Press. A compilation of articles and papers on common-pool resources.

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Additional references and historically significant references are available on this book's Companion Website: www.routledge.com/cw/Tietenberg

Chapter 3

Evaluating Trade-Offs

Benefit-Cost Analysis and Other Decision-Making Metrics

> No sensible decision can be made any longer without taking into account not only the world as it is, but the world as it will be.
>
> —Isaac Asimov, U.S. science fiction novelist and scholar (1920–1992)

Introduction

In the last chapter we noted that economic analysis has both positive and normative dimensions. The normative dimension helps to separate the policies that make sense from those that don't. Since resources are limited, it is not possible to undertake all ventures that might appear desirable, so making choices is inevitable.

Normative analysis can be useful in public policy in several different situations. It might be used, for example, to evaluate the desirability of a proposed new pollution control regulation or a proposal to preserve an area currently scheduled for development. In these cases, the analysis helps to provide guidance on the desirability of a program before that program is put into place. In other contexts, it might be used to evaluate how an already-implemented program has worked out in practice. Here the relevant question is: Was this a wise use of resources? In this chapter, we present and demonstrate the use of several decision-making metrics that can assist us in evaluating options.

Normative Criteria for Decision Making

Normative choices can arise in two different contexts. In the first context, we need simply to choose among options that have been predefined, while in the second we try to find the optimal choice among all the possible options.

Evaluating Predefined Options: Benefit-Cost Analysis

If you were asked to evaluate the desirability of some proposed action, you would probably begin by attempting to identify both the gains and the losses from that action. If the gains exceed the losses, then it seems natural to support the action.

That simple framework provides the starting point for the normative approach to evaluating policy choices in economics. Economists suggest that actions have both benefits and costs. If the benefits exceed the costs, then the action is desirable. On the other hand, if the costs exceed the benefits, then the action is not desirable. (Comparing benefits and costs across time will be covered later in this chapter.)

We can formalize this in the following way. Let B be the benefits from a proposed action and C be the costs. Our decision rule would then be

if $B > C$, support the action.

Otherwise, oppose the action.[1]

As long as B and C are positive, a mathematically equivalent formulation would be

if $B/C > 1$, support the action.

Otherwise, oppose the action.

So far so good, but how do we measure benefits and costs? In economics, the system of measurement is anthropocentric, which simply means human centered. All benefits and costs are valued in terms of their effects (broadly defined) on humanity. As shall be pointed out later, that does *not* imply (as it might first appear) that ecosystem effects are ignored unless they *directly* affect humans. The fact that large numbers of humans contribute voluntarily to organizations that are dedicated to environmental protection provides ample evidence that humans place a value on environmental preservation that goes well beyond any direct use they might make of it. Nonetheless, the notion that humans are doing the valuing is a controversial point that will be revisited and discussed in Chapters 4 and 13, along with the specific techniques for valuing these effects.

In benefit-cost analysis, benefits are measured simply as the relevant area under the demand curve since the demand curve reflects consumers' willingness to pay. Total costs are measured by the relevant area under the marginal cost curve.

It is important to stress that environmental services have costs even though they are produced without any human input. All costs should be measured as opportunity costs. To firm up this notion of opportunity cost, consider an example. Suppose a particular stretch of river can be used either for white-water rafting or to generate electric power. Since the dam that generates the power would flood the rapids, the two uses are incompatible. The opportunity cost of producing power is the foregone net benefit that would have resulted from the white-water rafting. The *marginal opportunity cost curve* defines the additional cost of producing another unit of electricity resulting from the associated incremental loss of net benefits due to reduced opportunities for white-water rafting.

Since net benefit is defined as the excess of benefits over costs, it follows that net benefit is equal to that portion of the area under the demand curve that lies above the supply curve.

Consider Figure 3.1, which illustrates the net benefits from preserving a stretch of river. Suppose that we are considering preserving a 4-mile stretch of river and that the benefits and costs of that action are reflected in Figure 3.1. Should that stretch be preserved? Why or why not? Hold on to your answer because we will return to this example later.

Finding the Optimal Outcome

In the preceding section, we examined how benefit-cost analysis can be used to evaluate the desirability of specific actions. In this section, we want to examine how this approach can be used to identify "optimal," or best, approaches.

In subsequent chapters, which address individual environmental problems, the normative analysis will proceed in three steps. First, we will identify an optimal outcome. Second, we will attempt to discern the extent to which our institutions produce optimal outcomes and, where divergences occur between actual and optimal outcomes, to attempt to uncover the behavioral sources of the problems. Finally, we can use both our knowledge of the nature of the problems and their underlying behavioral causes as a basis for designing appropriate policy solutions. Although applying these three steps to each of the environmental problems must reflect the uniqueness of each situation, the overarching framework used to shape that analysis remains the same.

To provide some illustrations of how this approach is used in practice, consider two examples: one drawn from natural resource economics and another from environmental economics. These are meant to be illustrative and to convey a flavor of the argument; the details are left to upcoming chapters.

Consider the rising number of depleted ocean fisheries. Depleted fisheries, which involve fish populations that have fallen so low as to threaten their viability as commercial fisheries, not only jeopardize oceanic biodiversity, but also pose a threat to both the individuals who make their living from the sea and the communities that depend on fishing to support their local economies.

How would an economist attempt to understand and resolve this problem? The first step would involve defining the optimal stock or the optimal rate of harvest of the fishery. The second step would compare this level with the actual stock and harvest levels. Once this economic framework is applied, not only does it become clear that stocks are much lower than optimal for many fisheries, but also the reason for excessive exploitation becomes clear. Understanding the nature of the problem has led quite naturally to some solutions. Once implemented, these

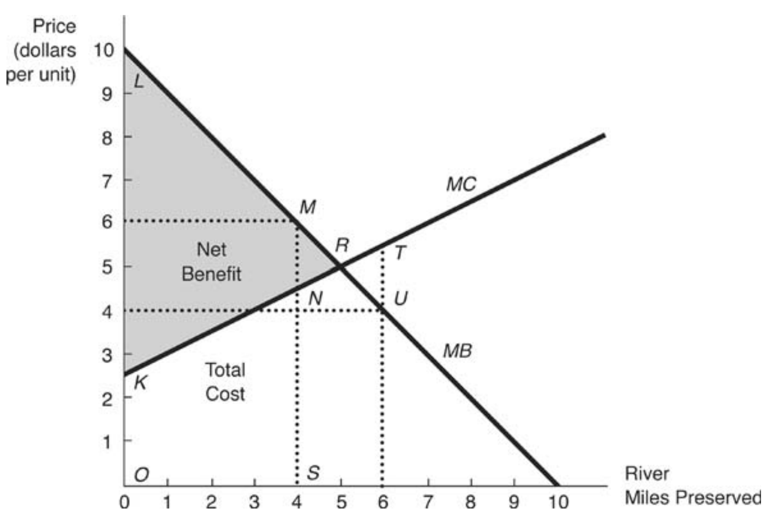

Figure 3.1 The Derivation of Net Benefits

policies have allowed some fisheries to begin the process of renewal. The details of this analysis and the policy implications that flow from it are covered in Chapter 12.

Another problem involves solid waste. As local communities run out of room for landfills in the face of an increasing generation of waste, what can be done?

Economists start by thinking about how one would define the optimal amount of waste. The definition necessarily incorporates waste reduction and recycling as aspects of the optimal outcome. The analysis not only reveals that current waste levels are excessive, but also suggests some specific behavioral sources of the problem. Based upon this understanding, specific economic solutions have been identified and implemented. Communities that have adopted these measures have generally experienced lower levels of waste and higher levels of recycling. The details are spelled out in Chapter 8.

In the rest of the book, similar analysis is applied to energy, minerals, water, pollution, climate change, and a host of other topics. In each case, the economic analysis helps to point the way toward solutions. To initiate that process, we must begin by defining "optimal."

Relating Optimality to Efficiency

According to the normative choice criterion introduced earlier in this chapter, desirable outcomes are those where the benefits exceed the costs. It is therefore a logical next step to suggest that optimal polices are those that maximize net benefits (benefits minus costs). The concept of *static efficiency*, or merely *efficiency*, was introduced in Chapter 2. An allocation of resources is said to satisfy the static efficiency criterion if the economic surplus from the use of those resources is maximized by that allocation. Notice that the net benefits area to be maximized in an "optimal outcome" for public policy is identical to the "economic surplus" that is maximized in an efficient allocation. Hence, efficient outcomes are also optimal outcomes.

Let's take a moment to show how this concept can be applied. Previously, we asked whether an action that preserved 4 miles of river was worth doing (Figure 3.1). The answer is yes because the net benefits from that action are positive. (Can you see why?)

Static efficiency, however, requires us to ask a rather different question, namely, what is the optimal (or efficient) number of miles to be preserved? We know from the definition that the optimal amount of preservation would maximize net benefits. Does preserving 4 miles maximize net benefits? Is it the efficient outcome?

We can answer that question by establishing whether it is possible to increase the net benefit by preserving more or less of the river. If the net benefit can be increased by preserving more miles, clearly, preserving 4 miles could not have maximized the net benefit and, therefore, could not have been efficient.

Consider what would happen if society were to choose to preserve 5 miles instead of 4. Refer back to Figure 3.1. What happens to the net benefit? It increases by area *MNR*. Since we can find another allocation with greater net benefit, 4 miles of preservation could not have been efficient. Could 5? Yes. Let's see why.

We know that 5 miles of preservation convey more net benefits than 4. If this allocation is efficient, then it must also be true that the net benefit is smaller for levels of preservation higher than 5. Notice that the additional cost of preserving the sixth unit (the area under the marginal cost curve) is larger than the additional benefit received from preserving it (the corresponding area under the demand curve). Therefore, the triangle *RTU* represents the reduction in net benefit that occurs if 6 miles are preserved rather than 5.

Since the net benefit is reduced, both by preserving less than 5 miles and by preserving more than 5, we conclude that five units is the preservation level that maximizes net benefit (the

shaded area). Therefore, from our definition, preserving 5 miles constitutes an efficient or optimal allocation.[2]

One implication of this example, which will be very useful in succeeding chapters, is what we shall call the "first equimarginal principle":

> First Equimarginal Principle (the "Efficiency Equimarginal Principle"): Social net benefits are maximized when the social marginal benefits from an allocation equal the social marginal costs.

The social marginal benefit is the increase in social benefits received from supplying one more unit of the good or service, while social marginal cost is the increase in cost incurred from supplying that additional unit of the good or service.

This criterion helps to minimize wasted resources, but is it fair? The ethical basis for this criterion is derived from a concept called *Pareto optimality*, named after the Italian-born Swiss economist Vilfredo Pareto, who first proposed it around the turn of the twentieth century.

> Allocations are said to be Pareto optimal if no other feasible allocation could benefit at least one person without any deleterious effects on some other person.

Allocations that do not satisfy this definition are suboptimal. Suboptimal allocations can always be rearranged so that some people can gain net benefits without the rearrangement causing anyone else to lose net benefits. Therefore, the gainers could use a portion of their gains to compensate the losers sufficiently to ensure they were at least as well off as they were prior to the reallocation.

Efficient allocations are Pareto optimal. Since net benefits are maximized by an efficient allocation, it is not possible to increase the net benefit by rearranging the allocation. Without an increase in the net benefit, it is impossible for the gainers to compensate the losers sufficiently; the gains to the gainers would necessarily be smaller than the losses to the losers.

Inefficient allocations are judged inferior because they do not maximize the size of the pie to be distributed. By failing to maximize net benefit, they are forgoing an opportunity to make some people better off without harming others.

Comparing Benefits and Costs across Time

The analysis we have covered so far is very useful for thinking about actions where time is not an important factor. Yet many of the decisions made now have consequences that persist well into the future. Time is a factor. Exhaustible energy resources, once used, are gone. Biological renewable resources (such as fisheries or forests) can be overharvested, leaving smaller and possibly weaker populations for future generations. Persistent pollutants can accumulate over time. How can we make choices when the benefits and costs occur at different points in time?

Incorporating time into the analysis requires an extension of the concepts we have already developed. This extension provides a way for thinking not only about the magnitude of benefits and costs, but also about their timing. In order to incorporate timing, the decision rule must provide a way to compare net benefits received in different time periods. The concept that allows this comparison is called *present value*. Before introducing this expanded decision rule, we must define present value.

Present value explicitly incorporates the time value of money. A dollar today invested at 10 percent interest yields \$1.10 a year from now (the return of the \$1 principal plus \$0.10

interest). The present value of \$1.10 received one year from now is therefore \$1, because, given \$1 now, you can turn it into \$1.10 a year from now by investing it at 10 percent interest. We can find the present value of any amount of money (X) received one year from now by computing $X/(1 + r)$, where r is the appropriate interest rate (10 percent in our above example).

What could your dollar earn in 2 years at r percent interest? Because of compound interest, the amount would be $\$1(1 + r)(1 + r) = \$1(1 + r)^2$. It follows then that the present value of X received 2 years from now is $X/(1 + r)^2$. The present value of X received in three years is $X/(1+r)^3$.

By now the pattern should be clear. The present value of a *one-time* net benefit received n years from now is

$$PV[B_n] = \frac{B_n}{(1 + r)^n}$$

The present value of a stream of net benefits $\{B_0, \ldots, B_n\}$ received over a period of n years is computed as

$$PV[B_0, \ldots, B_n] = \sum_{i=0}^{n} \frac{B_i}{(1 + r)^i}$$

where r is the appropriate interest rate and B_0 is the amount of net benefits received immediately. The process of calculating the present value is called *discounting*, and the rate r is referred to as the discount rate.

The number resulting from a present-value calculation has a straightforward interpretation. Suppose you were investigating an allocation that would yield the following pattern of net benefits on the last day of each of the next 5 years: \$3,000, \$5,000, \$6,000, \$10,000, and \$12,000. If you use an interest rate of 6 percent ($r = 0.06$) and the above formula, you will discover that this stream has a present value of \$29,205.92 (see Table 3.1). Notice how each amount is discounted back the appropriate number of years to the present and then these discounted values are summed.

What does that number mean? If you put \$29,205.92 in a savings account earning 6 percent interest and wrote yourself checks, respectively, for \$3,000, \$5,000, \$6,000, \$10,000, and \$12,000 on the last day of each of the next 5 years, your last check would just restore the account to a \$0 balance (see Table 3.2). Thus, you should be indifferent about receiving \$29,205.92 now or in the specific 5-year stream of benefits totaling \$36,000; given one, you can get the other. Hence, the method is called present value because it translates everything back to its current worth.

It is now possible to show how this analysis can be used to evaluate actions. Calculate the present value of net benefits from the action. If the present value is greater than zero, the action can be supported. Otherwise it should not.

Table 3.1 Demonstrating Present Value Calculations

Year	*1*	*2*	*3*	*4*	*5*	*Sum*
Annual Amounts	\$3,000	\$5,000	\$6,000	\$10,000	\$12,000	\$36,000
Present Value (r = 0.06)	\$2,830.19	\$4,449.98	\$5,037.72	\$7,920.94	\$8,967.10	\$29,205.92

Table 3.2 Interpreting Present Value Calculations

Year	*1*	*2*	*3*	*4*	*5*	*6*
Balance at Beginning of Year	$29,205.92	$27,958.28	$24,635.77	$20,113.92	$11,320.75	**$0.00**
Year-End Fund Balance before Payment (r = 0.06)	$30,958.28	$29,635.77	$26,113.92	$21,320.75	$12,000.00	
Payment	$3,000	$5,000	$6,000	$10,000	$12,000	

Dynamic Efficiency

The static efficiency criterion is very useful for comparing resource allocations when time is not an important factor. How can we think about optimal choices when the benefits and costs occur at different points in time?

The traditional criterion used to find an optimal allocation when time is involved is called *dynamic efficiency*, a generalization of the static efficiency concept already developed. In this generalization, the present-value criterion provides a way for comparing the net benefits received in one period with the net benefits received in another.

An allocation of resources across *n* time periods satisfies the dynamic efficiency criterion if it maximizes the present value of net benefits that could be received from all the possible ways of allocating those resources over the *n* periods.[3]

Applying the Concepts

Having now spent some time developing the concepts we need, let's take a moment to examine some actual studies in which they have been used.

Pollution Control

Benefit-cost analysis has been used to assess the desirability of efforts to control pollution. Pollution control certainly confers many benefits, but it also has costs. Do the benefits justify the costs? That was a question the U.S. Congress wanted answered, so in Section 812 of the Clean Air Act Amendments of 1990, it required the U.S. Environmental Protection Agency (EPA) to evaluate the benefits and costs of the U.S. air pollution control policy initially over the 1970–1990 period and subsequently over the 1990–2020 time period (see Example 3.1).

In responding to this congressional mandate, the EPA set out to quantify and monetize the benefits and costs of achieving the emissions reductions required by U.S. policy. Benefits quantified by this study included reduced death rates and lower incidences of chronic bronchitis, lead poisoning, strokes, respiratory diseases, and heart disease as well as the benefits of better visibility, reduced structural damages, and improved agricultural productivity.

We shall return to this study later in the book for a deeper look at how these estimates were derived, but a couple of comments are relevant now. First, despite the fact that this study

did not attempt to value all pollution damage to ecosystems that was avoided by this policy, the net benefits are still strongly positive. While presumably the case for controlling pollution would have been even stronger had all such avoided damage been included, the desirability of this form of control is evident even with only a partial consideration of benefits. An inability to monetize all benefits and costs does not necessarily jeopardize the ability to reach sound policy conclusions.

Although these results justify the conclusion that pollution control made economic sense, they do not justify the stronger conclusion that the policy was efficient. To justify that conclusion, the study would have had to show that the present value of net benefits was maximized, not merely positive. In fact, this study did not attempt to calculate the maximum net benefits outcome, and if it had, it would have almost certainly discovered that the policy during this period was not optimal. As we shall see in Chapters 15 and 17, the costs of the chosen policy approach were higher than necessary to achieve the desired emissions reductions. With an optimal policy mix, the net benefits would have been even higher.

EXAMPLE 3.1

Does Reducing Pollution Make Economic Sense? Evidence from the Clean Air Act

In its 1997 report to Congress, the EPA presented the results of its attempt to discover whether the Clean Air Act had produced positive net benefits over the period 1970–1990. The results suggested that the present value of benefits (using a discount rate of 5 percent) was $22.2 trillion, while the costs were $0.523 trillion. Performing the necessary subtraction reveals that the net benefits were therefore equal to $21.7 trillion. According to this study, U.S. air pollution control policy during this period made very good economic sense.

Soon after the period covered by this analysis, substantive changes were made in the Clean Air Act Amendments of 1990 (the details of those changes are covered in later chapters). Did those additions also make economic sense?

In August of 2010, the U.S. EPA issued a report of the benefits and costs of the Clean Air Act from 1990 to 2020. This report suggests that the costs of meeting the 1990 Clean Air Act Amendment requirements are expected to rise to approximately $65 billion per year by 2020 (2006 dollars). Almost half of the compliance costs ($28 billion) arise from pollution controls placed on cars, trucks, and buses, while another $10 billion arises from reducing air pollution from electric utilities.

These actions are estimated to cause benefits (from reduced pollution damage) to rise from roughly $800 billion in 2000 to almost $1.3 trillion in 2010, ultimately reaching approximately $2 trillion per year (2006 dollars) by 2020! For persons living in the United States, a cost of approximately $200 per person by 2020 produces approximately a $6,000 gain in benefits per person from the improvement in air quality. Many of the estimated benefits come from reduced risk of early mortality due to exposure to fine particulate matter. Table 3.3 provides a summary of the costs and benefits and includes a calculation of the benefit/cost ratio.

Table 3.3 Summary Comparison of Benefits and Costs from the Clean Air Act 1990–2020 (Estimates in Million 2006$)

	Annual Estimates			*Present Value Estimate*
	2000	*2010*	*2020*	*1990–2020*
Monetized Direct Costs:				
Low[1]				
Central	$20,000	$53,000	$65,000	$380,000
High[1]				
Monetized Direct Benefits:				
Low[2]	$90,000	$160,000	$250,000	$1,400,000
Central	$770,000	$1,300,000	$2,000,000	$12,000,000
High[2]	$2,300,000	$3,800,000	$5,700,000	$35,000,000
Net Benefits:				
Low	$70,000	$110,000	$190,000	$1,000,000
Central	$750,000	$1,200,000	$1,900,000	$12,000,000
High	$2,300,000	$3,700,000	$5,600,000	$35,000,000
Benefit/Cost Ratio:				
Low[3]	5/1	3/1	4/1	4/1
Central	39/1	25/1	31/1	32/1
High[3]	115/1	72/1	88/1	92/1

Notes:

1 The cost estimates for this analysis are based on assumptions about future changes in factors such as consumption patterns, input costs, and technological innovation. We recognize that these assumptions introduce significant uncertainty into the cost results; however, the degree of uncertainty or bias associated with many of the key factors cannot be reliably quantified. Thus, we are unable to present specific low and high cost estimates.

2 Low and high benefit estimates are based on primary results and correspond to 5th and 95th percentile results from statistical uncertainty analysis, incorporating uncertainties in physical effects and valuation steps of benefits analysis. Other significant sources of uncertainty not reflected include the value of unquantified or unmonetized benefits that are not captured in the primary estimates and uncertainties in emissions and air quality modeling.

3 The low benefit/cost ratio reflects the ratio of the low benefits estimate to the central costs estimate, while the high ratio reflects the ratio of the high benefits estimate to the central costs estimate. Because we were unable to reliably quantify the uncertainty in cost estimates, we present the low estimate as "less than X" and the high estimate as "more than Y," where X and Y are the low and high benefit/cost ratios, respectively.

Sources: U.S. Environmental Protection Agency. (1997). *The Benefits and Costs of the Clean Air Act, 1970 to 1990*. Washington, D.C.: Environmental Protection Agency, Table 18, p. 56; U.S. Environmental Protection Agency Office of Air and Radiation, *The Benefits and Costs of the Clean Air Act, 1990 to 2020—Summary Report, 8/16/2010*. Full Report available at www.epa.gov/clean-air-act-overview/benefits-and-costs-clean-air-act-1990-2010-first-prospective-study (accessed January 14, 2018).

Estimating Benefits of Carbon Dioxide Emission Reductions

Benefit-cost analysis is frequently complicated by the estimation of benefits and costs that are difficult to quantify. (Chapter 4 takes up the topic of nonmarket valuation in detail.) One such value is the benefit of reductions in carbon emissions.

Executive Order 12866 requires government agencies "to assess both the costs and the benefits of the intended regulation and, recognizing that some costs and benefits are difficult to quantify, propose or adopt a regulation only upon a reasoned determination that the benefits of the intended regulation justify its costs."[4] In order to include benefits from reducing carbon dioxide emission, agencies use what is called the "social cost of carbon" to reflect what those damages would have been in the absence of the reductions. The social cost of carbon is the marginal increase in the present value (in dollars) of the economic damages (e.g., sea level rise, floods, changes in agricultural productivity, and altered ecosystem services) resulting from a small increase (usually 1 metric ton) in carbon dioxide emissions. Since the social cost of carbon is a present value calculation, both the timing of the emission reduction and the discount rate play an important role.

The Interagency Working Group on Social Cost of Carbon presented the first set of estimates for the social cost of carbon in 2010. In 2013, these estimates were revised upwards with the estimate for the social cost of carbon increasing from $22 to approximately $37 per ton of carbon (using a discount rate of 3 percent). In 2016, they were revised again. Table 3.4 illustrates the 2016 revised social cost of carbon dioxide using 2.5, 3, and 5 percent discount rates for selected years. The fourth column presents the extreme case (95th percentile) using a 3 percent discount rate. Notice the importance of the discount rate in determining what value is used. (Can you explain why?)

The social cost of carbon is useful in making sure that the calculated benefits of carbon reductions reflect the reduced damages that can be expected. Example 3.2 demonstrates one way the social cost of carbon has been used in policy.

How much difference has it made in general? One study examined all economically significant federal regulations since 2008 to see what difference using a social cost of carbon would make. When they compared the ranking of the proposed policy to the status quo, they

Table 3.4 Revised Social Cost of CO_2, 2015–2050 (in 2007 dollars per metric ton of CO_2)

	Discount		*Rates*	
Year	*5% Avg*	*3% Avg*	*2.5% Avg*	*3% 95th*
2015	$11	$36	$56	$105
2020	$12	$42	$62	$123
2025	$14	$46	$68	$138
2030	$16	$50	$73	$152
2035	$18	$55	$78	$168
2040	$21	$60	$84	$183
2045	$23	$64	$89	$197
2050	$26	$69	$95	$212

Source: https://19january2017snapshot.epa.gov/climatechange/social-cost-carbon_.html (accessed May 15, 2017).

found little evidence that use of the social cost of carbon to include carbon reduction benefits into the calculation affected U.S. policy choices to date. The authors speculate that the absence of a discernible impact may be explained, in part, because U.S. regulators have succeeded in selecting the "low-hanging fruit," where the net benefits of early policies that reduce carbon are the largest and therefore most likely to pass a benefit-cost test even without using the social cost of carbon.[5]

In 2017, with the election of President Donald Trump everything changed. Soon after taking office President Trump signed an Executive Order that calls on agencies to disband the Interagency Working Group on Social Cost of Greenhouse Gases and to withdraw the documents that are the basis for the current calculation of the social cost of carbon. That EPA website has now been removed, although scientists have preserved the material (it can be found at https://19january2017snapshot.epa.gov/climatechange/social-cost-carbon_.html).

While this order does not abandon the concept completely, it does signal a desire to use a new approach to measuring and using the concept. What will this mean for future policies? Stay tuned.

EXAMPLE 3.2

Using the Social Cost of Capital: The DOE Microwave Oven Rule

In 2013, the Department of Energy (DOE) announced new rules for energy efficiency for microwave ovens in standby mode. By improving the energy efficiency of these ovens, this rule would reduce carbon emissions. In the regulatory impact analysis associated with this rule, it was necessary to value the reduced damages from this lower level of emissions. The social cost of carbon was used to provide this information.

Using the 2010 social cost of carbon produced a present value of net benefits for the microwave oven rule over the next 30 years of $4.2 billion. Since this value is positive, it means that implementing this rule would increase efficiency.

We know that using the revised 2013 number would increase the present value of net benefits, but by how much? According to the DOE, using the 2013 instead of the 2010 social cost of carbon increases the present value of net benefits to $4.6 billion. In this case the net benefits were large enough both before and after the new SCC estimates to justify implementing the rule, but it is certainly possible that in other cases these new estimates would justify rules that prior to the change would not have been justified.

Note that microwave purchasers will bear the cost of this set of rules (as prices rise to reflect the higher production costs), but they will not receive all of the benefits (those reflecting a reduction in external costs). However, the DOE notes that due to the increased energy efficiency of the appliances subject to these rules (and the resulting lower energy costs for purchasers), the present value of savings to consumers is estimated to be $3.4 billion over the next 30 years (DOE 2013), an amount that is larger than the costs. In this case the rules represent a win for both microwave consumers and the planet.

Sources: http://energy.gov/articles/new-energy-efficiency-standards-microwave-ovens-save-consumers-energy-bills; *Technical Update of the Social Cost of Carbon for Regulatory Impact Analysis—Under Executive Order 12866*, https://obamawhitehouse.archives.gov/sites/default/files/omb/inforeg/scc-tsd-final-july-2015.pdf

Issues in Benefit Estimation

The analyst charged with the responsibility for performing a benefit-cost analysis encounters many decision points requiring judgment. If we are to understand benefit-cost analysis, the nature of these judgments must be clear in our minds.

Primary versus Secondary Effects. Environmental projects usually trigger both primary and secondary consequences. For example, the primary effect of cleaning a lake will be an increase in recreational uses of the lake. This primary effect will cause a further ripple effect on services provided to the increased number of users of the lake. Are these secondary benefits to be counted?

The answer depends upon the employment conditions in the surrounding area. If this increase in demand results in employment of previously unused resources, such as labor, the value of the increased employment should be counted. If, on the other hand, the increase in demand is met by a shift in previously employed resources from one use to another, it is a different story. In general, secondary employment benefits should be counted in high unemployment areas or when the particular skills demanded are underemployed at the time the project is commenced. They should not be counted when the project simply results in a rearrangement of productively employed resources.

Accounting Stance. The accounting stance refers to the geographic scale or scope at which the benefits are measured. Scale matters because in a benefit-cost analysis only the benefits or costs affecting that specific geographic area are counted. Suppose, for example, that the federal government picks up many of the costs, but the benefits are received by only one region. Even if the benefit-cost analysis shows this to be a great project for the region, that will not necessarily be the case for the nation as a whole. Once the national costs are factored in, the national project benefits may not exceed the national project costs. Debate 3.1 examines this issue in relation to the social cost of carbon.

Aggregation. Related to accounting stance are challenges of aggregation. Estimates of benefits and costs must be aggregated in order to derive total benefits and total costs. How many people benefit and how many people incur costs are very important in any aggregation, but, additionally, *how* they benefit might impact that aggregation. Suppose, for example, those living closer to the project received more benefits per household than those living farther away. In this case these differences should be accounted for.

With and Without Principle. The "with and without" principle states that only those benefits that would result from the project should be counted, ignoring those that would have accrued anyway. Mistakenly including benefits that would have accrued anyway would overstate the benefits of the program.

Tangible versus Intangible Benefits. *Tangible benefits* are those that can reasonably be assigned a monetary value. *Intangible benefits* are those that cannot be assigned a monetary value, either because data are not available or reliable enough or because it is not clear how to measure the value even with data.[6] Quantification of intangible benefits is the primary topic of the next chapter.

How are intangible benefits to be handled? One answer is perfectly clear: they should not be ignored. To ignore intangible benefits is to bias the results. That benefits are intangible does not mean they are unimportant.

DEBATE 3.1

What Is the Proper Geographic Scope for the Social Cost of Carbon?

The Social Cost of Carbon is an estimate of the economic damages associated with a small increase in carbon dioxide (CO_2) emissions, conventionally 1 metric ton, in a given year. Any reduction in these damages resulting from a proposed regulation is used to estimate the climate benefits of U.S. rulemakings.

Because climate change is a global public good, the efficient damage estimate must include all damages, not just damages to the United States. Some critics argue that, because it is used in U.S. regulatory procedures, it should include only U.S. damages; otherwise it might justify regulations that impose costs on U.S. citizens for the purpose of producing benefits enjoyed by citizens of other countries who do not bear the cost.

Proponents of the global metric point out that the measure is designed to be a means of internalizing a marginal external cost and it cannot do that accurately and efficiently if it leaves out some of the costs. Calculating it only for U.S. damages would create a biased measure that would underestimate the damages and raise the possibility of biased regulatory decisions based upon it.

Furthermore, they argue that the characterization of this measure as allowing benefits created by American citizens to be enjoyed by foreign citizens is a bit misleading. These benefits do not reflect goods and services purchased by U.S. citizens that are enjoyed abroad. Rather they reflect a reduction in the damages that U.S. citizens would otherwise be imposing on others. American law typically does not allow someone to inflict damage on neighbors simply because they are on the other side of some boundary. Reducing damages imposed on others has a different moral context than spillover benefits.

Some regulatory analysts have now suggested that the “U.S.-only” measure should not replace the existing measure, but complement it. Both should be provided. What do you think?

Source: Dudley, Susan E., Fraas, Art, Gayer, Ted, Graham, John, Lutter, Randall, Shogren, Jason F., & Viscusi, W. Kip. (2016). How much will climate change rules benefit Americans? *Forbes* (February 9).

Intangible benefits should be quantified to the fullest extent possible. One frequently used technique is to conduct a sensitivity analysis of the estimated benefit values derived from less than perfectly reliable data. We can determine, for example, whether or not the outcome is sensitive, within wide ranges, to the value of this benefit. If not, then very little time has to be spent on the problem. If the outcome is sensitive, the person or persons making the decision bear the ultimate responsibility for weighing the importance of that benefit.

Approaches to Cost Estimation

Estimating costs is generally easier than estimating benefits, but it is not easy. One major problem for both derives from the fact that benefit-cost analysis is forward-looking and thus requires an estimate of what a particular strategy *will* cost, which is much more difficult than tracking down what an existing strategy *does* cost.

Two approaches have been developed to estimate these costs.

The Survey Approach. One way to discover the costs associated with a policy is to ask those who bear the costs, and presumably know the most about them, to reveal the magnitude of the costs to policymakers. Polluters, for example, could be asked to provide control-cost estimates to regulatory bodies. The problem with this approach is the strong incentive not to be truthful. An overestimate of the costs can trigger less stringent regulation; therefore, it is financially advantageous to provide overinflated estimates.

The Engineering Approach. The engineering approach bypasses the source being regulated by using general engineering information to catalog the possible technologies that could be used to meet the objective and to estimate the costs of purchasing and using those technologies. The final step in the engineering approach is to assume that the sources would use technologies that minimize cost. This produces a cost estimate for a "typical," well-informed firm.

The engineering approach has its own problems. These estimates may not approximate the actual cost of any particular firm. Unique circumstances may cause the costs of that firm to be higher, or lower, than estimated; the firm, in short, may not be typical.

The Combined Approach. To circumvent these problems, analysts frequently use a combination of survey and engineering approaches. The survey approach collects information on possible technologies, as well as special circumstances facing the firm. Engineering approaches are used to derive the actual costs of those technologies, given the special circumstances. This combined approach attempts to balance information best supplied by the source with that best derived independently.

In the cases described so far, the costs are relatively easy to quantify and the problem is simply finding a way to acquire the best information. This is not always the case, however. Some costs are not easy to quantify, although economists have developed some ingenious ways to secure monetary estimates even for those costs.

Take, for example, a policy designed to conserve energy by forcing more people to carpool. If the effect of this is simply to increase the average time of travel, how is this cost to be measured?

For some time, transportation analysts have recognized that people value their time, and a large amount of literature has now evolved to provide estimates of how valuable time savings or time increases would be. The basis for this valuation is opportunity cost—how the time might be used if it weren't being consumed in travel. Although the results of these studies depend on the amount of time involved, individual decisions seem to imply that travelers value their travel time at a rate not more than half their wage rates.

The Treatment of Risk

For many environmental problems, it is not possible to state with certainty what consequences a particular policy will have, because scientific estimates themselves often are imprecise. Determining the efficient exposure to potentially toxic substances requires obtaining results

at high doses and extrapolating to low doses, as well as extrapolating from animal studies to humans. It also requires relying upon epidemiological studies that infer a pollution-induced adverse human health impact from correlations between indicators of health in human populations and recorded pollution levels.

For example, consider the potential damages from climate change. While scientists agree on most of the potential impacts of climate change, such as sea level rise and species losses, the timing and extent of those losses are not certain.

The treatment of risk in the policy process involves two major dimensions: (1) identifying and quantifying the risks, and (2) deciding how much risk is acceptable. The former is primarily scientific and descriptive, while the latter is more evaluative or normative.

Benefit-cost analysis grapples with the evaluation of risk in several ways. Suppose we have a range of policy options *A*, *B*, *C*, *D* and a range of possible outcomes *E*, *F*, *G* for each of these policies depending on how the economy evolves over the future. These outcomes, for example, might depend on whether the demand growth for the resource is low, medium, or high. Thus, if we choose policy *A*, we might end up with outcomes *AE*, *AF*, or *AG*. Each of the other policies has three possible outcomes as well, yielding a total of 12 possible outcomes.

We could conduct a separate benefit-cost analysis for each of the 12 possible outcomes. Unfortunately, the policy that maximizes net benefits for *E* may be different from that which maximizes net benefits for *F* or *G*. Thus, if we only knew which outcome would prevail, we could select the policy that maximized net benefits; the problem is that we do not. Furthermore, choosing the policy that is best if outcome *E* prevails may be disastrous if *G* results instead.

When a dominant policy emerges, this problem is avoided. A *dominant policy* is one that confers higher net benefits for every outcome. In this case, the existence of risk concerning the future is not relevant for the policy choice. This fortuitous circumstance is exceptional rather than common, but it can occur.

Other options exist even when dominant solutions do not emerge. Suppose, for example, that we were able to assess the likelihood that each of the three possible outcomes would occur. Thus, we might expect outcome *E* to occur with probability 0.5, *F* with probability 0.3, and *G* with probability 0.2. Armed with this information, we can estimate the expected present value of net benefits. The *expected present value of net benefits* for a particular policy is defined as the sum over outcomes of the present value of net benefits for that policy where each outcome is weighted by its probability of occurrence. Symbolically this is expressed as

$$EPVNB_j = \sum_{i=0}^{I} P_i PVNB_{ij}, \quad j = 1, \ldots, J,$$

where

$EPVNBj$ = expected present value of net benefits for policy j,
Pi = probability of the ith outcome occurring,
$PVNBij$ = present value of net benefits for policy j if outcome i prevails,
J = number of policies being considered,
I = number of outcomes being considered.

The final step is to select the policy with the highest expected present value of net benefits.

This approach has the substantial virtue that it weighs higher probability outcomes more heavily. It also, however, makes a specific assumption about society's preference for risk. This approach is appropriate if society is risk-neutral. *Risk-neutrality* can be defined most easily by the use of an example. Suppose you were to choose between being given a definite $50 or entering a lottery in which you had a 50 percent chance of winning $100 and a 50 percent

chance of winning nothing. (Notice that the expected value of this lottery is $50 = 0.5($100) + 0.5($0).) You would be said to be risk-neutral if you would be indifferent between these two choices. If you view the lottery as more attractive, you would be exhibiting *risk-loving* behavior, while a preference for the definite $50 would suggest *risk-averse* behavior. Using the expected present value of net benefits approach implies that society is risk-neutral.

Is that a valid assumption? The evidence is mixed. The existence of gambling suggests that at least some members of society are risk-loving, while the existence of insurance suggests that, at least for some risks, others are risk-averse. Since the same people may gamble and own insurance policies, it is likely that the type of risk may be important.

Even if individuals were demonstrably risk-averse, this would not be a sufficient condition for the government to forsake risk-neutrality in evaluating public investments. One famous article (Arrow & Lind, 1970) argues that risk-neutrality is appropriate since "when the risks of a public investment are publicly borne, the total cost of risk-bearing is insignificant and, therefore, the government should ignore uncertainty in evaluating public investments." The logic behind this result suggests that as the number of risk bearers (and the degree of diversification of risks) increases, the amount of risk borne by any individual diminishes to zero.

When the decision is irreversible, as demonstrated by Arrow and Fisher (1974), considerably more caution is appropriate. Irreversible decisions may subsequently be regretted, but the option to change course will be lost forever. Extra caution also affords an opportunity to learn more about alternatives to this decision and its consequences before acting. Isn't it comforting to know that occasionally procrastination can be optimal?

There is a movement in national policy in both the courts and the legislature to search for imaginative ways to define acceptable risk. In general, the policy approaches reflect a case-by-case method. We shall see that current policy reflects a high degree of risk aversion toward a number of environmental problems.

Distribution of Benefits and Costs

Many agencies are now required to consider the distributional impacts of costs and benefits as part of any economic analysis. For example, the U.S. EPA provides guidelines on distributional issues in its "Guidelines for Preparing Economic Analysis." According to the EPA, distributional analysis "assesses changes in social welfare by examining the effects of a regulation across different sub-populations and entities." Distributional analysis can take two forms: *economic impact analysis* and *equity analysis*. Economic impact analysis focuses on a broad characterization of who gains and who loses from a given policy. Equity analysis examines impacts on disadvantaged groups or sub-populations. The latter delves into the normative issue of equity or fairness in the distribution of costs and benefits. Loomis (2011) outlines several approaches for incorporating distribution and equity into benefit-cost analysis. Some issues of the distribution of benefits and costs related to energy efficiency rules for appliances were highlighted in Example 3.2.

Choosing the Discount Rate

Recall that discounting allows us to compare all costs and benefits in current dollars, regardless of when the benefits accrue or costs are charged. Suppose a project will impose an immediate cost of $4,000,000 (today's dollars), but the $5,500,000 benefits will not be earned until 5 years out. Is this project a good idea? On the surface it might seem like it is, but recall that $5,500,000 in 5 years is not the same as $5,500,000 today. At a discount rate of 5 percent, the present value of benefits minus the present value of costs is positive. However, at a 10 percent

discount rate, this same calculation yields a negative value, since the present value of costs exceeds the benefits. Can you reproduce the calculations that yield these conclusions?

As Example 3.3 indicates, this has been, and continues to be, an important issue. When the public sector uses a discount rate lower than that in the private sector, the public sector

EXAMPLE 3.3

The Importance of the Discount Rate

Let's begin with an historical example. For years the United States and Canada had been discussing the possibility of constructing a tidal power project in the Passamaquoddy Bay between Maine and New Brunswick. This project would have heavy initial capital costs, but low operating costs that presumably would hold for a long time into the future. As part of their analysis of the situation, a complete inventory of costs and benefits was completed in 1959.

Using the same benefit and cost figures, Canada concluded that the project should not be built, while the United States concluded that it should. Because these conclusions were based on the same benefit-cost data, the differences can be attributed solely to the use of different discount rates. The United States used 2.5 percent while Canada used 4.125 percent. The higher discount rate makes the initial cost weigh much more heavily in the calculation, leading to the Canadian conclusion that the project would yield a negative net benefit. Since the lower discount rate weighs the lower future operating costs relatively more heavily, Americans saw the net benefit as positive.

In a more recent illustration of why the magnitude of the discount rate matters, on October 30, 2006, economist Nicholas Stern from the London School of Economics issued a report using a discount rate of 0.1 percent that concluded that the benefits of strong, early action on climate change would considerably outweigh the costs. Other economists, such as William Nordhaus of Yale University, who preferred a discount rate around 6 percent, found that optimal economic policies to slow climate change involve only modest rates of emissions reductions in the near term, followed by sharp reductions in the medium and long term.

In this debate, the desirability of strong current action is dependent (at least in part) on the size of the discount rate used in the analysis. Higher discount rates reduce the present value of future benefits from current investments in abatement, implying a smaller marginal benefit. Since the costs associated with those investments are not affected nearly as much by the choice of discount rate (remember that costs occurring in the near future are discounted less), a lower present value of marginal benefit translates into a lower optimal investment in abatement.

Far from being an esoteric subject, the choice of the discount rate is fundamentally important in defining the role of the public sector, the types of projects undertaken, and the allocation of resources across generations.

Sources: Stokey, E., & Zeckhauser, R. (1978). *A Primer for Policy Analysis*. New York: W. W. Norton, 164–165; Mikesell, R. (1977). *The Rate of Discount for Evaluating Public Projects*. Washington, D.C.: The American Enterprise Institute for Public Policy Research, 3–5; the Stern Report: http://webarchive.nationalarchives.gov.uk/20100407011151/http://www.hm-treasury.gov.uk/sternreview_index.htm; Nordhaus, W. (2007). A review of the Stern Review on the economics of climate change. *Journal of Economic Literature*, *XLV* (September), 686–702.

will find more projects with longer payoff periods worthy of authorization. And, as we have already seen, the discount rate is a major determinant of the allocation of resources among generations as well.

The discount rate can be defined conceptually as the social opportunity cost of capital. This cost of capital can be divided further into two components: (1) the riskless cost of capital, and (2) the risk premium. Traditionally, economists have used long-term interest rates on government bonds as one measure of the cost of capital, adjusted by a risk premium that would depend on the riskiness of the project considered. Unfortunately, the choice of how large an adjustment to make has been left to the discretion of the analysts. This ability to affect the desirability of a particular project or policy by the choice of discount rate led to a situation in which government agencies were using a variety of discount rates to justify programs or projects they supported. One set of hearings conducted by Congress during the 1960s discovered that, at one time, agencies were using discount rates ranging from 0 to 20 percent.

During the early 1970s, the Office of Management and Budget published a circular that required, with some exceptions, all government agencies to use a discount rate of 10 percent in their benefit-cost analysis. A revision issued in 1992 reduced the required discount rate to 7 percent. This circular also includes guidelines for benefit-cost analysis and specifies that certain rates will change annually.[7] This standardization reduces biases by eliminating the agency's ability to choose a discount rate that justifies a predetermined conclusion. It also allows a project to be considered independently of fluctuations in the true social cost of capital due to cycles in the behavior of the economy. On the other hand, when the social opportunity cost of capital differs from this administratively determined level, the benefit-cost analysis will not, in general, define the efficient allocation.

Example 3.3 highlights a different aspect of the choice of the discount rate for decisions involving long time horizons. It considers the question of whether or not discount rates should decline over time. Debate 3.2 explores this question.

DEBATE 3.2

Discounting over Long Time Horizons: Should Discount Rates Decline?

As you now recognize, the choice of the discount rate can significantly alter the outcome of a benefit-cost analysis. This effect is exacerbated over long time horizons and can become especially influential in decisions about spending now to mitigate damages from climate change, which may be uncertain in both magnitude and timing. What rate is appropriate? Recent literature and some evidence argue for declining rates of discount over long time horizons. Should a declining rate schedule be utilized? A blue-ribbon panel of experts recently gathered to debate this and related questions (Arrow et al., 2012).

An unresolved debate in the economics literature revolves around the question of whether discount rates should be positive ("descriptive"), reflecting actual market rates, or normative ("prescriptive"), reflecting ethical considerations. Those who argue for the descriptive approach prefer to use market rates of return since expenditures to mitigate climate change

are investment expenditures. Those who argue for the alternative prescriptive approach argue for including judgments about intergenerational equity. These rates are usually lower than those found in actual markets (Griffiths et al. 2012).

In the United States, the Office of Management and Budget (OMB) currently recommends a constant rate of discount for project analysis. The recommendation is to use 3 percent and 7 percent real discount rates in sensitivity analysis (OMB, 2003) with options for lower rates if future generations are impacted. The United Kingdom and France utilize discount rate schedules that decline over time. Is one of these methods better than the other for discounting over long time horizons? If a declining rate is appropriate, how fast should that rate decline?

The blue-ribbon panel agreed that theory provides strong arguments for a "declining certainty-equivalent discount rate" (Arrow et al., 2012, p. 21). Although the empirical literature also supports a rate that is declining over time (especially in the presence of uncertainty about future costs and/or benefits), the results from the empirical literature vary widely depending on the model assumptions and underlying data. If a declining rate schedule were to be adopted in the United States, this group of experts recommends that the EPA's Science Advisory Board be asked to develop criteria that could be used as the common foundation for determining what the schedule should look like.

Sources: Arrow, K., Maureen, J., Cropper, L., Gollier, C., Groom, B., Heal, G. M., et al. (December 2012). How should benefits and costs be discounted in an intergenerational context: The views of an expert panel. *RFF DP 12–53*; Griffiths, C., Kopits, E., Marten, A., Moore, C., Newbold, S., & Wolverton, A. (2012). The social cost of carbon: Valuing carbon reductions in policy analysis. In R. A. de Mooij, M. Keen, & I. W. H. Parry (Eds.). *Fiscal Policy to Mitigate Climate Change: A Guide for Policy Makers*. Washington, D.C.: IMF, 69–87; OMB (Office of Management and Budget). Circular A-4: Regulatory Analysis. Washington, DC Executive Office of the President. www.whitehouse.gov/omb/circulars_a004_a-4

Divergence of Social and Private Discount Rates

Earlier we concluded that producers, in their attempt to maximize producer surplus, also maximize the present value of net benefits under the "right" conditions, such as the absence of externalities, the presence of properly defined property rights, and the presence of competitive markets within which the property rights can be exchanged.

Now let's consider one more condition. If resources are to be allocated efficiently, firms must use the same rate to discount future net benefits as is appropriate for society at large. If firms were to use a higher rate, they would extract and sell resources faster than would be efficient. Conversely, if firms were to use a lower-than-appropriate discount rate, they would be excessively conservative.

Why might private and social rates differ? As noted above the social opportunity cost of capital can be separated into two components: the risk-free cost of capital and the risk

premium. The *risk-free cost of capital* is the rate of return earned when there is absolutely no risk of earning more or less than the expected return. The *risk premium* is an additional cost of capital required to compensate the owners of this capital when the expected and actual returns may differ. Therefore, because of differences in the risk premium, the cost of capital is higher in risky industries than in no-risk industries.

Another difference between private and social discount rates may stem from a difference in social and private risk premiums. If the risk of certain private decisions is different from the risks faced by society as a whole, then the social and private risk premiums may differ. One obvious example is the risk *caused* by the government.

If the firm is afraid its assets will be confiscated by the government, it may choose a higher discount rate to make its profits before nationalization occurs. From the point of view of society—as represented by government—this is not a risk and, therefore, a lower discount rate is appropriate. When private rates exceed social rates, current production is higher than is desirable to maximize the net benefits to society. Both energy production and forestry have been subject to this source of inefficiency.

Another divergence in discount rates may stem from different underlying rates of time preference. Such a divergence in time preferences can cause not only a divergence between private and social discount rates (as when firms have a higher rate of time preference than the public sector), but even between otherwise similar analyses conducted in two different countries.

Time preferences would be expected to be higher, for example, in a cash-poor, developing country than in an industrialized country. Since the two benefit-cost analyses in these two countries would be based upon two different discount rates, they might come to quite different conclusions. What is right for the developing country may not be right for the industrialized country and vice versa.

Although private and social discount rates do not always diverge, they may. When those circumstances arise, market decisions are not efficient.

A Critical Appraisal

We have seen that it is sometimes, but not always, difficult to estimate benefits and costs. When this estimation is difficult or unreliable, it limits the value of a benefit-cost analysis. This problem would be particularly disturbing if biases tended to increase or decrease net benefits systematically. Do such biases exist?

In the early 1970s, Robert Haveman (1972) conducted a major study that continues to shed some light on this question. Focusing on Army Corps of Engineers water projects, such as flood control, navigation, and hydroelectric power generation, Haveman compared the *ex ante* (before the fact) estimate of benefits and costs with their *ex post* (after the fact) counterparts. Thus, he was able to address the issues of accuracy and bias. He concluded that:

> In the empirical case studies presented, ex post estimates often showed little relationship to their ex ante counterparts. On the basis of the few cases and the a priori analysis presented here, one could conclude that there is a serious bias incorporated into agency ex ante evaluation procedures, resulting in persistent overstatement of expected benefits. Similarly in the analysis of project construction costs, enormous variance was found among projects in the relationship between estimated and realized costs. Although no persistent bias in estimation was apparent, nearly 50 percent of the projects displayed realized costs that deviated by more than plus or minus 20 percent from ex ante projected costs.[8]

In the cases examined by Haveman, at least, the notion that benefit-cost analysis is purely a scientific exercise was clearly not consistent with the evidence; the biases of the analysts were merely translated into numbers.

Does their analysis mean that benefit-cost analysis is fatally flawed? Absolutely not! Valuation methods have improved considerably since the Haveman study, but problems remain. This study does, however, highlight the enduring importance of calculating an accurate value and of including all of the potential benefits and costs (e.g., nonmarket values). As elementary as it might seem including both the benefits and the costs is necessary. As Example 3.4 illustrates, that is not always the case in practice.

EXAMPLE 3.4

Is the Two for One Rule a Good Way to Manage Regulatory Overreach?

Environmental regulations can be costly, but they also produce economic benefits. Efficiency suggests that regulations whose benefits exceed their costs should be pursued and that is the path followed by previous Executive Orders (EOs) from Presidents Reagan (EO 12291), Clinton (EO 12866), and Obama (EO 13563).

In 2017, the Trump administration abandoned business as usual and issued EO 13771, mandating that for every new regulation issued, two must be thrown out.[9] What does economic analysis and, in particular, benefit-cost analysis have to say about this one-in, two-out prescription?

Executive Order 13771 reads, in part: "(c) . . . any new incremental costs associated with new regulations shall, to the extent permitted by law, be offset by the elimination of existing costs associated with at least two prior regulations."

In his attempt to reduce regulatory overreach President Trump's approach seems to suggest that only the costs are important when evaluating current and new regulations. Benefits don't matter. Since most of the current regulations were put into place based on benefits and costs, removing them based solely on costs would be a "blunt instrument"—one that is poorly targeted on making efficient choices.

Economist Robert Shiller further argues that regulation is in the public interest in many areas and "the world is far too complex to make it possible to count up regulations meaningfully and impose a two-for-one rule."

Alan Krupnick, economist at Resources for the Future, points out that even if a "cost-only" approach were justified, it would not be easy to implement. For example "what is a cost? Is it a projected cost in the rule or actual costs as implemented? Is it present discounted costs or something else to account for cost streams over time? Is it direct costs or do indirect costs (say, to consumers) count? Is it private costs or costs to society?"

Regardless of the answer to those questions, however, benefits do matter. As Krupnick notes, "How do we determine which regulations are ineffective and unnecessary without considering their benefits? The answer is simple—we cannot."

Imagine if we only saved endangered species that cost the least to save, or cleaned up only the least expensive oil spills. Making decisions based solely on costs is misguided economics.

Sources: www.rff.org/research/publications/trump-s-regulatory-reform-process-analytical-hurdles-and-missing-benefits; www.nytimes.com/2017/02/17/upshot/why-trumps-2-for-1-rule-on-regulations-is-no-quick-fix.html; www.env-econ.net/2017/02/two-for-one-too-blunt-an-instrument-for-good-governance.html

Haveman's analysis also serves to remind us that benefit-cost analysis is not a stand-alone technique. It should be used in conjunction with other available information. Economic analysis including benefit-cost analysis can provide useful information, but it should not be the only determinant for all decisions.

Benefit-cost analysis also limited in that it does not really address the question of who reaps the benefits and who pays the cost. It is quite possible for a particular course of action to yield high net benefits, but to have the benefits borne by one societal group and the costs borne by another. This scenario serves to illustrate a basic principle—ensuring that a particular policy is efficient provides an important, but not always the sole, basis for public policy. Other aspects, such as who reaps the benefit or bears the burden, are also important considerations. Distributional benefit-cost analysis can help illuminate potential inequities.

In summary, on the positive side, benefit-cost analysis is frequently a very useful part of the policy process. Even when the underlying data are not strictly reliable, the outcomes may not be sensitive to that unreliability. In other circumstances, the data may be reliable enough to give indications of the consequences of broad policy directions, even when they are not reliable enough to fine-tune those policies. Benefit-cost analysis, when done correctly, can provide a useful complement to the other influences on the political process by clarifying what choices yield the highest net benefits to society.

On the negative side, benefit-cost analysis has been attacked as seeming to promise more than can actually be delivered, particularly in the absence of solid benefit information. This concern has triggered two responses. First, regulatory processes have been developed that can be implemented with very little information and yet have desirable economic properties. The recent reforms in air pollution control, which we cover in Chapters 14 and 15, provide some powerful examples.

The second involves techniques that supply useful information to the policy process without relying on controversial techniques to monetize environmental services that are difficult to value. The rest of this chapter deals with the two most prominent of these—cost-effectiveness analysis and impact analysis.

Even when benefits are difficult or impossible to quantify, economic analysis has much to offer. Policymakers should know, for example, how much various policy actions will cost and what their impacts on society will be, even if the efficient policy choice cannot be identified with any certainty.

Other Decision-Making Metrics

Cost-Effectiveness Analysis

What can be done to guide policy when the requisite valuation for benefit-cost analysis is either unavailable or not sufficiently reliable? Without a good measure of benefits, making an efficient choice is no longer possible.

In such cases, however, it is often possible to set a policy target on some basis other than a strict comparison of benefits and costs. One example is pollution control. What level of pollution should be established as the maximum acceptable level? In many countries, studies of the effects of a particular pollutant on human health have been used as the basis for establishing that pollutant's maximum acceptable concentration. Researchers attempt to find a threshold level below which no damage seems to occur. That threshold is then further lowered to provide a margin of safety and that becomes the pollution target.

Approaches could also be based upon expert opinion. Ecologists, for example, could be enlisted to define the critical numbers of certain species or the specific critical wetlands resources that should be preserved.

Once the policy target is specified, however, economic analysis can have a great deal to say about the cost consequences of choosing a means of achieving that objective. The cost consequences are important not only because eliminating wasteful expenditures is an appropriate goal in its own right, but also to assure that they do not trigger a political backlash.

Typically, several means of achieving the specified objective are available; some will be relatively inexpensive, while others turn out to be very expensive. The problems are frequently complicated enough that identifying the cheapest means of achieving an objective cannot be accomplished without a rather detailed analysis of the choices.

Cost-effectiveness analysis frequently involves an *optimization procedure*. An optimization procedure, in this context, is merely a systematic method for finding the lowest-cost means of accomplishing the objective. This procedure does not, in general, produce an efficient allocation because the predetermined objective may not be efficient. All efficient policies are cost-effective, but not all cost-effective policies are efficient.

Earlier in this chapter we introduced the concept of the efficiency equimarginal principle. According to that principle, net benefits are maximized when the marginal benefit is equal to the marginal cost.

A similar, and equally important, equimarginal principle exists for cost-effectiveness:

> Second Equimarginal Principle (the Cost-Effectiveness Equimarginal Principle): The least-cost means of achieving an environmental target will have been achieved when the marginal costs of all possible means of achievement are equal.

Suppose we want to achieve a specific emissions reduction across a region, and several possible techniques exist for reducing emissions. How much of the control responsibility should each technique bear? The cost-effectiveness equimarginal principle suggests that the techniques should be used such that the desired reduction is achieved and the cost of achieving the last unit of emissions reduction (in other words, the marginal control cost) should be the same for all sources.

To demonstrate why this principle is valid, suppose that we have an allocation of control responsibility where marginal control costs are much higher for one set of techniques than for another. This cannot be the least-cost allocation since we could lower cost while retaining the same amount of emissions reduction. To be specific, costs could be lowered by allocating more control to the lower marginal cost sources and less to the high marginal cost sources. Since it is possible to find a way to lower cost while holding emissions constant, then clearly the initial allocation could not have minimized cost. Once marginal costs are equalized, it becomes impossible to find any lower-cost way of achieving the same degree of emissions reduction; therefore, that allocation must be the allocation that minimizes costs.

In our pollution control example, cost-effectiveness can be used to find the least-cost means of meeting a particular standard and its associated cost. Using this cost as a benchmark case, we can estimate how much costs could be expected to increase from this minimum level if policies that are not cost effective are implemented. Cost-effectiveness analysis can also be used to determine how much compliance costs can be expected to change if the EPA chooses a more stringent or less stringent standard. In Chapters 14 and 15, we shall examine in detail the current movement toward cost-effective polices, a movement that was triggered in part by studies showing that the cost reductions from reform could be substantial.

Impact Analysis

What can be done when the information needed to perform a benefit-cost analysis or a cost-effectiveness analysis is not available? The analytical technique designed to deal with this problem is called *impact analysis*. An impact analysis, regardless of whether it focuses on economic impact or environmental impact or both, attempts to quantify the consequences of various actions.

In contrast to benefit-cost analysis, a pure impact analysis makes no attempt to convert all these consequences into a one-dimensional measure, such as dollars, to ensure comparability. In contrast to cost-effectiveness analysis, impact analysis does not necessarily attempt to optimize. Impact analysis places a large amount of relatively undigested information at the disposal of the policymaker. It is up to the policymaker to assess the importance of the various predicted consequences and act accordingly.

On January 1, 1970, President Nixon signed the National Environmental Policy Act of 1969. This act, among other things, directed all agencies of the federal government to:

> include in every recommendation or report on proposals for legislation and other major Federal actions significantly affecting the quality of the human environment, a detailed statement by the responsible official on—
>
> i. the environmental impact of the proposed action;
> ii. any adverse environmental effects which cannot be avoided should the proposal be implemented;
> iii. alternatives to the proposed action;
> iv. the relationships between local short-term uses of man's environment and the maintenance and enhancement of long-term productivity; and
> v. any irreversible and irretrievable commitments of resources which would be involved in the proposed action should it be implemented.[10]

This was the beginning of the environmental impact statement, which is now a familiar, if controversial, part of environmental policy making.

Current environmental impact statements are more sophisticated than their early predecessors and may contain a benefit-cost analysis or a cost-effectiveness analysis in addition to other more traditional impact measurements. Historically, however, the tendency has been to issue huge environmental impact statements that are virtually impossible to comprehend in their entirety.

In response, the Council on Environmental Quality, which, by law, administers the environmental impact statement process, has set content standards that are now resulting in shorter, more concise statements. To the extent that they merely quantify consequences, statements can avoid the problem of "hidden value judgments" that sometimes plague benefit-cost analysis, but they do so only by bombarding the policymakers with masses of noncomparable information.

Summary

Finding a balance in the relationship between humanity and the environment requires many choices. Some basis for making rational choices is absolutely necessary. If not made by design, decisions will be made by default.

Normative economics uses benefit-cost analysis for judging the desirability of the level and composition of provided services. Cost-effectiveness analysis and impact analysis offer alternatives to benefit-cost analysis. All of these techniques offer valuable information for decision making and all have shortcomings.

A static efficient allocation is one that maximizes the net benefit over all possible uses of those resources. The dynamic efficiency criterion, which is appropriate when time is an important consideration, is satisfied when the outcome maximizes the present value of net benefits from all possible uses of the resources. Later chapters examine the degree to which our social institutions yield allocations that conform to these criteria.

Because benefit-cost analysis is both very powerful and very controversial, in 1996 a group of economists of quite different political persuasions got together to attempt to reach some consensus on its proper role in environmental decision making. Their conclusion is worth reproducing in its entirety:

> Benefit-cost analysis can play an important role in legislative and regulatory policy debates on protecting and improving health, safety, and the natural environment. Although formal benefit-cost analysis should not be viewed as either necessary or sufficient for designing sensible policy, it can provide an exceptionally useful framework for consistently organizing disparate information, and in this way, it can greatly improve the process and, hence, the outcome of policy analysis. If properly done, benefit-cost analysis can be of great help to agencies participating in the development of environmental, health and safety regulations, and it can likewise be useful in evaluating agency decision-making and in shaping statutes.[11]

Even when benefits are difficult to calculate, however, economic analysis in the form of cost-effectiveness can be valuable. This technique can establish the least expensive ways to accomplish predetermined policy goals and to assess the extra costs involved when policies other than the least-cost policy are chosen. What it cannot do is answer the question of whether those predetermined policy goals are efficient.

At the other end of the spectrum is impact analysis, which merely identifies and quantifies the impacts of particular policies without any pretense of optimality or even comparability of the information generated. Impact analysis does not guarantee an efficient outcome.

All three of the techniques discussed in this chapter are useful, but none of them can stake a claim as being universally the "best" approach. The nature of the information that is available and its reliability make a difference.

Discussion Questions

1. Is risk-neutrality an appropriate assumption for benefit-cost analysis? Why or why not? Does it seem more appropriate for some environmental problems than others? If so, which ones? If you were evaluating the desirability of locating a hazardous waste incinerator in a particular town, would the Arrow–Lind rationale for risk-neutrality be appropriate? Why or why not?
2. Was the executive order issued by President George W. Bush mandating a heavier use of benefit-cost analysis in regulatory rule making a step toward establishing a more rational regulatory structure, or was it a subversion of the environmental policy process? Why?

Self-Test Exercises

1. Suppose a proposed public policy could result in three possible outcomes: (1) present value of net benefits of \$4,000,000, (2) present value of net benefits of \$1,000,000, or (3) present value of net benefits of –\$10,000,000 (i.e., a loss). Suppose society is risk-neutral and the probability of occurrence of each of these three outcomes are, respectively, 0.85, 0.10, and 0.05, should this policy be pursued or trashed? Why?

2. a. Suppose you want to remove ten fish of an exotic species that have illegally been introduced to a lake. You have three possible removal methods. Assume that q_1, q_2, and q_3 are, respectively, the amount of fish removed by each method that you choose to use so that the goal will be accomplished by any combination of methods such that $q_1 + q_2 + q_3 = 10$. If the marginal costs of each removal method are, respectively, $\$10q_1$, $\$5q_2$, and $\$2.5q_3$, how much of each method should you use to achieve the removal cost-effectively?
   b. Why isn't an exclusive use of method 3 cost-effective?
   c. Suppose that the three marginal costs were constant (not increasing as in the previous case) such that $MC_1 = \$10$, $MC_2 = \$5$, and $MC_3 = \$2.5$. What is the cost-effective outcome in that case?

3. Consider the role of discount rates in problems involving long time horizons such as climate change. Suppose that a particular emissions abatement strategy would result in a \$500 billion reduction in damages 50 years into the future. How would the maximum amount spent now to eliminate those damages change if the discount rate is 2 percent, rather than 10 percent?

Notes

1 Actually if $B = C$, it wouldn't make any difference if the action occurs or not; the benefits and costs are a wash.
2 The monetary worth of the net benefit is the sum of two right triangles, and it equals (1/2)(\$5)(5) + (1/2)(\$2.50)(5) or \$18.75. Can you see why?
3 The mathematics of dynamic efficiency are presented in the appendix to Chapter 5.
4 Interagency Working Group on the Social Cost of Carbon, August 2016. www.epa.gov/sites/production/files/2016-12/documents/sc_co2_tsd_august_2016.pdf (accessed November 18, 2017).
5 Hahn, Robert W., & and Ritz, Robert A. (2015). Does the social cost of carbon matter? Evidence from US policy. *The Journal of Legal Studies*, *44*(1) (January), 229–248.
6 The division between tangible and intangible benefits changes as our techniques improve. Recreation benefits were, until the advent of the travel-cost model, treated as intangible. The travel-cost model will be discussed in the next chapter.
7 Annual rates can be found at www.whitehouse.gov/omb/. 2010 rates can be found at www.whitehouse.gov/omb/circulars_a094/a94_appx-c
8 A more recent assessment of costs (Harrington et al., 1999) found evidence of both overestimation and underestimation, although overestimation was more common. The authors attributed the overestimation mainly to a failure to anticipate technical innovation.
9 Executive Order 13371, "Reducing Regulation and Controlling Regulatory Cost."
10 83 Stat. 853.
11 From Arrow, Kenneth, et al. (1996). Is there a role for benefit-cost analysis in environmental, health and safety regulation? *Science*, *272* (April 12), 221–222. Reprinted with permission from AAAS.

Chapter 4

Valuing the Environment

Methods

> For it so falls out / That what we have we prize not to the worth / Whiles we enjoy it, but being lack'd and lost / Why, then we rack the value, then we find / The virtue that possession would not show us / Whiles it was ours.
>
> —William Shakespeare, *Much Ado About Nothing*

Introduction

Soon after the *Exxon Valdez* oil tanker ran aground on the Bligh Reef in Prince William Sound off the coast of Alaska on March 24, 1989, spilling approximately 11 million gallons of crude oil, the Exxon Corporation (now Exxon Mobil) accepted the liability for the damage caused by the leaking oil. This liability consisted of two parts: (1) the cost of cleaning up the spilled oil and restoring the site insofar as possible, and (2) compensation for the damage caused to the local ecology. Approximately $2.1 billion was spent in cleanup efforts and Exxon also spent approximately $303 million to compensate fishermen whose livelihoods were greatly damaged for the 5 years following the spill.[1] Litigation on environmental damages settled with Exxon agreeing to pay $900 million over 10 years. The punitive damages phase of this case began in May 1994. In January 2004, after many rounds of appeals, the U.S. District Court for the State of Alaska awarded punitive damages to the plaintiffs in the amount of $4.5 billion.[2] This amount was later cut almost in half to $2.5 billion and in 2008 the Supreme Court ruled that even those punitive damages were excessive based on maritime law and further argued that the punitive damages should not exceed the $507 million in compensatory damages already paid.[3]

In the spring of 2010, the Deepwater Horizon, a BP well in the Gulf of Mexico, exploded and began spewing an *Exxon Valdez*-sized oil spill every 4 to 5 days. By the time the leaking well was capped in August 2010, an estimated 134 million gallons had been spread through the Gulf of Mexico, almost 20 times greater than the *Exxon Valdez* spill, and the largest

maritime spill in U.S. history. In 2016, a settlement was reached calling for total payments of $20.8 billion; $8.8 billion of this was for natural resource damages.[4] This amount is over and above the approximately $30 billion already spent on cleanup and other claims after the spill.[5] How can the economic damages from oil spills, like these that caused substantial economic and environmental harm, be calculated? Thousands of birds have been found dead in the Gulf since the BP spill, for example. What are they "worth?" Interestingly, the *Exxon Valdez* spill triggered pioneering work focused on providing monetary estimates of environmental damages, setting the stage for what is today considered standard practice for *nonmarket valuation*—the monetization of those goods and services without market prices.

In Chapter 3, we examined the basic concepts economists use to calculate these damages. Yet implementing these concepts is far from a trivial exercise. While the costs of cleanup are fairly transparent, estimating the damage is far more complex. For example, how were the $900 million in damages in the Exxon case and the $20 billion in the BP case determined?

In this chapter, we explore how we can move from the general concepts to the actual estimates of compensation required by the courts. A series of special techniques has been developed to value the benefits from environmental improvement or, conversely, to value the damage done by environmental degradation. Special techniques are necessary because most of the normal valuation techniques that have been used over the years cannot be applied to environmental resources. Benefit-cost analysis requires the monetization of all relevant benefits and costs of a proposed policy or project, not merely those where the values can be derived from market transactions As such, it is also important to monetize those environmental goods and services that are not traded in any market. Even more difficult to grapple with are those nonmarket benefits associated with values unrelated to use, topics explored below.

Why Value the Environment?

While it may prove difficult, if not impossible, to place a completely accurate value on certain environmental amenities, not making the attempt leaves us valuing them by default at $0. Will valuing them at $0 lead us to the best policy decisions? Probably not, but that does not prevent controversy from arising over attempts to replace $0 with a more appropriate value (Debate 4.1).

Many federal agencies depend on benefit-cost analyses for decision-making. Ideally, the goal is to choose the most economically desirable projects, given limited budgets. Estimation of benefits and costs is used for such diverse actions as follows:

- natural resources damage assessments, such as for oil spills (National Oceanic and Atmospheric Administration);
- the designation of critical habitat under the Endangered Species Act (U.S. Fish and Wildlife Service);
- dam relicensing applications (The Federal Energy Regulatory Commission);
- estimating the costs and benefits of the Clean Air Act and the Clean Water Act.

These analyses, however, frequently fail to incorporate important nonmarket values. If the analysis does not include all the appropriate values, the results will be flawed. Is this exercise worth it?

DEBATE 4.1

Should Humans Place an Economic Value on the Environment?

Arne Naess, the late Norwegian philosopher, used the term *deep ecology* to refer to the view that the nonhuman environment has "intrinsic" value, a value that is independent of human interests. Intrinsic value is contrasted with "instrumental" value, in which the value of the environment is derived from its usefulness in satisfying human wants.

Two issues are raised by the Naess critique: (1) what is the basis for the valuing of the environment? and (2) how is the valuation accomplished? The belief that the environment may have a value that goes beyond its direct usefulness to humans is in fact quite consistent with modern economic valuation techniques. As we shall see in this chapter, economic valuation techniques now include the ability to quantify a wide range of "nonuse" values as well as the more traditional "use" values.

Controversies over how the values are derived are less easily resolved. As described in this chapter, economic valuation is based firmly upon human preferences. Proponents of deep ecology, on the other hand, would argue that allowing humans to determine the value of other species would have no more moral basis than allowing other species to determine the value of humans. Rather, deep ecologists argue, humans should only use environmental resources when necessary for survival; otherwise, nature should be left alone. And, because economic valuation is not helpful in determining survival necessity, deep ecologists argue that it contributes little to environmental management.

Those who oppose all economic valuation face a dilemma: when humans fail to value the environment, it may be assigned a default value of zero in calculations designed to guide policy. A value of zero, however derived, will tend to justify a great deal of environmental degradation that could not be justified with proper economic valuation. Support seems to be growing for the proposition that economic valuation can be a very useful means of demonstrating when environmental degradation is senseless, even when judged from a limited anthropomorphic perspective.

Sources: Costanza, R., et al. (1998). The value of ecosystem services: Putting the issues in perspective. *Ecological Economics*, *25*(1), 67–72; Daily, Gretchen, & Ellison, Katherine. (2003). *The New Economy of Nature: The Quest to Make Conservation Profitable.* Washington, D.C.: Island Press.

Valuation

While the valuation techniques we shall cover can be applied to both the damage caused by pollution and the services provided by the environment, each context offers its own unique

aspects. We begin our investigation of valuation techniques by exposing some of the special challenges posed by the first of those contexts, pollution control.

In the United States, damage estimates are not only used in the design of policies, but, as indicated in the opening paragraphs of this chapter, they have also become important to the courts, who need some basis for deciding the magnitude of liability awards.[6]

The damage caused by pollution can take many different forms. The first, and probably most obvious, is the effect on human health. Polluted air and water can cause disease when ingested. Other forms of damage include loss of enjoyment from outdoor activities and damage to vegetation, animals, and materials.

Assessing the magnitude of this damage requires (1) identifying the affected categories, (2) estimating the physical relationship between the pollutant emissions (including natural sources) and the damage caused to the affected categories, (3) estimating responses by the affected parties toward mitigating some portion of the damage, and (4) placing a monetary value on the unmitigated physical damages. Each step is often difficult to accomplish.

Because the data used to track down causal relationships do not typically come from controlled experiments, identifying the affected categories is a complicated matter. Obviously, we cannot run large numbers of people through controlled experiments. If people were subjected to different levels of some pollutant, such as carbon monoxide, so that we could study the short-term and long-term effects, some might become ill and even die. Ethical concern precludes human experimentation of this type.

This leaves us essentially two choices. We can try to infer the impact on humans from controlled laboratory experiments on animals, or we can do statistical analysis of differences in mortality or disease rates for various human populations living in polluted environments to see the extent to which they are correlated with pollution concentrations. Neither approach is completely acceptable.

Animal experiments are expensive, and the extrapolation from effects on animals to effects on humans is tenuous at best. Many significant exposure effects do not appear for a long time. To determine these effects in a reasonable period of time, test animals are commonly subjected to large doses for relatively short periods. The researcher then extrapolates from the results of these high-dosage, short-duration experiments to estimate the effects of low-dose, long-duration exposure to pollution on a human population. Because these extrapolations move well beyond the range of experimental observations, many scientists disagree on how the extrapolations should be accomplished. Ethical concerns also arise with animal experiments.

Statistical studies, on the other hand, deal with human populations exposed to low doses for long periods, but, unfortunately, they have another set of problems—correlation does not imply causation. To illustrate, the fact that death rates are higher in cities with higher pollution levels does not prove that the higher pollution caused the higher death rates. Perhaps those same cities averaged older populations or perhaps they had more smokers. Existing studies have been sophisticated enough to account for many of these other possible influences but, because of the relative paucity of data, they have not been able to cover them all.

The problems discussed so far arise when identifying whether a particular observed effect results from pollution. The next step is to estimate how strong the relationship is between the effect and the pollution concentrations. In other words, it is necessary not only to discover *whether* pollution causes an increased incidence of respiratory disease, but also to estimate *how much* reduction in respiratory illness could be expected from a given reduction in pollution.

The nonexperimental nature of the data makes this a difficult task. It is not uncommon for different researchers analyzing the same data to come to remarkably different conclusions. Diagnostic problems are compounded when the effects are synergistic—that is, when the effect

depends, in a nonadditive way, on contributing factors such as the victims' smoking habits or the presence of other harmful substances in the air or water.

Once physical damages have been identified, the next step is to place a monetary value on them. It is not difficult to see how complex an undertaking this is. Think about the difficulties in assigning a value to extending a human life by several years or to the pain, suffering, and grief borne by both a cancer victim and the victim's family.

How can these difficulties be overcome? What valuation techniques are available not only to value pollution damage, but also to value the large number of services that the environment provides?

Types of Values

Economists have decomposed the total economic value conferred by resources into three main components: (1) use value, (2) option value, and (3) nonuse or passive-use values. *Use value* reflects the direct use of the environmental resource. Examples include fish harvested from the sea, timber harvested from the forest, water extracted from a stream for irrigation, even the scenic beauty conferred by a natural vista. If you used one of your senses to experience the resource—sight, sound, touch, taste, or smell—then you have *used* the resource. Pollution can cause a loss of use value, such as when air pollution increases the vulnerability to illness, an oil spill adversely affects a fishery, or smog enshrouds a scenic vista.

Option value reflects the value people place on a future ability to use the environment. It reflects the willingness to pay to preserve the option to use the environment in the future even if one is not currently using it. Whereas use value reflects the value derived from current use, option value reflects the desire to preserve the potential for possible future use. Are you planning to go to Yellowstone National Park next summer? Perhaps not, but would you like to preserve the option to go someday?

Passive-use or *nonconsumptive use values* arise when the resource is not actually consumed in the process of experiencing it. These types of values reflect the common observation that people are more than willing to pay for improving or preserving resources that they will never use. One type of nonuse value is a *bequest value*. Bequest value is the willingness to pay to ensure a resource is available for your children and grandchildren. A second type of nonuse value, a pure nonuse value, is called *existence value*. Existence value is measured by the willingness to pay to ensure that a resource continues to exist in the absence of any interest in future use. The term existence value was coined by economist John Krutilla in his now-famous quote, "There are many persons who obtain satisfaction from mere knowledge that part of wilderness North America remains even though they would be appalled by the prospect of being exposed to it."[7] These values are "independent of any present or future use these people might make of those resources."[8]

When the Bureau of Reclamation began looking at sites for dams near the Grand Canyon, groups such as the Sierra Club rose up in protest of the potential loss of this unique resource. When Glen Canyon was flooded by Lake Powell, even those who never intended to visit recognized this potential loss. Because this value does not derive either from direct use or potential use, it represents a very different category of value.

These categories of value can be combined to produce the total willingness to pay (TWP):

$$\text{TWP} = \text{Use Value} + \text{Option Value} + \text{Nonuse Value}$$

Since nonuse or passive-use values are derived from motivations other than personal use, they are obviously less tangible than use values. Total willingness to pay estimated without

nonuse values, however, will be less than the minimum amount that would be required to compensate individuals if they are deprived of this environmental asset. Furthermore, estimated nonuse values can be quite large. Therefore, it is not surprising that they are controversial. Indeed when the U.S. Department of Interior drew up its regulations on the appropriate procedures for performing natural resource damage assessment, it prohibited the inclusion of nonuse values unless use values for the incident under consideration were zero. A subsequent 1989 decision by the District of Columbia Court of Appeals (880 F. 2nd 432) overruled this decision and allowed nonuse values to be included as long as they could be measured reliably.

Classifying Valuation Methods

Typically, the researcher's goal is to estimate the total willingness to pay for the good or service in question. This is the area under the demand curve up to the quantity consumed (recall discussion from Chapter 2). For a market good, this calculation is relatively straightforward. However, nonmarket goods and services, the focus of this chapter, require the estimation of willingness to pay either through examining behavior, drawing inferences from the demand for related goods, or through responses to surveys. And, as highlighted above, capturing all components of value is challenging.

This section will provide a brief overview of some of the methods available to estimate these values and to convey some sense of the range of possibilities and how they are related. Subsequent sections will provide more specific information about how they are actually used.

Valuation methods can be separated into two broad categories: stated preference and revealed preference methods. Revealed preference methods are based on actual observable choices that allow resource values to be directly inferred from those choices. For example, in calculating how much local fishermen lost from the oil spill, the revealed preference method might calculate how much the catch declined and the resulting diminished value of the catch. In this case, prices are directly observable, and their use allows the direct calculation of the loss in value. Or, more indirectly, in calculating the value of an occupational environmental risk (such as some exposure to a substance that could pose some health risk), we might examine the differences in wages across industries in which workers take on different levels of risk.

Compare this with the direct stated preference method that can be used when the value is not directly observable, such as the value of preserving a species. Analysts derive this value by using a survey that attempts to elicit the respondents' willingness to pay (their "stated preference") for preserving that species.

Each of these broad categories of methods includes both indirect and direct techniques. The possibilities are presented in Table 4.1. We start with an examination of stated preference survey methods.

Stated Preference Methods

Stated preference methods use survey techniques to elicit willingness to pay for a marginal improvement or for avoiding a marginal loss. These methods are typically of two types, contingent valuation surveys and choice experiments. *Contingent valuation*, the most direct approach, provides a means of deriving values that cannot be obtained in more traditional ways. The simplest version of this approach merely asks respondents what they would be willing to pay for a change in environmental quality (such as an improvement in wetlands or reduced exposure to pollution) or on preserving the resource in its current state. Typically this question is framed as, "What is the maximum you are willing to pay for the change?"

Table 4.1 Economic Methods for Measuring Environmental and Resource Values

Methods	*Revealed Preference*	*Stated Preference*
Direct	Market Price Simulated Markets	Contingent Valuation
Indirect	Travel Cost Hedonic Property Values Hedonic Wage Values Avoidance Expenditures	Choice Experiments Conjoint Analysis Attribute-Based Models Contingent Ranking

Source: Modified by the authors from Mitchell and Carson, 1989.

Alternative versions ask a "yes" or "no" question, such as whether or not the respondent would pay $\$X$ to prevent the change or preserve the species. The answers reveal either an upper bound (in the case of a "no" answer) or a lower bound (in the case of a "yes" answer).

Choice experiments, on the other hand, present respondents with a set of options. Each set consists of various levels of attributes or characteristics of the good. One of the characteristics will be the "price" of that bundle of attributes. Each choice set typically includes the status quo bundle which includes a price of $0 since it represents no change. Respondents choose their preferred option.

Contingent Valuation Method

The contingent valuation survey approach creates a hypothetical market and asks respondents to consider a willingness-to-pay question *contingent* on the existence of this market. Contingent valuation questions come with their own set of challenges. The major concern with the use of the contingent valuation method has been the potential for survey respondents to give biased answers. Six types of potential bias have been the focus of a large amount of research: (1) strategic bias, (2) information bias, (3) starting-point bias, (4) hypothetical bias, (5) payment vehicle bias (protest bids), and (6) the observed discrepancy between willingness to pay (WTP) and willingness to accept (WTA).

Strategic bias arises when the respondent intentionally provides a biased answer in order to influence a particular outcome. If a decision to preserve a stretch of river for fishing, for example, depends on whether or not the survey produces a sufficiently large value for fishing, the respondents who enjoy fishing may be tempted to provide an answer that ensures a high value, rather than the lower value that reflects their true valuation. Another variation on strategic bias is social desirability bias, which occurs when respondents try to present themselves in a favorable light; one common example is when voters claim to have voted when they did not.

Information bias may arise whenever respondents are forced to value attributes with which they have little or no experience. For example, the valuation by a recreationist of a loss in water quality in one body of water may be based on the ease of substituting recreation on another body of water. If the respondent has no experience using the second body of water, the valuation could be based on an entirely false perception.

Visual aids have been shown to reduce uncertainty and unfamiliarity with the good or service being valued, but the nature of the visual aid may affect the response. Labao et al. (2008) found that colored photographs, as opposed to black-and-white photographs, influenced respondent

willingness to pay for the Philippine Eagle. The colored photographs resulted in a higher willingness to pay than black-and-white photos. Why? The authors suggest that the higher willingness to pay could be explained by photographs in color simply providing more information or by “enhancing respondents’ ability to assimilate information.” In any case, the nature of the visual aide seems important for revealing preferences.

Starting-point bias may arise in those survey instruments in which a respondent is asked to check off their WTP from a predefined range of possibilities. How that range is defined by the designer of the survey may affect the resulting answers. A range of $0–$100 may produce a valuation by respondents different from, for example, a range of $10–$100, even if no responses are in the $0–$10 range. Ladenburg and Olsen (2008), in a study of willingness to pay to protect nature areas in Denmark from new highway development, found that the starting-point bias in their choice experiment was gender specific, with female respondents exhibiting the greatest sensitivity to the starting point.

Hypothetical bias can enter the picture because the respondent is being confronted by a contrived, rather than an actual, set of choices. Since he or she will not actually have to pay the estimated value, the respondent may treat the survey casually, providing ill-considered answers. One early survey (Hanemann, 1994) found ten studies that directly compared willingness-to-pay estimates derived from surveys with actual expenditures. Although some of the studies found that the willingness-to-pay estimates derived from surveys exceeded actual expenditures, the majority of those found that the differences were not statistically significant. Subsequently, Ehmke, Lusk, and List (2008) tested whether hypothetical bias depends on location and/or culture. In a study based on student experiments in China, France, Indiana, Kansas, and Niger, they found significant differences in bias across locations. Given that policymakers frequently rely on existing benefits estimates from other locations when making decisions, this finding should not be taken lightly. The strengths and weaknesses of using estimates derived in one setting to infer benefits in another, a technique known as *benefit transfer*, are discussed below.

Increasingly, environmental economists are using these types of experiments to try to determine the severity of some of these biases as well as to learn how to reduce bias. Some of these experiments are conducted in a laboratory setting, such as a computer lab or a classroom designed for this purpose. In one such experiment on voluntary provision of public goods (donations), Landry et al. (2006) found that for door-to-door interviews, an increase in physical attractiveness of the interviewer led to sizable increases in giving. Interestingly, physical attractiveness also led to increases in response rates, particularly by male households. Sometimes called *interviewer bias*, biases like these can be kept small through well-designed and pretested surveys.

Another challenge, *payment vehicle bias*, can arise when respondents react negatively to the choice of the payment vehicle. The payment vehicle represents how the stated WTP would be collected. Common choices include donations, taxes or increases to utility bills. If a respondent is averse to taxes or has a negative perception of the agency collecting the (hypothetical) payment, they may state $0 for their willingness to pay. If their true willingness to pay is actually greater than zero, but they are “protesting” the question or payment vehicle, this zero must be excluded from the analysis. Determining which zero bids are valid and which are protests is important.

The final source of bias addresses observed gaps between two supposedly closely related concepts—willingness to pay (WTP) and willingness to accept (WTA) compensation. Respondents to contingent valuation surveys tend to report much higher values when asked for their willingness to accept compensation for a specified loss of some good or service than if asked for their willingness to pay for a specified increase of that same good or service. Economic theory suggests the two should be equal. Debate 4.2 explores some of the reasons offered for the difference.

Measuring willingness to pay or willingness to accept in the presence of price changes makes two new concepts relevant—compensating variation and equivalent variation. *Compensating variation* is the amount of money it would take to *compensate* for a price increase in order to make a consumer just as well off as she or he was before the price increase. How much the consumer was "hurt" by the price increase can be measured by the compensating variation. *Equivalent variation*, on the other hand, is the amount of money it would take to make a consumer indifferent (same income) between the money and the price increase. In other words, how much money would she or he pay to avoid the price increase?

If the compensating variation is greater than zero, that amount represents willingness to pay. If it is negative, it represents willingness to accept. In other words, for increases in environmental quality, compensating variation should be positive (WTP). For decreases in environmental quality, it should be negative (WTA). Equivalent variation is just the opposite—the amount of money the household would need to be given to be just as well off as before the environmental change. Equivalent variation will be positive for increases in environmental quality (WTA) and negative for decreases (WTP). In theory, in the absence of any income effects, these measures (along with consumer surplus) should be equivalent.

Much experimental work has been done on contingent valuation to determine how serious a problem biases may present. One early survey (Carson et al., 1994) uncovered 1,672 contingent valuation studies., A much more recent one gives annotations for more than 7,500 studies in 130 countries (Carson, 2011)! Are the results from these surveys reliable enough for the policy process?

DEBATE 4.2

Willingness to Pay versus Willingness to Accept: Why So Different?

Many contingent valuation studies have found that respondents tend to report much higher values for questions that ask what compensation the respondent would be willing to accept (WTA) to give something up than for questions that ask for the willingness to pay (WTP) for an incremental improvement in the same good or service. Economic theory suggests that differences between WTP and WTA should be small, but experimental findings both in environmental economics and in other microeconomic studies have found large differences. Why?

Some economists have attributed the discrepancy to a psychological endowment effect; the psychological value of something you own is greater than something you do not. In other words, you would require more compensation to be as well off without it than you would be willing to pay to get that same good, and as such, you would be less willing to give it up (WTA > WTP) (Kahneman, Knetsch, & Thaler, 1990). This is a form of what behavioral economists call loss aversion—the psychological premise that losses are more highly valued than gains.

Others have suggested that the difference may be explainable in terms of the market context. In the absence of good substitutes, large differences between WTA and WTP would be the expected outcome. In the presence of

close substitutes, WTP and WTA should not be that different, but the divergence between the two measures should increase as the degree of substitution decreases (Hanemann, 1991; Shogren et al., 1994).

The characteristics of the good may matter as well. In their review of the evidence provided by experimental studies, Horowitz and McConnell (2002) find that for "ordinary goods" the ratio of WTA/WTP is smaller than the ratio of WTA/WTP for public and nonmarket goods. Their results support the notion that the nature of the property rights involved is not neutral.

The moral context of the valuation may matter as well. Croson et al. (Draft 2005) show that the amount of WTA compensation estimated in a damage case increases with the culpability of the party causing the damage as long as that party is also paying the damages. If, however, a third party is paying, WTA is insensitive to culpability. This difference suggests that the valuation implicitly includes an amount levied in punishment for the party who caused the damage (the valuation becomes the lost value plus a sanction).

It may also be the case that, in dynamic settings, respondents are uncertain about the value of the good. Zhao and Kling (2004) argue that in intertemporal settings, the equivalence of compensating variation/equivalent variation and WTP/WTA breaks down, in part because WTP and WTA have a behavioral component and the timing of a decision will be impacted by the consumer's rate of time preference and willingness to take risks. A buyer or seller, by committing to a purchase or sale, must forgo opportunities for additional information. These "commitment costs" reduce WTP and increase WTA. The larger the commitment costs, the larger is the divergence between the two measures.

Ultimately, the choice of which concept to use in environmental valuation comes down to how the associated property right is allocated. If someone owns the right to the resource, asking how much compensation they would require to give it up is the appropriate question. If the respondent does not have the right, using WTP to estimate the value of acquiring it is the right approach. However, as Horowitz and McConnell point out, since the holders and nonholders of "rights" value them differently, the initial allocation of property rights can have a strong influence on valuation decisions for environmental amenities. And, as Zhao and Kling note, the timing of the decision can also be an important factor.

Sources: Croson, R., Rachlinski, J. J., & Johnston, J. (Draft 2005). Culpability as an explanation of the WTA–WTP discrepancy in contingent valuation; Hanemann, W. M. (1991). Willingness to pay and willingness to accept: How much can they differ? *American Economic Review*, *81*, 635–647; Horowitz, J. K., & McConnell, K. E. (2002). A review of WTA/WTP studies. *Journal of Environmental Economics and Management*, *44*, 426–447; Kahneman, D., Knetsch, J., & Thaler, R. (1990). Experimental tests of the endowment effect and the Coase theorem. *Journal of Political Economy*, *98*, 1325–1348; Shogren, J. F., Shin, Senung Y., Hayes, D. J., & Kliebenstein, J. B. (1994). Resolving differences in willingness to pay and willingness to accept. *American Economic Review*, *84*(1), 255–270; Zhao, Jinhua, & Kling, Catherine. (2004). Willingness to pay, compensating variation, and the cost of commitment. *Economic Inquiry*, *42*(3), 503–517.

Faced with the need to compute damages from oil spills, the National Oceanic and Atmospheric Administration (NOAA) convened a panel of independent economic experts (including two Nobel prize laureates) to evaluate the use of contingent valuation methods for determining lost passive-use or nonuse values. Their report, issued on January 15, 1993 (58 FR 4602), was cautiously supportive.

The committee made clear that it had several concerns with the technique. Among those concerns, the panel listed (1) the tendency for contingent valuation willingness-to-pay estimates to seem unreasonably large; (2) the difficulty in assuring the respondents have understood and absorbed the issues in the survey; and (3) the difficulty in assuring that respondents are responding to the specific issues in the survey rather than reflecting general warm feelings about public-spiritedness, known as the "warm glow" effect.[9]

But the panel also made clear its conclusion that suitably designed surveys could eliminate or reduce these biases to acceptable levels and it provided, in an appendix, specific guidelines for determining whether a particular study was suitably designed. The panel suggested that when practitioners follow these guidelines, they:

> can produce estimates reliable enough to be the starting point of a judicial process of damage assessment, including lost passive-use values. . . . [A well-constructed contingent valuation study] contains information that judges and juries will wish to use, in combination with other estimates, including the testimony of expert witnesses.

Specifically, they suggested the use of referendum-type (yes/no) willingness-to-pay questions, personal interviews when possible, clear scenario descriptions, and follow-up questions.

These guidelines have been influential in shaping subsequent studies. For example, Example 4.1 shares the results of a large contingent valuation survey, designed to estimate the value of preventing future spills. While influential, these guidelines have become dated, and, in 2017, new "contemporary guidelines" were published (Johnston et al., 2017). These guidelines provide best practice recommendations for both contingent valuation and choice experiments using what has been learned in the approximately 8000 stated preference studies published since the NOAA guidelines were first published. The authors offer 23 recommendations including designing a survey that clearly describes the status quo or baseline scenario, selecting a random sample of the affected population and choosing an appropriate survey mode. They also recommend pretesting of the survey instrument. They give extensive guidance on when to choose contingent valuation over a choice experiment and vice versa as well recommendations for reducing and addressing response bias.

Choice Experiments

Indirect hypothetical stated preference methods include several attribute-based methods. Attribute-based methods, such as choice experiments, are useful when project options have multiple levels of different attributes. Like contingent valuation, choice experiments are also survey based, but instead of asking respondents to state a willingness to pay, they are asked to choose among alternate bundles of goods. Each bundle has a set of attributes and the levels of each attribute vary across bundles. Since one of the attributes in each bundle is a price measure, willingness to pay can be identified.

Consider an example (Landry and Mires, 2017) that surveyed North Carolina residents on their preferences and willingness to pay for marine cultural heritage sites (e.g., shipwrecks). The choice experiment included five attributes including the preservation zone, the availability of public programs and whether or not there was a walking, virtual, or diving trail. Table 4.2

EXAMPLE 4.1

Leave No Behavioral Trace: Using the Contingent Valuation Method to Measure Passive-Use Values

Until the *Exxon Valdez* tanker spilled 11 million gallons of crude oil into Prince William Sound in Alaska, the calculation of nonuse (or passive-use) values was not a widely researched topic. However, following the 1989 court ruling in *Ohio v. U.S. Department of the Interior* that said lost passive-use values could now be compensated within natural resources damages assessments and the passage of the Oil Pollution Act of 1990, the estimation of nonuse and passive-use values became not only a topic of great debate, but also a rapidly growing research area within the economics community.

One study (Carson et al., 2003) discusses the design, implementation, and results of a large survey designed to estimate the passive-use values related to large oil spills. In particular, the survey asked respondents their willingness to pay to prevent a similar disaster in the future by funding an escort ship program that would help prevent and/or contain a future spill. The survey was conducted for the State of Alaska in preparation for litigation in the case against the *Exxon Valdez*.

The survey followed the recommendations made by the NOAA panel for conducting contingent valuation surveys and for ensuring reliable estimates. It relied upon face-to-face interviews and the sample was drawn from the national population. The study used a binary discrete-choice (yes/no) question where the respondent was asked whether he or she would be willing to pay a specific amount, with the amount varying across four versions of the survey. A one-time increase in taxes was the chosen method of payment. They also avoided potential embedding bias (where respondents may have difficulty valuing multiple goods) by using a survey that valued a single good. The survey contained pictures, maps, and background information to make sure the respondent was familiar with the good he/she was being asked to value.

Using the survey data, the researchers were able, statistically, to estimate a valuation function by relating the respondent's willingness to pay to respondent characteristics. After multiplying the estimate of the median willingness to pay by the population sampled, they reported aggregate lost passive-use values at $2.8 billion (in 1990 dollars). They point out that this number is a lower bound, not only because willingness to accept compensation would be a more appropriate measure of actual lost passive-use from the spill (see Debate 4.2), but also because their median willingness to pay was less than the mean.

The *Exxon Valdez* spill sparked a debate about the measurement of nonuse and passive-use values. Laws put into place after the spill now ensure that passive-use values will be included in natural resource damage assessments. Should other parts of the world follow suit?

Source: Carson, Richard T., Mitchell, Robert C., Hanemann, Michael, Kopp, Raymond J., Presser, Stanley, & Ruud, Paul A. (2003). Contingent valuation and lost passive use: Damages from the *Exxon Valdez* oil spill. *Environmental and Resource Economics*, *25*, 257–286.

Table 4.2 Choice Experiment: Attributes and Levels

Attributes	*Levels*
Preservation Zone	• Status quo; 30 shipwrecks protected • 38 more shipwrecks (68 total; 127% increase); 2192 m^2 of bottomland • 56 more shipwrecks (124 total; 313% increase); 13,498 m^2 of bottomland
Public Programs	• No change • Increase museum exhibits and provide educational workshops • Increase museum exhibits and provide educational workshops, plus public television series about shipwrecks and creation of boating tours to shipwrecks
Walking Trail	• Yes/No
Virtual Trail	• Yes/No
Diving Trail	• Yes/No

reproduces the attributes and levels. With three attributes with three levels and two with two levels, there are 216 possible different profiles. Best practices suggests the use of a fractional factorial design and the authors chose eight versions with three choice sets per version of the survey, resulting in 24 choice sets.

Respondents were given a choice set of three different alternative management plans and the status quo (no purchase). Table 4.3 demonstrates a sample survey question. The researchers found a willingness to pay of $98 per household for a moderate level of public programs on Maritime Archeology and Shipwrecks; $90/household for the Walking Trails for *the Graveyard of the Atlantic*; and $84/household for Virtual Trails for *the Graveyard of the Atlantic*: (Landry and Mires 2017).

Choice experiments have evolved from both contingent valuation and marketing studies. This approach allows the respondent to make a familiar choice (choose a bundle) and allows the researcher to derive marginal willingness to pay for an attribute from that choice.

In another example, Haefele et al. (2016) present the results of a choice experiment in which they estimated the total economic value of U.S. National Parks (Example 4.2). Including both visitation values and passive-use values for U.S. residents, the total value of U.S. National Parks and Programs more than pays for itself at $92 billion dollars. This estimate is considered a minimum bound since international visitation values were not included.

Contingent ranking, another survey method, also falls within this final category. Respondents are given a set of hypothetical situations that differ in terms of the environmental amenity available (instead of a bundle of attributes) and are asked to rank-order them. These rankings can then be compared to see the implicit tradeoffs between more of the environmental amenity and less of the other characteristics. When one or more of these characteristics is expressed in terms of a monetary value, it is possible to use this information and the rankings to impute a value to the environmental amenity.

Sometimes more than one of these techniques may be used simultaneously. In some cases, using multiple techniques is necessary to capture the total economic value; in other cases, it may be used to provide independent estimates of the value being sought as a check on the reliability of the estimate. Chapter 13 will provide several additional examples.

Table 4.3 Sample Choice Experiment Question

I: Here is the first voting opportunity
(Please chose one of the four options below by putting an "X" in one of the empty boxes)

26.		Program 1		Program 2		Program 3		Status Quo
Preservation Zone		Yellow Zone		Yellow Zone		Red Zone		Red Zone
Public Programs		Large Investment		No Investment		Large Investment		No Investment
Walking Trails		Yes		No		No		No
Virtual Trails		No		Yes		Yes		No
SCUBA Diving Trails		Yes		No		No		No
One-time Tax		$12		$55		$145		$0
*put an "X" in **one** of the boxes to the right*		☐		☐		☐		☐

27. How confident are you about this choice from these options? (Please select one)
❍ Very Certain ❍ Somewhat Certain ❍ Somewhat Uncertain ❍ Very Uncertain ❍ Don't Know

II: Now consider another voting opportunity with different choices
(Please choose one of the four options below by putting an "X" in one of the empty boxes)

28.		Program 4		Program 5		Program 6		Status Quo
Preservation Zone		Orange Zone		Orange Zone		Yellow Zone		Red Zone
Public Programs		Large Investment		No Investment		No Investment		No Investment
Walking Trails		No		Yes		No		No
Virtual Trails		No		Yes		No		No
SCUBA Diving Trails		Yes		No		Yes		No
One-time Tax		$145		$12		$55		$0
*put on "X" in **one** of the boxes to the right*		☐		☐		☐		☐

29. How confident are you about this choice from these options? (Please select one)
❍ Very Certain ❍ Somewhat Certain ❍ Somewhat Uncertain ❍ Very Uncertain ❍ Don't Know

III: Finally, consider this third opportunity with different choices
(Please chose one of the four options below by putting an "X" in one of the empty boxes)

30.		Program 7		Program 8		Program 9		Status Quo
Preservation Zone		Red Zone		Red Zone		Yellow Zone		Red Zone
Public Programs		No Investment		Moderate Investment		Moderate Investment		No Investment
Walking Trails		Yes		No		Yes		No
Virtual Trails		Yes		No		Yes		No
SCUBA Diving Trails		No		Yes		No		No
One-time Tax		$12		$145		$55		$0
*put on "X" in **one** of the boxes to the right*		☐		☐		☐		☐

31. How confident are you about this choice from these options? (Please select one)
❍ Very Certain ❍ Somewhat Certain ❍ Somewhat Uncertain ❍ Very Uncertain ❍ Don't Know

EXAMPLE 4.2

The Value of U.S. National Parks

In 2016, the National Park Service in the United States turned 100 years old. As federal budget deficits loom, there has been some talk of selling off some of these sites. What is the value of the National Park lands, waters, and historic sites? According to the first ever comprehensive estimate, it is, at a minimum, valued at $92 billion.

Haefele et al. (2016) present the results of a survey of American households focused on estimating the total economic value (TEV) of National Parks and Programs. Previous studies have focused on the value of specific National Park or monument sites, but none had attempted to estimate the value of all of these national treasures. The goal was to calculate total economic value; visitation values and passive-use (or nonuse) values.

Using the population of all U.S. households from which to draw a sample, researchers used a mixed mode approach that utilized both mail and internet surveys with phone call reminders. Two rounds of surveys were implemented between 2013 and 2015.

In the survey, participants were asked whether protecting National Parks was important to them. Nearly 95 percent of the sample said they were, even if they did not visit them. Moreover, 93.5 percent thought it was important to protect trails, parks, and open spaces for current and future generations whether they use them or not. The language in these questions suggests bequest and passive-use values. Only 6.2 percent thought the U.S. should sell off some National Parks. The survey also included questions on respondents' political point of view. The sample of respondents leaned to the conservative side of the aisle.

The stated preference survey design was a choice experiment in which respondents chose among bundles that included the size of cuts to programs as well as the percentages of lands sold. Choice experiments typically allow respondents to choose a status quo bundle for which the price is $0. In order to minimize hypothetical bias (respondents stating a higher willingness to pay than they would actually pay), the choice question was followed by reminders to consider their budgets. This "cheap talk" technique has been shown to significantly reduce hypothetical bias.

Respondents were asked their willingness to pay a specific amount of money to pay for the National Park Service Programs. The payment vehicle utilized was an increase in federal income tax for each of the next 10 years. As we have discussed in this chapter, protest responses must be omitted from the data since those answers do not represent willingness to pay, instead representing a scenario (usually payment vehicle) protest. Since the payment vehicle chosen was federal income tax, there was some initial concern that protest zeros would be problematic, however, only 7.5 percent of the responses were considered to be protests.

Using econometric analysis, the marginal willingness to pay (or implicit price) for each type of National Park or National Park Service Program were estimated. These values are reproduced in Table 4.4.

Table 4.4 Per-household total economic value (TEV) for the National Park system and NPS Programs

National Parks	*Estimated value*
Nature-focused National Parks (79,096,632 acres)	$1,113.24
History-focused National Parks (226 sites)	$874.72
Water-focused National Parks (4,818,275 acres)	$977.93
Per household value for all National Park acres/sites	**$2,967.00**
NPS Programs	
Historic sites and buildings protected each year (2000)	$316.31
Acres transferred to communities each year (2700)	$98.41
National landmarks protected each year (114)	$347.98
Schoolchildren served by NPS educational programs (4.1 million)	$682.62
Per household value for all NPS programs	**$1,445.00**

Source: Table 4 in Haefele et al. (2016)

These household values were then multiplied by the total number of households in the population to determine the total economic value. In order to present a minimum bound (or very conservative estimate), they assumed that households that did not return a survey were willing to pay $0.

The final tally of $92 billion includes both use values for visitors and passive-use or existence values, $62 billion of which (or two-thirds) is for the National Park Service lands and waters and historic sites, with $30 billion for programs. Of the $62 billion, the authors suggest that approximately half of that value is passive-use value. Of course, these values do not even include the willingness to pay of the millions of international tourists that visit U.S. National Parks each year or those who hold passive-use values for these locations. Thus, the $92 billion TEV also represents "the minimum amount that U.S. households are willing to pay to avoid the loss of the NPS and its programs" (Haelfele et al., 2016, p. 25).

According to one of the authors of the study, Linda Bilmes at Harvard University, the study shows that "Americans value the National Park Service at least 30 times more than the government spends on them." It is a happy 100th birthday indeed.

Sources: Haefele, Michelle, Loomis, John, & Bilmes, Linda. (2016). Total economic valuation of the National Park Service Lands and Programs: Results of a survey of the American public. Faculty Research Working Paper Series. RWP16-024 (June); Haefele, Michelle, Loomis, John, & and Bilmes, Linda. (2016). Total economic valuation of US National Park Service estimated to be $92 billion: Implications for policy. *The George Wright Forum*, *33*(3): 335–345; National Park Foundation Press Release. (June 30, 2016). National Park Foundation announces study determining value of America's National Parks to be $92 billion.

Revealed Preference Methods

Revealed preference methods are "observable" because they involve actual behavior and expenditures and "indirect" because they infer a value rather than estimate it directly. Suppose, for example, a particular sport fishery is being threatened by pollution, and one of the damages caused by that pollution is a reduction in sportfishing. How is this loss to be valued when access to the fishery is free?

Travel-Cost Method. One way to derive this loss is through *travel-cost* methods. Travel-cost methods may infer the value of a recreational resource (such as a sport fishery, a park, or a wildlife preserve where visitors hunt with a camera) by using information on how much visitors spend in getting to the site to construct a demand curve representing willingness to pay for a "visitor day."

Freeman et al. (2014) identify two variants of this approach. In the first, analysts examine the number of trips visitors make to a site. In the second, the analysts examine whether people decide to visit a site and, if so, which site. This second variant includes using a special class of models, known as random utility models, to value quality changes.

The first variant allows the construction of a travel cost demand function. The value of the flow of services from that site is the area under the estimated demand curve for those services or for access to the site, aggregated over all who visit the site. Using this variant, individual consumer surplus can be estimated. The area below the demand curve but above the travel cost (price) is the consumer surplus.

The second variant enables an analysis of how specific site characteristics influence choice and, therefore, indirectly how valuable those characteristics are. Knowledge of how the value of each site varies with respect to its characteristics allows the analyst to value how degradation of those characteristics (e.g., from pollution) would lower the value of the site.

Travel-cost models have been used to value National Parks, mountain climbing, recreational fishing, and beaches. Travel-cost models have also been used to value losses from events such as beach closures during oil spills, fish consumption advisories, and the cost of development that has eliminated a recreation area. The methodology for both variants is detailed in Parsons (2003).

In the random utility model, a person choosing a particular site takes into consideration site characteristics and its price (trip cost). Characteristics affecting the site choice include ease of access and environmental quality. Each site results in a unique level of utility and a person is assumed to choose the site giving the highest level of utility to that person. Welfare losses from an event such as an oil spill can then be measured by the resulting change in utility should the person have to choose an alternate, less desirable site.

Example 4.3 looks at the use of travel cost methods to estimate the economic impacts of beach closures due to oil spills in Minorca, Spain.

One interesting paradox that arises with the travel cost model is that those who live closest to the site, and may actually visit frequently, will have low travel costs. These users will appear to have a lower value for that site even if their (unmeasured) willingness to pay for the experience is very high. Another challenge in this model is how to incorporate the opportunity cost of time. Usually, this is represented by wages, but that approach is not universally accepted.

Hedonic Property Value and Hedonic Wage Methods. Two other revealed preference methods are the *hedonic property value* and *hedonic wage* methods. They share the characteristic that they use a statistical technique, known as multiple regression analysis, to

EXAMPLE 4.3

Using the Travel Cost Method to Estimate Recreational Value: Beaches in Minorca, Spain

Minorca, an island in the Mediterranean Sea, is a very popular tourist destination. Minorca's population doubles in the summer months from about 80,000 year-round residents to between 150,000 and 175,000 in the summer. The island's beaches are a major attraction.

Just how valuable are those beaches? To provide an estimate, researchers considered a hypothetical scenario in which an oil spill resulted in closure of certain beaches on the island. The analysis involved using a random utility model based upon survey data to estimate the economic impacts of these closures.

In 2008, 573 face-to-face individual surveys were conducted at 51 different beaches on the island using a discrete choice travel-cost survey. Respondents were asked some typical travel-cost survey questions such as where the trip originated, how they got to the site, how many people they were traveling with and their ages, and some questions to collect socio-economic demographics on the respondents. After being asked about their attitudes toward different beach attributes, they completed a questionnaire on the characteristics of the beach they were visiting. The characteristics included a measure of how urban the area was, the type of sand, how clean the beach was, how crowded it was, whether or not there was a toilet, presence of drink vendors, water temperature, calmness of the water, environmental quality, presence of a life guard, the direction the beach faced, and whether or not nudism was present on the beach. Travel costs included the cost of fuel and tolls plus travel time. Travel time varied by mode of transportation—using average walking and average driving speeds.

The random utility model allowed researchers to estimate the impacts on utility of the various beach characteristics identified by the surveys. Those characteristics positively affecting utility included north facing, presence of a life guard, presence of toilets and drink vendors, thin sand, presence of nudism, warm water temperatures, and good environmental quality. Characteristics negatively affecting utility included non-northern beaches, urban beaches, crowding, algae, and calm water.

Because some beach attributes were more highly valued than others, the range of estimates was dramatically affected by the details in the scenario. For example, for a closure affecting beaches on the west coast, the willingness to pay to avoid this loss was .24 euros (2008) per day per person with peak visitation of 25,000 visitors. Aggregating the per-visitor value across visitors produced a daily welfare loss from these closures of 6,000 euros. On the other extreme, a spill forcing closure of the more valuable northern beaches would cause the welfare loss to rise to 1.73 euros per day per person for a total of 43,250 euros during peak visitation.

It is easy to take highly enjoyable recreational sites for granted since they are freely provided by nature. As a result they may not be given their due when resources are allocated for their protection and enhancement. The travel-cost method can help to

inform policy not only by demonstrating how truly valuable they are, but also by allowing useful distinctions to be made among various recreation resources.

Source: Pere, Riera, McConnell, Kenneth E., Giergiczny, Marek, & Mahieu, Pierre-Alexandre. (2011). Applying the travel-cost method to Minorca beaches: Some policy results. In Jeff Bennett (Ed.), *International Handbook on Non-Market Environmental Valuation*. Cheltenham, U.K.: Edward Elgar, 60–73.

"tease out" the environmental component of value in a related market. For example, it is possible to discover that, all other things being equal, property values are lower in polluted neighborhoods than in clean neighborhoods. (Property values fall in polluted neighborhoods because they are less desirable places to live.)

Hedonic property value models use market data (house prices) and then break down the house sales price into its attributes, including the house characteristics (e.g., number of bedrooms, lot size, and features), the neighborhood characteristics (e.g., crime rates, school quality, and so on), and environmental characteristics (e.g., air quality, percentage of open space nearby, distance to a local landfill, etc.).

Hedonic models allow for the measurement of the marginal willingness to pay for discrete changes in an attribute. Numerous studies have utilized this approach to examine the effect on property value of things such as distance to a hazardous waste site (Michaels & Smith, 1990), large farm operations (Palmquist et al., 1997), open space and land use patterns (Bockstael, 1996; Geoghegan et al., 1997; Acharya & Bennett, 2001), dams and rivers (Bohlen & Lewis, 2009; Lewis and Landry, 2017), brownfields (Mihaescu & vom Hofe, 2012), and shale oil production facilities (Gopalakrishnan & Klaiber, 2013). This approach has become commonplace with the use of geographic information systems (discussed below).[10]

Hedonic wage approaches are similar except that they attempt to isolate the environmental risk component of wages, which serves to isolate the amount of compensation workers require in order to work in risky occupations. It is well known that workers in high-risk occupations demand higher wages in order to be induced to undertake the risks. When the risk is environmental (such as exposure to a toxic substance), the results of the multiple regression analysis can be used to construct a willingness to pay to avoid this kind of environmental risk. Additionally, the compensating wage differential can be used to calculate the value of a statistical life (Taylor, 2003). Techniques for valuing reductions in life-threatening risks will be discussed later in this chapter.

Benefit Transfer and Meta-Analysis

The NOAA panel report has created an interesting dilemma. Although it legitimized the use of contingent valuation for estimating passive-use (nonconsumptive use) and nonuse values, the panel has also set some rather rigid guidelines that reliable studies should follow. The cost of completing an "acceptable" contingent valuation study could well be so high that they will only be useful for large incidents, those for which the damages are high enough to justify their use. Yet, due to the paucity of other techniques, the failure to use contingent valuation may, by default, result in passive-use values of zero. That is not a very appealing alternative.[11]

One key to resolving the dilemma created by the possible expense of implementing the NOAA panel's recommendations may be provided by a technique called benefit transfer. Since original studies are time consuming and expensive, benefit transfer allows the estimates for

the site of interest to be based upon estimates from other sites or from an earlier time period to provide the foundation for a current estimate.

Benefit transfer methods can take one of three forms: value transfers, benefit function transfers, or meta-analysis. Sometimes the actual benefit values derived from point estimates can simply be directly transferred from one context to another, usually adjusted for differences between the study site and the policy site. Function transfer involves using a previously estimated benefit function that relates site characteristics to site values. In this case, the differentiating characteristics of the site of interest are entered into the previously derived benefit function in order to derive newer, more site-specific values (Johnston et al., 2006).

Most recently, meta-analysis has been utilized. *Meta-analysis*, sometimes called the "analysis of analyses," takes empirical estimates from a sample of studies, statistically relates them to the characteristics of the studies, and calculates the degree to which the reported differences can be attributed to differences in location, subject matter, or methodology. For example, meta-analysis has been used with cross sections of contingent valuation studies as a basis for isolating and quantifying the determinants of nonuse value. Once these determinants have been isolated and related to specific policy contexts, it may be possible to transfer estimates from one context to another by finding the value consistent with the new context without incurring the time and expense of conducting new surveys each time.

Benefit transfer methods have been widely used in situations for which financial, time, or data constraints preclude original analysis. Policymakers frequently look to previously published studies for data that could inform a prospective decision. Benefit transfer has the advantage of being quick and inexpensive, but the accuracy of the estimates deteriorates as the new context tends to deviate (either temporally or spatially) the further it is from the context used to derive the estimates. Benefit transfer has not escaped controversy. Johnston and Rosenberger (2010) and Johnston et al. (2015) provide a comprehensive discussion of benefit transfer and outline some of the potential problems with the use of benefit transfer, including a lack of studies that are both of sufficiently high quality and policy relevant. Additionally, many of the published studies do not provide enough information on the attributes to allow an assessment of how they might have affected the derived value.

In response to some of these concerns, a valuation inventory database has emerged. The Environmental Valuation Reference Inventory (EVRI) is an online searchable database of over 4000 empirical studies on the economic value of environmental benefits and human health effects. It was specifically developed as a tool for use in benefit transfer.[12]

Benefit transfers are also subject to large errors. A few studies have tested the validity of environmental value transfer across sites. In those that have, the transfer errors have been sizable and wide ranging, sometimes over 100 percent for stated preference survey transfers (Brouwer, 2000, and Rosenberg and Stanley, 2006). Using meta-data from 31 empirical studies, Kaul et al. (2013) find a median transfer error of 39 percent. Lewis and Landry (2017) compare original hedonic property value model results to a test of transferring those results via benefit function transfer and find errors ranging from 29 percent to 1000 percent! These results suggest caution with the use of benefit transfer.

Using Geographic Information Systems to Enhance Valuation

Geographic information systems (GIS) are computerized mapping models and analysis tools. A GIS map is made up of layers such that many variables can be visualized simultaneously using overlays. GIS offers a powerful collection of tools for depicting and examining spatial relationships. Most simply, GIS can be used to produce compelling measurements and graphics that communicate the spatial structure of data and analytic results with a force and clarity

otherwise impossible. But the technology's real value lies in the potential it brings to ask novel questions and enrich our understanding of social and economic processes by explicitly considering their spatial structure. Models that address environmental externalities have, almost by definition, a strong spatial component.[13]

Fundamentally spatial in nature, use of GIS in hedonic property models is a natural fit. Housing prices vary systematically and predictably from neighborhood to neighborhood. Spatial characteristics, from air quality to the availability of open space, can influence property values of entire neighborhoods; if one house enjoys abundant open space or especially good air quality, it is highly likely that its neighbors do as well.

In a 2008 paper, Lewis, Bohlen, and Wilson used GIS and statistical analysis to evaluate the impacts of dams and dam removal on local property values. In a unique "experiment," they collected data on property sales for 10 years before and after the Edwards Dam on the Kennebec River in Maine was removed. The Edwards Dam was the first federally licensed hydropower dam in the United States to be removed primarily for the purpose of river restoration. They also collected data on property sales approximately 20 miles upstream where two dams were still in place. GIS technology enhanced this study by facilitating the calculation of the distance from each home to both the river and the nearby dams. Lewis et al., 2008 found that homeowners pay a price penalty for living close to a dam. In other words, willingness to pay for identical housing is higher the further away from the dam the house is located. They also found that the penalty near the Edwards Dam site dropped to nearly zero after the dam was removed. Interestingly, the penalty upstream also dropped significantly. While a penalty for homes close to the dams upstream remains, it fell after the downstream dam was removed. Can you think of reasons why?[14]

Example 4.4 shows how the use of GIS can enable hedonic property value models to investigate how the view from a particular piece of property might affect its value.

Averting Expenditures. A final example of an indirect observable method involves examining "averting" or "avoidance" expenditures. Averting expenditures are those designed to reduce the damage caused by pollution by taking some kind of averting or defensive action. Examples include installing indoor air purifiers in response to an influx of polluted air or relying on bottled water as a response to the pollution of local drinking water supplies. Since people would not normally spend more to prevent a problem than would be caused by the problem itself, averting expenditures can provide a lower-bound estimate of the damage caused by pollution. They also cause a disproportionate hardship on poor households that cannot afford such coping expenditures. Dickie (2016) argues that ignoring averting expenditures or behavior may underestimate damages. He offers a simple example using contaminated drinking water. Suppose contaminated drinking water increases waterborne illness by 4 percent. If half the population avoids the contamination by some form of averting action such as using an alternate source of water, frequency of illness will drop to 2 percent. Only half the population is now exposed, thus reducing damages. However, the avoidance expenses must be included in the damage estimate. If they are not, the damages will be underestimated (Dickie, 2016). Example 4.5 illustrates the impact of coping or averting expenditures on residents of Kathmandu, Nepal.

Challenges

Aggregation. As you have probably figured out by now, nonmarket valuation faces several challenges. One challenge involves the aggregation of estimated values into a total value that can be used in benefit-cost analysis. How large is the relevant population? Do benefits change with distance to the resource in question? Debate 4.3 explores some of these challenging issues.

EXAMPLE 4.4

Using GIS to Inform Hedonic Property Values: Visualizing the Data

GIS offers economists and others powerful tools for analyzing spatial data and spatial relationships. For nonmarket valuation, GIS has proven to be especially helpful in enhancing hedonic property value models by incorporating both the proximity of environmental characteristics and their size or amount. GIS studies have also allowed for the incorporation of variables that reflect nearby types and diversity of land use.

Geo-coding housing transactions assign a latitude and longitude coordinate to each sale. GIS allows other spatial data, such as land use, watercourses, and census data, to be "layered" on top of the map. By drawing a circle of the desired circumference around each house, GIS can help us to calculate the amount of each amenity that is in that circle as well as the density and types of people who live there. Numerous census data are available on variables such as income, age, education, crime rates, and commuting time. GIS also makes it relatively easy to calculate straight-line distances to desired (or undesired) locations, such as parks, lakes, schools, or landfills.

In a 2002 paper entitled "Out of Sight, Out of Mind? Using GIS to Incorporate Visibility in Hedonic Property Value Models," Paterson and Boyle use GIS to measure the extent to which visibility measures affect house prices in Connecticut. In their study, visibility is measured as the percentage of land visible within one kilometer of the property, both in total and broken out for various land use categories. Finally, they added variables that measured the percentage of area in agriculture or in forest, or covered by water within one kilometer of each house.

They find that visibility is indeed an important environmental variable in explaining property values, but the nature of the viewshed matters. While simply having a view is not a significant determinant of property values, viewing certain types of land uses is. Proximity to development reduces property values only if the development is visible, for example, suggesting that out of sight really does mean out of mind! They conclude that any analysis that omits variables that reflect nearby environmental conditions can lead to misleading or incorrect conclusions about the impacts of land use on property values. GIS is a powerful tool for helping a researcher include these important variables.

Source: Paterson, Robert, & Boyle, Kevin. (2002). Out of sight, out of mind? Using GIS to incorporate visibility in hedonic property value models. *Land Economics*, *78*(3), 417–425.

Partial Values. Another large challenge for nonmarket valuation is that most studies only capture a portion of the total value of an environmental good or service. For example, ecosystems are bundles of values, but the methods outlined in this chapter are only capable of capturing a portion of the value.

Figure 4.1 illustrates the different methods environmental economists use to capture different types of value. Each of these methods relies on different data and, many times, different experts. Rarely is the available time or money sufficient to apply all methods to a particular question.

Debate 4.4 illustrates the challenges and importance of attempts to capture the total economic value by examining a specific case study—polar bears in Canada.

EXAMPLE 4.5

Valuing the Reliability of Water Supplies: Coping Expenditures in Kathmandu Valley, Nepal

Nepal, like many other poor developing countries, experiences chronic shortages of safe drinking water. The Kathmandu Valley is no exception. The National Water Supply Corporation serves 70 percent of the population, but the public water supply is neither reliable nor safe. Shortages are frequent and the water quality is frequently contaminated with fecal coliform and nitrogen-ammonia (Pattanayak et al., 2005).

How much should be invested in improving water quality depends on how valuable clean water is to this population. Quantifying those benefits requires establishing how much residents would be willing to pay for cleaner water. One pathway for quantifying willingness to pay in this context can be found in analyzing how much households spend to cope with the unreliable water supply. It turns out they purchase water from water vendors, collect water from public taps, invest in wells or storage tanks, purchase filtration systems, and/or boil water. All of these coping mechanisms have both a financial cost and a cost associated with the time devoted to coping. Using coping costs as a proxy for willingness to pay can serve as the basis for constructing a lower-bound estimate of the demand curve for water provision in settings where other more direct valuation strategies are simply not practical to implement.

In a survey of 1500 households in five municipalities, researchers found that for households in the Kathmandu Valley, coping or averting behaviors cost the average household about 1 percent of monthly income, most of this attributed to the time spent collecting water. The authors note that these coping costs are almost twice as much as the current monthly bills paid to the water utility.

Some demographic factors were found to have influenced household coping expenditures.

- Wealthier households were found to have higher coping expenditures. As the authors note, this confirms the intuition that relatively rich households have more resources and therefore invest more in water treatment, storage, and purchases.
- More educated respondents also had higher coping costs, perhaps because these households better understood the risks of contaminated water.

If, as suggested by these two findings, the poor face higher financial and educational barriers in their quest for cleaner water, water policy in this region faces an environmental justice issue as well as an efficiency issue.

Even though averting expenditures represent only a lower bound of willingness to pay for water supply, they can provide valuable information for the estimation of benefits of water provision. In addition, these data imply that the common assertion that in poor countries the costs of supplying clean water are so high that they necessarily exceed the benefits received by water users may be a misconception—the value of water in this valley was found to be at least twice the current per unit charge even when the lower bound estimating technique was used.

Source: Pattanayak, Subhrendu K., Yang, Jui-Chen, Whittington, Dale, & Bal Kumar, K. C. (2005). Coping with unreliable public water supplies: Averting expenditures by households in Kathmandu, Nepal. *Water Resources Research*, *41*(2), doi:10.1029/2003WR002443.

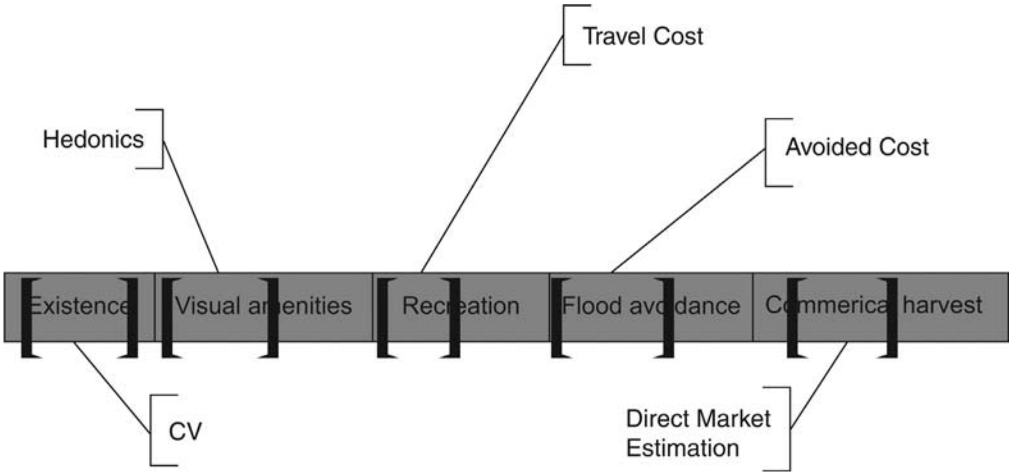

FIGURE 4.1 Different Methods, Different Experts, Different Data

Source: Courtesy of James Boyd, Resources for the Future

DEBATE 4.3

Distance Decay in Willingness to Pay: When and How Much Does Location Matter?

One challenge in performing benefit-cost analysis is accurately choosing the "extent of the market." The extent of the market refers to ***who benefits*** from the resource in question. Loomis (1996) argues that not accounting for the full extent of the market (i.e., including everyone who gains some benefit) can lead to **under**estimates of willingness to pay and aggregate value.

On the other hand, a more inclusive design might include respondents with vastly lower willingness to pay simply because of their location. For some resources, distant respondents have a lower willingness to pay for its improvement. It seems reasonable to expect, for example, that the benefits from a reduction in river pollution to an individual household would probably depend on its proximity to the river. Those closest to the river place the highest value on the improvement. In other words, since it seems reasonable to expect that some types of values do experience a "distance decay," in aggregating benefits this deterioration should certainly be taken into account.

Bateman et al. (2006) argue that not accounting for distance decay can lead to **over**estimates of willingness to pay. Those who are further away still benefit and should be counted, but at some kind of decreasing rate. Recently, the number of stated preference studies (contingent valuation and choice

experiments) that focus on distance decay has increased so we have learned more about it.

What do these studies say about the circumstances that give rise to distance decay?

Interestingly, the empirical evidence suggests that both the type of value being measured (use or nonuse value) as well as the type of willingness to pay question (compensating versus equivalent variation) matter. Hanley et al. (2003) and Bateman et al. (2006) both find that distance decay does arise for use value, but very little or not at all for nonuse values. If, however, some of the current nonusers become users under the proposed scenario, their valuation would experience some distance decay. This result follows the intuition that if the willingness to pay question is framed as a marginal improvement in quality (compensating variation), then some of the nonusers might become users and that possibility would be reflected in their valuations. If the question is framed as equivalent variation (willingness to pay to avoid loss), nonuser valuations experience no distance decay, since they will remain nonusers.

These studies suggest that spatial patterns in nonmarket values have important implications not only for how benefit-cost analysis should be conducted and interpreted but also for how that analysis affects the policy evaluations. Different design choices as to the extent of the market and whether to aggregate across particular political or economic jurisdiction can lead to very different results. As Schaafsma et al. (2012) suggest, these spatial patterns should be taken into account both when drawing samples for willingness to pay surveys, and when aggregating the results.

Sources: Bateman, Ian, Day, Brett H., Georgiou, Stavros, & Lake, Ian. (September 2006). The aggregation of environmental benefit values: Welfare measures, distance decay and total WTP. Discussion paper; Hanley, Nick, Schlapfer, Felix, & Spurgeon, James. (2003). Aggregating the benefits of environmental improvements: Distance-decay functions for use and nonuse values. *Journal of Environmental Management*, *68*, 297–304; Loomis, John B. (1996). How large is the extent of the market for public goods: Evidence from a nationwide contingent valuation survey. *Applied Economics*, 28, 779–782; Schaafsma, Marije, Brouwer, Roy, & Rose, John. (2012). Directional heterogeneity in WTP models for environmental valuation. *Ecological Economics*, *79*(1), 21–31.

Valuing Human Life

One fascinating public policy area where these various approaches have been applied is in the valuation of human life. Many government programs, from those controlling hazardous pollutants in the workplace or in drinking water, to those improving nuclear power plant safety, are designed to save human life as well as to reduce illness. How resources should be allocated among these programs depends crucially on the value of human life. In order to answer this question, an estimate of the value of that life to society is necessary and federal regulations require such estimates for benefit-cost analysis. How is life to be valued?

DEBATE 4.4

What Is the Value of a Polar Bear?

Because polar bears are such a charismatic species, they have obviously attracted lots of popular support, but exactly how valuable are they? In 2011, the Canadian government issued a report in which it attempted to estimate the different socio-economic values of polar bears in Canada.

They commissioned the study in part to determine the economic impact of adding the polar bear to a list of at-risk species. This study represents one of the few studies to try to estimate the value of polar bears and the only one that tries to do it in a comprehensive fashion.

The authors tried to capture active use values (subsistence and sport hunting, polar bear viewing, and value in scientific research), as well as passive-use values (existence and bequest values). Multiple nonmarket valuation methods were used in this study including travel cost (viewing), market prices (hunting), meta-analysis, and benefit transfer (passive-use values). Time and budgetary constraints precluded the use of stated preference methods such as contingent valuation or choice experiments. The summary of their findings is reproduced in Figure 4.2. Note that the direct use values actually comprise a relatively small portion of the total value.

An effort to document the value of a species like this produces a value that is no doubt much closer to the truth than the default value of zero, but how close are these numbers to the true value? There are several caveats to consider:

- Consider the calculation for the value of polar bear meat. For this the cost, the next best substitute, which in this case was beef (for humans) and dog food was used. One could certainly argue for alternatives.
- Sport values were estimated using the benefit transfer method. Recall the challenges for using benefit transfer, in particular for a unique species like the polar bear. The study closest to this one was conducted in 1989 and focused on big game and grizzly bear hunting. For the polar bear study, the 1989 values were translated into 2009 dollars. The authors suggest their number might be an underestimate since hunting for a polar bear is such a unique experience. On the other hand, they also acknowledge that the number could just as easily be an overestimate if the charismatic image of the polar bear reduces willingness to pay for hunting.
- Finally, passive-use values were also calculated using benefit transfer. Since no study has been done on the preservation value of the polar bear in Canada, the researchers used a meta-analysis of species at risk (Richardson & Loomis 2009). While that study calculated a total economic value, for the polar bear study the benefit transfer was specifically designed to capture only preservation value. It was relatively straightforward to remove direct uses (visitors) from the transferred value, but not the indirect use benefits such as scientific value.

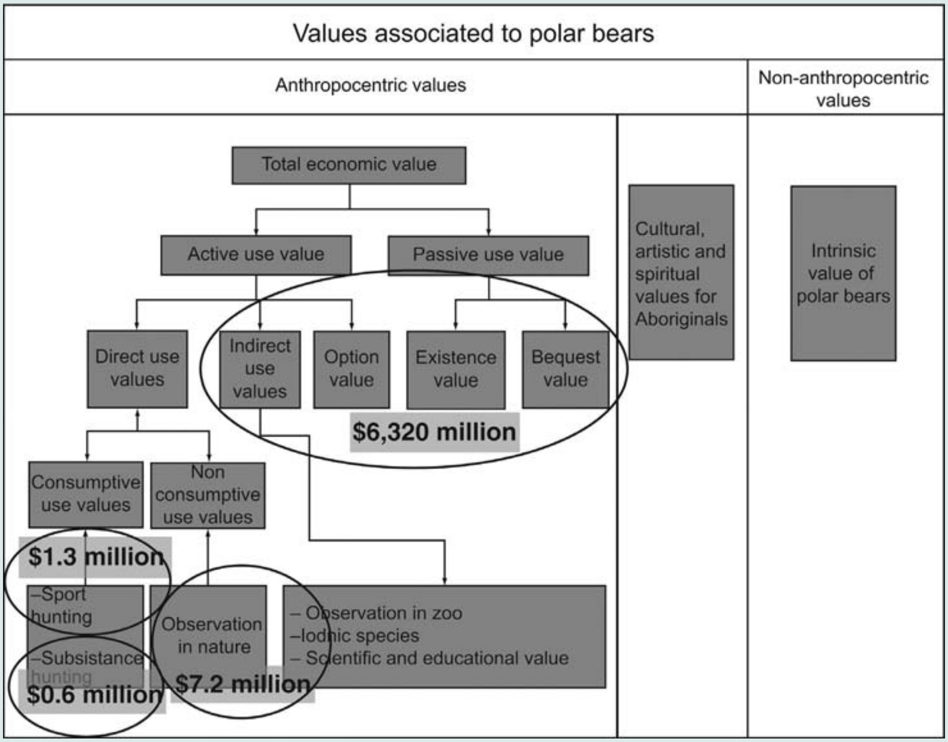

FIGURE 4.2 Monetary Values Associated with Polar Bears in Canada, by Value Category (Aggregate Amounts for Canada)

Source: ÉcoRessources Consultants, for Environment Canada, 2011, p. 32

Using any of these values as inputs into others creates a potential to double count, a common mistake that will be discussed further in Chapter 13. In fact, scientific values were calculated separately for the polar bear study as well as being included in the preservation value estimated via benefit transfer. As such, these numbers could overestimate the value.

What would you be willing to pay to protect the polar bear? As we have seen in this chapter, these types of questions are challenging to answer.

Source: ÉcoRessources Consultants. (2011). Evidence of the socio-economic importance of Polar bears for Canada. Report for Environment Canada. Full report is accessible at http://publications.gc.ca/site/archivee-archived.html?url=http://publications.gc.ca/collections/collection_2012/ec/CW66-291-2011-eng.pdf; Richardson, Leslie, & Loomis, John. (2009). Total economic valuation of endangered species: A summary and comparison of the United States and the rest of the world estimates. In K. N. Ninan (Ed.), *Conserving and Valuing Ecosystem Services and Biodiversity: Economic, Institutional and Social Challenges*. London: Earthscan, 25–46.

The simple answer, of course, is that life is priceless, but that turns out to be not very helpful. Because the resources used to prevent loss of life are scarce, choices must be made. The economic approach to valuing lifesaving reductions in environmental risk is to calculate the change in the probability of death resulting from the reduction in environmental risk and to place a value on the change. Thus, it is not life itself that is being valued, but rather a reduction in the probability that some segment of the population could be expected to die earlier than others. This *value of statistical life* (VSL) represents an individual's willingness to pay for small changes in mortality risks. It does not represent a willingness to pay to prevent certain death. It is measured as the "marginal rate of substitution between mortality risk and money (i.e., other goods and services)" (Cameron, 2010) and as such is also called *mortality risk valuation*. Debate 4.5 examines the controversy associated with valuing changes in these mortality risks.

DEBATE 4.5

Is Valuing Human Life Immoral?

In 2004, economist Frank Ackerman and lawyer Lisa Heinzerling teamed up to write a book that questions the morality of using benefit-cost analysis to evaluate regulations designed to protect human life. In *Priceless: On Knowing the Price of Everything and the Value of Nothing* (2004), they argue that benefit-cost analysis is immoral because it represents a retreat from the traditional standard that all citizens have an absolute right to be free from harm caused by pollution. When it justifies a regulation that will allow some pollution-induced deaths, benefit-cost analysis violates this absolute right.

Economist Maureen Cropper responds that it would be immoral not to consider the benefits of lifesaving measures. Resources are scarce and they must be allocated so as to produce the greatest good. If all pollution were reduced to zero, even if that were possible, the cost would be extremely high and the resources to cover that cost would have to be diverted from other beneficial uses. Professor Cropper also suggests that it would be immoral to impose costs on people about which they have no say—for example, the costs of additional pollution controls—without at least trying to consider what choices people would make themselves. Like it or not, hard choices must be made.

Cropper also points out that people are always making decisions that recognize a trade-off between the cost of more protection and the health consequences of not taking the protection. Thinking in terms of trade-offs should be a familiar concept. She points out that people drive faster to save time, thereby increasing their risk of dying. They also decide how much money to spend on medicines to lower their risk of disease or they may take jobs that pose morbidity or even mortality risks.

In her response to Ackerman and Heinzerling, Cropper acknowledges that benefit-cost analysis has its flaws and that it should never be the only decision-making guide. Nonetheless, she argues that it does add useful information to

the process and throwing that information away could prove to be detrimental to the very people that Ackerman and Heinzerling seek to protect.

Sources: Ackerman, Frank, & Heinzerling, Lisa. (2004). *Priceless: On Knowing the Price of Everything and the Value of Nothing.* New York: The New Press; Ackerman, Frank. (2004). Morality, cost-benefit and the price of life. *Environmental Forum*, *21*(5), 46–47; Cropper, Maureen. (2004). Immoral not to weigh benefits against costs. *Environmental Forum*, *21*(5), 47–48.

It is possible to translate the value derived from this procedure into an "implied value of statistical life." This is accomplished by dividing the amount each individual is willing to pay for a specific reduction in the probability of death by the probability reduction. Suppose, for example, that a particular environmental policy could be expected to reduce the average concentration of a toxic substance to which 1 million people are exposed. Suppose further that this reduction in exposure could be expected to reduce the risk of death from 1 out of 100,000 to 1 out of 150,000. This implies that the number of expected deaths would fall from 10 to 6.67 in the exposed population as a result of this policy. If each of the 1 million persons exposed is willing to pay $5 for this risk reduction (for a total of $5 million), then the implied value of a statistical life is approximately $1.5 million ($5 million divided by 3.33). Alternatively, the VSL can be calculated using the change in WTP divided by the change in risk. For this example, that would be $5 divided by the change in risk of death (1/100,000–1/150,000), or $1.5 million. Thus, the VSL is capturing the rate of trade-off between money and a very small risk of death.

What actual values have been derived from these methods? One early survey (Viscusi, 1996) of a large number of studies examining reductions in a number of life-threatening risks found that most implied values for human life (in 1986 dollars) were between $3 million and $7 million. This same survey went on to suggest that the most appropriate estimates were probably closer to the $5 million estimate. In other words, all government programs resulting in risk reductions costing less than $5 million per life saved would be justified in benefit-cost terms. Those costing more might or might not be justified, depending on the appropriate value of a life saved in the particular risk context being examined.

In a meta-analysis, Mrozek and Taylor (2002) found much lower values for VSL. Using over 40 labor market studies, their research suggests that a range of $1.5 million to $2.5 million for VSL is more appropriate. What about age? Does the VSL change with age? Apparently so. Viscusi (2008) finds an inverted U-shape relationship between VSL and age. Specifically, using the hedonic wage model, they estimate a VSL of $3.7 million for persons ages 18–24, $9.7 million for persons ages 35–44, and $3.4 million for persons ages 55–62. According to their study, VSL rises with age, peaks, and then declines.

What about the value of statistical life across populations or countries with different incomes? Most agencies in the United States use VSLs between $5 million and $8 million.[15] These estimates are based largely on hedonic wage studies that have been conducted in the United States or in other high-income countries.[16] How might those results be translated into settings featuring populations with lower incomes?

Adjustments for income are typically derived using an estimate of the income elasticity of demand. Recall that income elasticity is the percent change in consumption given a 1 percent change in income. Hammitt and Robinson (2011) note that applying income elasticities, derived for countries like the United States, might result in nonsensical VSL

estimates if blindly applied to lower-income countries. While U.S. agencies typically assume a 0.4 to 0.6 percent change in VSL for a 1 percent change in real income over time, elasticities closer to 1.0 or higher are more realistic for transfers of these values between high- and low-income countries. Using the higher income elasticity number is merited since willingness to pay for mortality risk reduction as a percentage of income drops at very low incomes; what limited income is available in poorer households is reserved for basic needs.

Summary: Nonmarket Valuation Today

In this chapter, we have examined the most prominent, but certainly not the only, techniques available to supply policymakers with the information needed to implement efficient policy. Finding the total economic value of the service flows requires estimating three components of value: (1) use value, (2) option value, and (3) nonuse or passive-use values.

Our review of these various techniques included direct observation, contingent valuation, contingent choice experiments, travel cost, hedonic property and wage studies, and averting or defensive expenditures. When time or funding precludes original research, benefit transfer or meta-analysis provide alternate methods for the estimation of values. In January 2011, a panel of experts gathered at the annual meeting of the American Economics Association to reflect on nonmarket valuation 20 years after the *Exxon Valdez* spill and, unknown to any of them when the panelists were asked to participate, 8 months after the *Deepwater Horizon* spill. The panelists had all worked on estimation of damages from the *Exxon Valdez* spill. The consensus among panelists was that while many of the issues with bias have been addressed in the literature, many unanswered questions remain and some areas still need work. While they all agreed that it is “hard to underestimate the powerful need for values” (i.e., some number is definitely better than no number), and we now have in place methods that can be easily utilized by all researchers, they also emphasized several problem areas. First, the value of time in travel cost models has not been resolved. What is the opportunity cost of time if you are unemployed, for example?

Second, in discussing other revealed preference methods, they asked the question, “How do the recent numerous foreclosures in the real estate market affect hedonic property value model assumptions?”[17] Third, choice experiments do not resolve all of the potential problems with contingent valuation. While choice experiments do seem to better represent actual market choices, some of the issues that arise in contingent valuation, such as the choice of the payment vehicle, also arise with choice experiments. In addition, some new challenges, such as how the sequencing of choices in choice experiments might affect outcomes, arise. The panel highlighted how this area of research has been enhanced by the field of behavioral economics, an emerging research area that combines economics and psychology to examine human behavior. And finally, they suggested that the NOAA panel recommendations be updated to reflect the new body of research. In 2017, a new set of guidelines was published to do just that. The 23 recommendations in those guidelines address these questions regarding stated preference surveys and attempt to synthesize the now large body of research that informs nonmarket valuation (Johnston et al., 2017).

Some of these same experts, along with several others, implemented a nationwide survey following the BP spill to assess what U.S. households would pay to avoid damages from another spill. Using state of the art techniques for stated preference surveys, they found that U.S. households would be willing to pay $17.2 billion to avoid the damages from another spill (Bishop et al., 2017). One author claimed, “this is proof that our natural resources have an immense monetary value to citizens of the United States who visit the Gulf and to those who simply care that this valuable resource is not damaged.”

Discussion Question

1. Certain environmental laws prohibit the EPA from considering the costs of meeting various standards when the levels of the standards are set. Is this a good example of appropriately prioritizing human health or simply an unjustifiable waste of resources? Why?

Self-Test Exercises

1. In Mark A. Cohen, "The Costs and Benefits of Oil Spill Prevention and Enforcement," *Journal of Environmental Economics and Management* Vol. 13 (June 1986), an attempt was made to quantify the marginal benefits and marginal costs of U.S. Coast Guard enforcement activity in the area of oil spill prevention. His analysis suggests (p. 185) that the marginal per-gallon benefit from the current level of enforcement activity is $7.50, while the marginal per-gallon cost is $5.50. Assuming these numbers are correct, would you recommend that the Coast Guard increase, decrease, or hold at the current level their enforcement activity? Why?
2. Professor Kip Viscusi estimated that the cost per life saved by current government risk-reducing programs ranges from $100,000 for unvented space heaters to $72 billion for a proposed standard to reduce occupational exposure to formaldehyde.
   a. Assuming these values to be correct, how might efficiency be enhanced in these two programs?
   b. Should the government strive to equalize the marginal costs per life saved across all lifesaving programs?
3. a. Suppose that hedonic wage studies indicate a willingness to pay $50 per person for a reduction in the risk of a premature death from an environmental hazard of 1/100,000. If the exposed population is 4 million people, what is the implied value of a statistical life?
   b. Suppose that an impending environmental regulation to control that hazard is expected to reduce the risk of premature death from 6/100,000 to 2/100,000 per year in that exposed population of 4 million people. Your boss asks you to tell her what is the maximum this regulation could cost and still have the benefits be at least as large as the costs. What is your answer?

Notes

1 U.S. District Court for the State of Alaska, Case Number A89-0095CV, January 28, 2004.
2 Ibid.
3 *Exxon Shipping Company v. Baker.*
4 Bishop et al. (2017).
5 In 2017, the United States Department of the Interior released the Deepwater Horizon Response and Restoration Administrative Record, which included an estimate of the total value of damages (see Example 18.4).
6 The rules for determining these damages are defined in Department of Interior regulations. See 40 Code of Federal Regulations 300:72–74.
7 Krutilla, John V. (1967). Conservation reconsidered. *American Economic Review*, *57*(4), 777–786.
8 Ibid. p. 779.

9 A more detailed description of the methodological issues and concerns with contingent valuation with respect to the actual *Exxon Valdez* contingent valuation survey can be found in Mitchell (2002).
10 There are many examples in this category. These are just a few.
11 Whittington (2002) examines the reasons why so many contingent valuation studies in developing countries are unhelpful. Poorly designed or rapidly implemented surveys could result in costly policy mistakes on topics that are very important in the developing world. The current push for cheaper, quicker studies is risky and researchers need to be very cautious.
12 www.evri.ca
13 For examples see Bateman et al. (2002), who describe the contributions of GIS in incorporating spatial dimensions into economic analysis, including benefit-cost analysis; and Clapp et al. (1997), who discuss the potential contributions GIS can make for urban and real estate economics.
14 Interestingly, after this study was complete, one of the two upstream dams, the Fort Halifax Dam, was removed in July 2008 after years of litigation about its removal.
15 See, for example, www.epa.gov/environmental-economics/mortality-risk-valuation
16 Many labor market estimates of VSL average near $7 million (Viscusi, 2008).
17 This question was taken up by another panel of experts at the 2012 Association of Environmental and Resource Economics annual conference and later published by Boyle et al. (2012).

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Chapter 5

Dynamic Efficiency and Sustainable Development

We usually see only the things we are looking for—so much so that we sometimes see them where they are not.

—Eric Hoffer, *The Passionate State of Mind* (1993)

Introduction

In previous chapters, we have developed two specific criteria for identifying allocation problems. The first, static efficiency, allows us to evaluate those circumstances where time is not a crucial aspect of the allocation problem. Typical examples might include allocating resources such as an annually replenished water supply or solar energy, where next year's flow is independent of this year's choices. The second, more complicated criterion, dynamic efficiency, is suitable for those circumstances where time is a crucial aspect and subsequent choices are dependent on earlier choices. The combustion of depletable energy resources such as oil would be a typical example, since supplies used now are unavailable for future generations.

After defining these criteria and showing how they could be operationally invoked, we demonstrated how helpful they can be. They are useful not only in identifying the misuse of environmental resources and ferreting out their behavioral sources, but also in providing a basis for identifying different types of remedies. These criteria even help design optimal policy instruments for restoring some sense of balance between the economy and the environment.

But the fact that these are powerful and useful tools in the quest for a sense of balance does not imply that they are the only criteria in which we should be interested. In a general sense, the efficiency criteria are designed to prevent wasteful use of environmental and natural resources. That is a desirable attribute, but it is not the only possible desirable attribute. We might care, for example, not only about the value of the environment (the size of the pie), but also how this value is shared (the size of each piece to recipients). In other words, fairness or justice concerns should accompany efficiency considerations.

In this chapter, we investigate one particular fairness concern—the treatment of future generations. We begin by considering a specific, ethically challenging situation—the allocation of a depletable resource over time.

Specifically, we trace out the temporal allocation of a depletable resource that satisfies the dynamic efficiency criterion and show how this allocation is affected by changes in the discount rate. To lay the groundwork for our evaluation of fairness, we define what we mean by a just allocation among generations. Finally, we consider not only how this theoretical definition can be made operationally measurable, but also how it relates to dynamic efficiency. To what degree is dynamic efficiency compatible with intergenerational fairness?

A Two-Period Model

Dynamic efficiency balances present and future uses of a depletable resource by maximizing the present value of the net benefits derived from its use. This implies a particular allocation of the resource across time. We can illustrate the properties of this allocation with the aid of a simple numerical example. We begin with the simplest of models—deriving the dynamic efficient allocation across two time periods. In subsequent chapters, we show how these conclusions generalize to longer time periods and to more complicated situations.

Assume that we have a fixed supply of a depletable resource to allocate between two periods. Assume further that the demand function is the same in each of the two periods, the marginal willingness to pay is given by the formula $P = 8 - 0.4q$, and the marginal cost of supplying that resource is constant at \$2 per unit (see Figure 5.1).

Note that if the total supply (Q) were 30 or greater, and we were concerned only with these two periods, an efficient allocation would allocate 15 units to each period, *regardless of the discount rate*. Thirty units would be sufficient to cover the demand in both periods;

Figure 5.1 The Allocation of an Abundant Depletable Resource: (a) Period 1 and (b) Period 2

the consumption in Period 1 would not reduce the consumption in Period 2. In this case the static efficiency criterion is sufficient because the allocations are not temporally interdependent—abundance eliminates the scarcity.

Consider, however, what happens when the available supply is less than 30. Suppose it equals 20. How do we determine the efficient allocation? According to the dynamic efficiency criterion, the efficient allocation is the one that maximizes the present value of the net benefit. The present value of the net benefit for both periods is simply the sum of the present values in each of the two periods. To take a concrete example, consider the present value of a particular allocation—15 units in the first period and five in the second. How would we compute the present value of that allocation?

The present value in the first period would be that portion of the geometric area under the demand curve that is over the supply curve—\$45.00.[1] The present value in the second period is that portion of the area under the demand curve that is over the supply curve from the origin to the five units received, multiplied by $1/(1 + r)$. If we use $r = 0.10$, then the present value of the net benefit received in the second period is \$22.73,[2] and the present value of the net benefits for the 2 years is \$67.73.

Having learned how to find the present value of net benefits for any allocation, how does one find the allocation that *maximizes* present value? One way, with the aid of a computer, is to try all possible combinations of q_1 and q_2 that sum to 20. The one yielding the maximum present value of net benefits can then be selected. That is tedious and, for those who have the requisite mathematics, unnecessary.

It turns out that the dynamically efficient allocation of this resource has to satisfy the condition that the present value of the marginal net benefit from the last unit in Period 1 equals the present value of the marginal net benefit from the last unit in Period 2 (see appendix at the end of this chapter for the derivation). Even without the mathematics, this principle is easy to understand, as can be demonstrated with the use of a simple graphical representation of the two-period allocation problem.

Figure 5.2 depicts the present value of the marginal net benefit for each of the two periods. The net benefit curve for Period 1 is to be read from left to right. The net benefit curve intersects the vertical axis at \$6; demand would be zero at \$8 and the marginal cost is \$2, so the difference (marginal net benefit) is \$6. The marginal net benefit for the first period goes to zero at 15 units because, at that quantity, the marginal willingness to pay for that unit exactly equals its marginal cost. Can you verify those numbers?

The only challenging aspect of drawing the graph involves constructing the curve for the present value of net benefits in Period 2. Two aspects of Figure 5.2 are worth noting. First, the zero axis for the Period 2 net benefits is on the right, rather than the left, side. Therefore, increases in Period 2 are recorded from right to left. By drawing the two periods this way, all points along the horizontal axis yield a total of 20 units allocated between the two periods. Any point on that axis picks a unique allocation between the two periods.[3]

Second, the present value of the marginal net benefit curve for Period 2 intersects the vertical axis at a different point than does the comparable curve in Period 1. Can you see why? This intersection is lower because the marginal benefits in the second period need to be discounted (multiplied by $1/(1 + r)$) to convert them into present value. This follows from the fact that they are received one year later. Thus, with the 10 percent discount rate we are using, the marginal net benefit on the right hand axis is \$6 and its present value is \$6/1.10 = \$5.45. Note that larger discount rates ($r > .10$) would rotate the Period 2 marginal benefit curve around the point of zero net benefit ($q_1 = 5$, $q_2 = 15$) toward the right-hand axis. We shall use this fact in a moment.

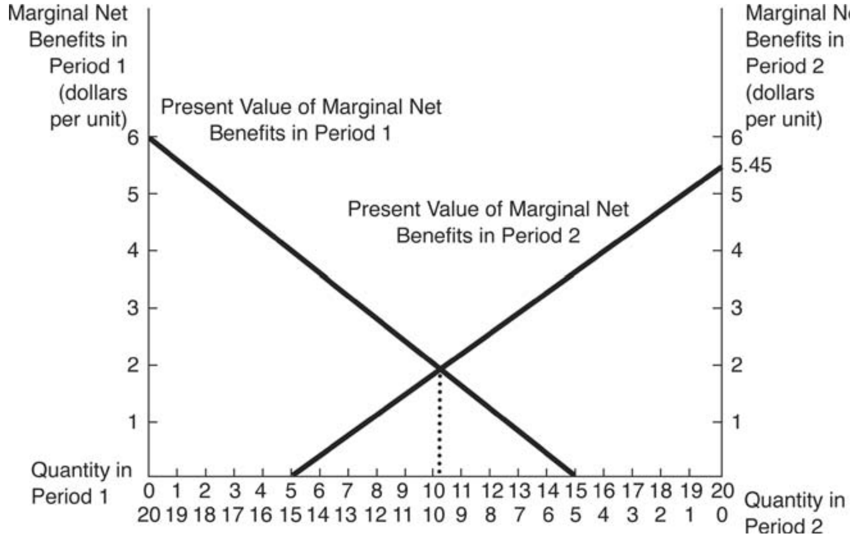

Figure 5.2 The Dynamically Efficient Allocation

The efficient allocation is now readily identifiable as the point where the two curves representing present value of marginal net benefits cross (since that is the allocation where the two marginal present values of net benefits for the two periods are equal). The total present value of net benefits is then the area under the marginal net benefit curve for Period 1 up to the efficient allocation, plus the area under the present value of the marginal net benefit curve for Period 2 from the right-hand axis up to its efficient allocation. Because we have an efficient allocation, the sum of these two areas is maximized.[4]

Since we have developed our efficiency criteria independent of an institutional context, these criteria are equally appropriate for evaluating resource allocations generated by markets, government rationing, or even the whims of a dictator. While *any* efficient allocation method must take scarcity into account, the details of precisely how that is done depend on the context.

Intertemporal scarcity imposes an opportunity cost that we henceforth refer to as the *marginal user cost*. When resources are scarce, greater current use diminishes future opportunities. The marginal user cost is the present value of these forgone future opportunities at the margin. To be more specific, uses of those resources, which would have been appropriate in the absence of scarcity, may no longer be appropriate once scarcity is present.

Consider a practical example. Using large quantities of water to keep lawns lush and green may be wholly appropriate for an area with sufficiently large replenishable water supplies, but quite inappropriate when it denies drinking water to future generations. Failure to take the higher future scarcity value of water into account in the present would lead to inefficiency due to the additional cost resulting from the increased scarcity imposed on the future. This additional marginal value created by scarcity is the marginal user cost.

We can illustrate this concept by returning to our numerical example. With 30 or more units, each period would be allocated 15 units, the resource would not be scarce, and the marginal user cost would therefore be zero.

With 20 units, however, scarcity emerges. No longer can 15 units be allocated to each period; each period will have to be allocated less than would be the case with abundance.

Due to this scarcity, the marginal user cost for this case is not zero. As can be seen from Figure 5.2, the present value of the marginal user cost—the additional value created by scarcity—is graphically represented by the vertical distance between the quantity (horizontal) axis and the intersection of the two present-value curves. Notice that the present value of the marginal net benefit for Period 1 is equal to the present value of the marginal net benefit for Period 2. This common value can either be read off the graph or determined more precisely, as demonstrated in the chapter appendix, to be $1.905.

We can make this concept of marginal user cost even more concrete by considering its use in a market context. An efficient market would have to consider not only the marginal cost of extraction for this resource but also the marginal user cost. Whereas in the absence of scarcity, the price would equal only the marginal cost of extraction, with scarcity, the price would equal the sum of marginal extraction cost and marginal user cost.

To see this, solve for the prices that would prevail in an efficient market facing scarcity over time. Inserting the efficient quantities for the two periods (10.238 and 9.762, respectively) into the willingness-to-pay function ($P = 8 - 0.4q$) yields $P_1 = 3.905$ and $P_2 = 4.095$. The corresponding supply-and-demand diagrams are given in Figure 5.3. Compare Figure 5.3 with Figure 5.1 to see the impact of scarcity on price.

Note that marginal user cost is zero in Figure 5.1, as expected from the absence of scarcity. In an efficient allocation involving scarcity, the marginal user cost for each period is the difference between the price and the marginal cost of extraction. Notice that it takes the value $1.905 in the first period and $2.095 in the second. In both periods, the <u>present value</u> of the marginal user cost is $1.905. In the second period, the actual marginal user cost is $1.905 ($1 + r$). Since $r = 0.10$ in this example, the actual (as opposed to present value) marginal user cost for the second period is $2.095.[5] Thus, while the *present value* of marginal user cost is equal in both periods, the actual marginal user cost rises over time.

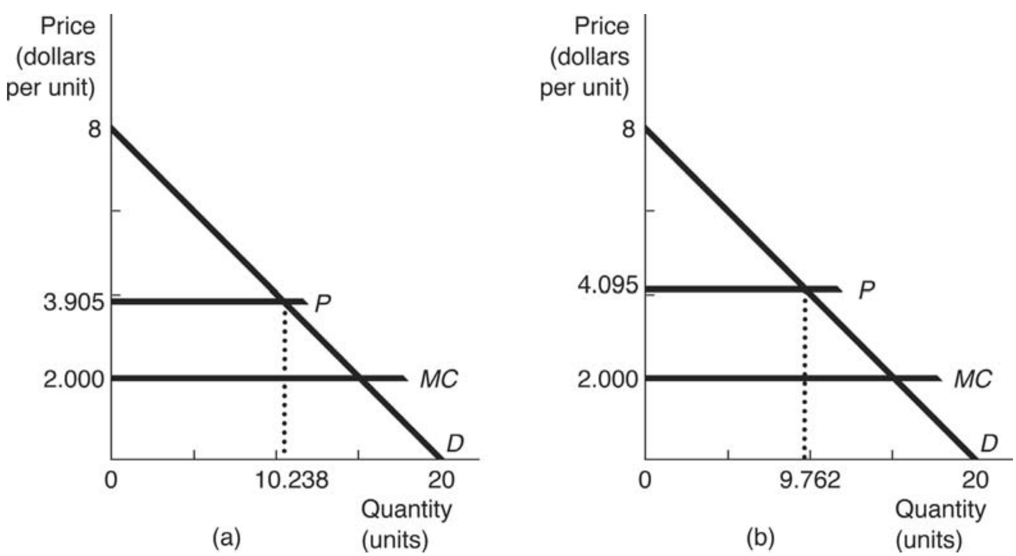

Figure 5.3 The Efficient Market Allocation of a Depletable Resource: The Constant-Marginal-Cost Case: (a) Period 1 and (b) Period 2

Both the size of the marginal user cost and the allocation of the resource between the two periods are affected not only by the degree of scarcity, but also by the discount rate. In Figure 5.2, because of discounting, the efficient allocation allocates somewhat more to Period 1 than to Period 2. A discount rate larger than 0.10 would be incorporated in this diagram by rotating (not shifting) the Period 2 curve an appropriate amount toward the right-hand axis, holding fixed the point at which it intersects the horizontal axis. (Can you see why?) The larger the discount rate, the greater the amount of rotation required.

The implication is clear—the amount allocated to the second period would be necessarily smaller with larger discount rates. The general conclusion, which holds for all models we consider, is that higher discount rates tend to skew resource extraction toward the present because they give the future less weight in balancing the relative value of present and future resource use. The choice of what discount rate to use, then, becomes a very important consideration for decision makers.

Defining Intertemporal Fairness

While no generally accepted standards of fairness or justice exist, some have more prominent support than others. One such standard concerns the treatment of future generations. What legacy should earlier generations leave to later ones? This is a particularly difficult issue because, in contrast to other groups for which we may want to ensure fair treatment, future generations cannot articulate their wishes, much less negotiate with current generations. ("We'll accept your radioactive wastes if you leave us plentiful supplies of titanium.")

One starting point for intergenerational equity is provided by philosopher John Rawls in his monumental work *A Theory of Justice*. Rawls suggests that one way to derive general principles of justice is to place, hypothetically, all people into an original position behind a "veil of ignorance." This veil of ignorance would prevent them from knowing their eventual position in society. Once behind this veil, people would be asked to decide on rules to govern the society that they would, after the decision, be forced to inhabit.

In our context, this approach would suggest a hypothetical meeting of all members of present and future generations to decide on rules for allocating resources among generations. Because these members are prevented by the veil of ignorance from knowing the generation to which they will belong after the rules are defined, they will not be excessively conservationist (lest they turn out to be a member of an earlier generation) or excessively exploitative (lest they become a member of a later generation).

What kind of rule would emerge from such a meeting? One possibility is the sustainability criterion. The *sustainability criterion* suggests that, at a minimum, future generations should be left no worse off than current generations. Allocations that impoverish future generations in order to enrich current generations are, according to this criterion, patently unfair.

In essence, the sustainability criterion suggests that earlier generations are at liberty to use resources that would thereby be denied to future generations as long as the well-being of future generations remains just as high as that of all previous generations. On the other hand, diverting resources from future use would violate the sustainability criterion if it reduced the well-being of future generations below the level enjoyed by preceding generations.

One of the implications of this definition of sustainability is that it is possible for the current generation to use resources (even depletable resources) as long as the interests of future generations could be protected. Do our institutions provide adequate protection for future generations? We begin with examining the conditions under which efficient allocations satisfy the sustainability criterion. Are all efficient allocations sustainable?

Are Efficient Allocations Fair?

In the numerical example we have constructed, it certainly does not appear that the efficient allocation satisfies the sustainability criterion. In the two-period example, more resources are allocated to the first period than to the second. Therefore, net benefits in the second period are lower than in the first. Sustainability does not allow earlier generations to profit at the expense of later generations, and this example certainly appears to be a case where that is happening.

Yet appearances can be deceiving. Choosing this particular extraction path does not prevent those in the first period from saving some of the net benefits for those in the second period. If the allocation is dynamically efficient, it will always be possible to set aside sufficient net benefits accrued in the first period for those in the second period, so that those in both periods will be at least as well off as they would have been with any other extraction profile and one of the periods will be better off.

We can illustrate this point with a numerical example that compares a dynamic efficient allocation with sharing to an allocation where resources are committed equally to each generation. Suppose, for example, you believe that setting aside half (10 units) of the available resources for each period would be a better allocation than the dynamic efficient allocation. The net benefits to each period from this alternative scheme would be $40. Can you see why?

Now let's compare this to an allocation of net benefits that could be achieved with the dynamic efficient allocation. For the dynamic efficient allocation to satisfy the sustainability criterion, we must be able to show that it can produce an outcome such that each generation would be at least as well off as it would be with the equal allocation and one will be better off. Can that be demonstrated?

In the dynamic efficient allocation with no sharing, the net benefits to the first period were 40.466, while those for the second period were 39.512.[6] Clearly, in the absence of sharing between the periods, this example would violate the sustainability criterion; the second generation is worse off than it would be with equal sharing. (While it would receive 40.00 from equal resource allocation across the two periods, it receives only 39.512 from the dynamic efficient allocation in the absence of any benefit sharing.)

But suppose the first generation was willing to share some of the net benefits from the extracted resources with the second generation. If the first generation keeps net benefits of $40 (thereby making it just as well off as if equal amounts were extracted in each period) and saves the extra $0.466 (the $40.466 net benefits earned during the first period in the dynamic efficient allocation minus the $40 reserved for itself) at 10 percent interest for those in the next period, this saving would grow to $0.513 by the second period [0.466(1.10)]. Add this to the net benefits received directly from the dynamic efficient allocation ($39.512), and the second generation would receive $40.025. Those in the second period would be better off by accepting the dynamic efficient allocation with sharing than they would if they demanded that resources be allocated equally between the two periods.

This example demonstrates that, although dynamic efficient allocations do not automatically satisfy sustainability criteria, they could be compatible with sustainability, even in an economy relying heavily on depletable resources. The possibility that the second period can be better off is not a guarantee; the required degree of sharing must take place. Example 5.1 points out that under some conditions this sharing does take place, although, as we shall see, such sharing is more likely to be the exception rather than the norm. In subsequent chapters, we shall examine both the conditions under which we could expect the appropriate degree of sharing to take place and the conditions under which it would not.

EXAMPLE 5.1

The Alaska Permanent Fund

One interesting example of an intergenerational sharing mechanism currently exists in the state of Alaska. Extraction from Alaska's oil fields generates significant income, but it also depreciates one of the state's main environmental assets. To protect the interests of future generations as the Alaskan pipeline construction neared completion in 1976, Alaska voters approved a constitutional amendment that authorized the establishment of a dedicated fund: the Alaska Permanent Fund. This fund was designed to capture a portion of the rents received from the sale of the state's oil to share with future generations. The amendment requires:

> At least 25 percent of all mineral lease rentals, royalties, royalty sales proceeds, federal mineral revenue-sharing payments and bonuses received by the state be placed in a permanent fund, the principal of which may only be used for income-producing investments.

The principal of this fund cannot be used to cover current expenses without a majority vote of Alaskans.

The fund is fully invested in capital markets and diversified among various asset classes. It generates income from interest on bonds, stock dividends, real estate rents, and capital gains from the sale of assets. To date, the legislature has used some of these annual earnings to provide dividends to every eligible Alaska resident, while retaining the rest in the fund in order to increase the size of the endowment, thereby assuring that it is not eroded by inflation. As of 2015, the market value of the fund was $52.8 billion and the dividend to every resident in that year was $2,072.00.

Although this fund does preserve some of the revenue for future generations, two characteristics are worth noting. First, the principal could be used for current expenditures if a majority of current voters agreed. To date, that has not happened, but it has been discussed. Second, only 25 percent of the oil net revenue is placed in the fund; assuming that net revenue reflects scarcity rent, full sustainability would require dedicating 100 percent of it to the fund. Because the current generation not only gets its share of the income from the permanent fund, but also receives 75 percent of the proceeds from current oil sales, this sharing arrangement falls short of full sustainability.

Source: The fund is managed by the Alaska Permanent Fund Corporation, www.apfc.org/home/Content/home/index.cfm (accessed January 19, 2017); The Alaska Permanent Fund Website: www.pfd.state.ak.us/ (accessed January 19, 2017).

Applying the Sustainability Criterion

One of the difficulties in assessing the fairness of intertemporal allocations using this version of the sustainability criterion is that it is so difficult to apply. Discovering whether the well-being of future generations would be lower than that of current generations requires us not only to know something about the allocation of resources over time, but also to know

something about the preferences of future generations (in order to establish how valuable various resource streams are to them). That is a tall (impossible?) order!

Is it possible to develop a version of the sustainability criterion that is more operational? Fortunately it is, thanks to what has become known as the "Hartwick Rule." In an early article, John Hartwick (1977) demonstrated that a constant level of consumption could be maintained perpetually from an environmental endowment if all the scarcity rent derived from resources extracted from that endowment were invested in capital. That level of investment would be sufficient to assure that the value of the total capital stock would not decline.

Two important insights flow from this reinterpretation of the sustainability criterion. First, with this version, it is possible to judge the sustainability of an allocation by examining whether or not the value of the total capital stock is declining—a declining capital stock violates the rule. That test can be performed each year without knowing anything about future allocations or preferences. Second, this analysis suggests the specific degree of sharing that would be necessary to produce a sustainable outcome, namely, all scarcity rent must be invested.

Let's pause to be sure we understand what is being said and why it is being said. Although we shall return to this subject later in the book, it is important now to have at least an intuitive understanding of the implications of this analysis. Consider an analogy. Suppose a grandparent left you an inheritance of $10,000 and you put it in a bank where it earns 10 percent interest.

What are the choices for allocating that money over time and what are the implications of those choices? If you withdrew exactly $1,000 per year, the amount in the bank would remain $10,000 and the income would last forever; you would be spending only the interest, leaving the principal intact. If you spend more than $1,000 per year, the principal would necessarily decline over time and eventually the balance in the account would go to zero. In the context of this discussion, spending $1,000 per year or less would satisfy the sustainability criterion, while spending more would violate it.

What does the Hartwick Rule mean in this context? It suggests that one way to tell whether an allocation (spending pattern) is sustainable or not is to examine what is happening to the value of the principal over time. If the principal is declining, the allocation (spending pattern) is not sustainable. If the principal is increasing or remaining constant, the allocation (spending pattern) is sustainable.

How do we apply this logic to the environment? In general, the Hartwick Rule suggests that the current generation has been given an endowment. Part of the endowment consists of environmental and natural resources (known as "natural capital") and another part consists of physical capital (such as buildings, equipment, schools, and roads). Sustainable use of this endowment implies that we should keep the principal (the value of the natural and physical endowment) intact and live off only the flow of services provided. We should not, in other words, chop down all the trees and use up all the oil, leaving future generations to fend for themselves. Rather, we need to assure that the value of the total capital stock is maintained, not depleted.

The desirability of this version of the sustainability criterion depends crucially on how substitutable the two forms of capital are. If physical capital can readily substitute for natural capital, then maintaining the value of the sum of the two is sufficient. If, however, physical capital cannot completely substitute for natural capital, investments in physical capital alone may not be enough to assure sustainability.

How tenable is the assumption of complete substitutability between physical and natural capital? Clearly, it is untenable for certain essential categories of environmental resources. Although we can contemplate the replacement of natural breathable air with universal air-conditioning in domed cities, both the expense and the artificiality of this approach make it an

EXAMPLE 5.2

Nauru: Weak Sustainability in the Extreme

The weak sustainability criterion is used to judge whether the depletion of natural capital is offset by sufficiently large increases in physical or financial capital so as to prevent total capital from declining. It seems quite natural to suppose that a violation of that criterion does demonstrate *unsustainable* behavior. But does fulfillment of the weak sustainability criterion provide an adequate test of *sustainable* behavior? Consider the case of Nauru.

Nauru is a small Pacific island that lies some 3000 kilometers northeast of Australia. It contains one of the highest grades of phosphate ever discovered. Phosphate is a prime ingredient in fertilizers.

Over the course of a century, first colonizers and then, after independence, the citizens of Nauru decided to extract massive amounts of this deposit. This decision has simultaneously enriched the remaining inhabitants (including the creation of a trust fund believed to contain over $1 billion) and destroyed most of the local ecosystems. Local needs are now mainly met by imports financed by the sales of the phosphate.

However wise or unwise the choices made by the people of Nauru were, they could not be replicated globally. An entire population cannot subsist solely on imports financed with trust funds; every import must be exported by someone! The story of Nauru demonstrates the value of complementing the weak sustainability criterion with other, more demanding criteria. Satisfying the weak sustainability criterion may be a necessary condition for sustainability, but it is not always sufficient.

Source: Gowdy, J. W., & McDaniel, C. N. (1999). The physical destruction of Nauru: An example of weak sustainability. *Land Economics*, *75*(2), 333–338.

absurd compensation device. Obviously, intergenerational compensation must be approached carefully (see Example 5.2).

Recognizing the weakness of the constant total capital definition in the face of limited substitution possibilities has led some economists to propose a new, additional definition. According to this new definition, an allocation is sustainable if it maintains the value of the stock of *natural* capital. This definition assumes that it is natural capital that drives future well-being and further assumes that little or no substitution between physical and natural capital is possible. To differentiate these two definitions, the maintenance of the value of total capital is known as the “weak sustainability” (less restrictive) definition, while maintaining the value of natural capital is known as the “strong sustainability” (more restrictive) definition.

A final, additional definition, known as “environmental sustainability,” requires that certain *physical flows* of certain key *individual* resources be maintained. This definition suggests that it is not sufficient to maintain the *value* of an *aggregate*. For a fishery, for example, this definition would require catch levels that did not exceed the growth of the biomass for the fishery. For a wetland, it would require the preservation of specific ecological functions.

Implications for Environmental Policy

In order to be useful guides to policy, our sustainability and efficiency criteria must be neither synonymous nor incompatible. Do these criteria meet that test?

They do. As we shall see later in the book, not all efficient allocations are sustainable and not all sustainable allocations are efficient. Yet some sustainable allocations are efficient and some efficient allocations are sustainable. Furthermore, market allocations may be either efficient or inefficient and either sustainable or unsustainable.

Do these differences have any policy implications? Indeed they do. In particular they suggest a specific strategy for policy. Among the possible uses for resources that fulfill the sustainability criterion, we choose the one that maximizes either dynamic or static efficiency as appropriate. In this formulation the sustainability criterion acts as an overriding constraint on social decisions. Yet, by itself, the sustainability criterion is insufficient because it fails to provide any guidance on which of the infinite number of sustainable allocations should be chosen. That is where efficiency comes in. It provides a means for maximizing the wealth derived from all the possible sustainable allocations.

This combination of efficiency with sustainability turns out to be very helpful in guiding policy. Many unsustainable allocations are the result of inefficient behavior. Correcting the inefficiency can either restore sustainability or move the economy a long way in that direction. Furthermore, and this is important, correcting inefficiencies can frequently produce win-win situations. In win-win changes, the various parties affected by the change can <u>all</u> be made better off after the change than before. This contrasts sharply with changes in which the gains to the gainers are smaller than the losses to the losers.

Win-win situations are possible because moving from an inefficient to an efficient allocation increases net benefits. The increase in net benefits provides a means for compensating those who might otherwise lose from the change. Compensating losers reduces the opposition to change, thereby making change more likely. Do our economic and political institutions normally produce outcomes that are both efficient and sustainable? In upcoming chapters we will provide explicit answers to this important question.

Summary

Both efficiency and ethical considerations can guide the desirability of private and social choices involving the environment. Whereas the former is concerned mainly with eliminating waste in the use of resources, the latter is concerned with assuring the fair treatment of all parties.

Previous chapters have focused on the static and dynamic efficiency criteria. Chapter 19 will focus on the environmental justice implications of environmental degradation and remediation for members of the current generation. The present chapter examines one globally important characterization of the obligation previous generations owe to generations that follow, and the policy implications that flow from acceptance of that obligation.

The specific obligation examined in this chapter—sustainable development—is based upon the notion that earlier generations should be free to pursue their own well-being as long as in so doing they do not diminish the welfare of future generations. This notion gives rise to three alternative definitions of sustainable allocations:

Weak Sustainability. Resource use by previous generations should not exceed a level that would prevent subsequent generations from achieving a level of well-being at least as great. One operational implication of this definition is that the value of the capital stock (natural

plus physical capital) should not decline. Individual components of the aggregate could decline in value as long as other components were increased in value (normally through investment) sufficiently to leave the aggregate value unchanged.

Strong Sustainability. According to this interpretation, the value of the remaining stock of natural capital should not decrease. This definition places special emphasis on preserving natural (as opposed to total) capital under the assumption that natural and physical capital offer limited substitution possibilities. This definition retains two characteristics of the previous definition: it preserves value (rather than a specific level of physical flow) and it preserves an aggregate of natural capital (rather than any specific component).

Environmental Sustainability. Under this definition, the *physical* flows of *individual* resources should be maintained, not merely the *value* of the *aggregate*. For a fishery, for example, this definition would emphasize maintaining a constant fish catch (referred to as a sustainable yield), rather than a constant value of the fishery. For a wetland, it would involve preserving specific ecological functions, not merely their aggregate value.

It is possible to examine and compare the theoretical conditions that characterize various allocations (including market allocations and efficient allocations) to the necessary conditions for an allocation to be sustainable under these definitions. According to the theorem that is now known as the "Hartwick Rule," if all of the scarcity rent from the use of scarce resources is invested in capital, the resulting allocation will satisfy the first definition of sustainability.

In general, not all efficient allocations are sustainable and not all sustainable allocations are efficient. Furthermore, market allocations can be (1) efficient, but not sustainable; (2) sustainable, but not efficient; (3) inefficient and unsustainable; and (4) efficient and sustainable. One class of situations, known as "win-win" situations, provides an opportunity to increase simultaneously the welfare of both current and future generations.

We shall explore these themes much more intensively as we proceed through the book. In particular, we shall inquire into when market allocations can be expected to produce allocations that satisfy the sustainability definitions and when they cannot. We shall also see several specific examples of how the skillful use of economic incentives can allow policymakers to exploit "win-win" situations to promote a transition onto a sustainable path for the future.

Discussion Question

1. The environmental sustainability criterion differs in important ways from both strong and weak sustainability. Environmental sustainability frequently means maintaining a constant physical flow of individual resources (e.g., fish from the sea or wood from the forest), while the other two definitions call for maintaining the *aggregate value* of those service flows. When might the two criteria lead to different choices? Why?

Self-Test Exercises

1. In the numerical example given in the text, the inverse demand function for the depletable resource is $P = 8 - 0.4q$ and the marginal cost of supplying it is \$2. (a) If 20 units are to be allocated between two periods, in a dynamic efficient allocation how much would be allocated to the first period and how much to the second period when the discount rate

is zero? (b) Given this discount rate, what would be the efficient price in the two periods? (c) What would be the marginal user cost in each period?

2. Assume the same demand conditions as stated in Problem 1, but let the discount rate be 0.10 and the marginal cost of extraction be $4. How much would be produced in each period in an efficient allocation? What would be the marginal user cost in each period? Would the static and dynamic efficiency criteria yield the same answers for this problem? Why?
3. Compare two versions of the two-period depletable resource model that differ only in the treatment of marginal extraction cost. Assume that in the second version the constant marginal extraction cost is lower in the second period than the first (perhaps due to the anticipated arrival of a new, superior extraction technology). The constant marginal extraction cost is the same in both periods in the first version and is equal to the marginal extraction cost in the first period of the second version. In a dynamic efficient allocation, how would the extraction profile in the second version differ from the first? Would relatively more or less be allocated to the second period in the second version than in the first version? Would the marginal user cost be higher or lower in the second version? Why?
4. a. Consider the general effect of the discount rate on the dynamic efficient allocation of a depletable resource across time. Suppose we have two versions of the two-period model discussed in this chapter. The two versions are identical except for the fact that the second version involves a higher discount rate than the first version. What effect would the higher discount rate have on the allocation between the two periods and the magnitude of the present value of the marginal user cost?
   b. Explain the intuition behind your results.
5. a. Consider the effect of population growth on the allocation on the dynamic efficient allocation of a depletable resource across time. Suppose we have two versions of the two-period model, discussed in this chapter, that are identical except for the fact that the second version involves a higher demand for the resource in the second period (e.g., the demand curve shifts to the right due to population growth) than the first version. What effect would the higher demand in the second period have on the allocation between the two periods and the magnitude of the present value of the marginal user cost?
   b. Explain the intuition behind your results.

Notes

1 The height of the triangle is $6 [$8 – $2] and the base is 15 units. The area is therefore (1/2)($6)(15) = $45.
2 The undiscounted net benefit is $25. The calculation is ((6 – 2) × 5) + (1/2 × (8 – 6) × 5) = $25. The discounted net benefit is therefore 25/1.10 = 22.73.
3 Note that the sum of the two allocations in Figure 5.2 is always 20. The left-hand axis represents an allocation of all 20 units to Period 2 and the right-hand axis represents an allocation entirely to Period 1.
4 Demonstrate that this point is the maximum by first allocating slightly more to Period 2 (and therefore less to Period 1) and showing that the total area decreases. Conclude by allocating slightly less to Period 2 and showing that, in this case as well, total area declines.
5 You can verify this by taking the present value of $2.095 and showing that it equals $1.905.
6 The supporting calculations are (1.905)(10.238) + 0.5(4.095)(10.238) for the first period and (2.095)(9.762) + 0.5(3.905)(9.762) for the second period.

Further Reading

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Kiron, D., Kruschwitz, N., Haanæs, K., & Velken, I. V. S. (2012). Sustainability nears a tipping point. *MIT Sloan Management Review*, *53*(2), 69–74. What is the role for the private sector in sustainable development? Is concern over the "bottom line" consistent with the desire to promote sustainable development?

Lopez, R., & Toman, M. A. (Eds.). (2006). *Economic Development and Environmental Sustainability.* New York: Oxford University Press. Thirteen essays that explore how the principles of sustainability can be implemented in the context of reducing poverty through development.

Pezzey, J. V. C., & Toman, M. A. (2002). Progress and problems in the economics of sustainability. In T. Tietenberg & H. Folmer (Eds.), *The International Yearbook of Environmental and Resource Economics: A Survey of Current Issues*. Cheltenham, U.K.: Edward Elgar, 165–232. An excellent technical survey of the economics literature on sustainable development.

USEPA Sustainability website: www.epa.gov/sustainability/learn-about-sustainability#what. A description of how the concept of sustainability affects the work of the United States Environmental Protection Agency.

World Bank. (2011). *The Changing Wealth of Nations: Measuring Sustainable Development in the New Millennium*. Washington, D.C.: World Bank. This study presents, for the first time, a set of "wealth accounts" for over 150 countries for 1995, 2000, and 2005. This set of accounts allows a longer-term assessment of global, regional and country performance within the weak sustainability context.

Additional references and historically significant references are available on this book's Companion Website: www.routledge.com/cw/Tietenberg

Appendix

The Simple Mathematics of Dynamic Efficiency

Assume that the demand curve for a depletable resource is linear and stable over time. Thus, the inverse demand curve in year t can be written as

$$P_t = a - bq_t$$

The total benefits from extracting an amount q_t in year t are then the integral of this function (the area under the inverse demand curve):

$$\begin{aligned}(\text{Total benefits})t &= \int_0^{qt} (a - bq)\,dq \\ &= aq_t - \frac{b}{2}q_t^2\end{aligned}$$

Further assume that the marginal cost of extracting that resource is a constant c and therefore the total cost of extracting any amount q_t in year t can be given by

$$(\text{Total cost})_t = cq_t$$

If the total available amount of this resource is $\overline{Q}$, then the dynamic allocation of a resource over n years is the one that satisfies the maximization problem:

$$Max_q \sum_{i=1}^{n} \frac{aq_i - bq_i^2/2 - cq_i}{(1+r)^{i-1}} + \lambda\left[\overline{Q} - \sum_{i=1}^{n} q_i\right]$$

Assuming that $\overline{Q}$ is less than would normally be demanded, the dynamic efficient allocation must satisfy

$$\frac{a - bq_i - c}{(1+r)^{i-1}} - \lambda = 0,\ i = 1, \ldots, n$$

$$\overline{Q} - \sum_{i=1}^{n} q_i = 0$$

An implication of the first of these two equations is that $(P - MC)$ increases over time at rate r. This difference, which is known as the marginal user cost, will play a key role in our thinking about allocating depletable resources over time.

An exact solution to the two-period model can be illustrated using these solution equations and some assumed values for the parameters.

The following parameter values are assumed by the two-period example:

$$a = 8,\ c = 2,\ b = 0.4,\ Q = 20, \text{ and } r = 0.10.$$

Inserting these parameters into the two equations (one for each period), we obtain

$$8 - 0.4q_1 - 2 - \lambda = 0,$$

$$\frac{8 - 0.4q_2 - 2}{1.10} - \lambda = 0$$

$$q_1 + q_2 = 20.$$

It is now readily verified that the solution (accurate to the third decimal place) is

$$q_1 = 10.238,\ q_2 = 9.762,\ \lambda = 1.905.$$

We can now demonstrate the propositions discussed in this text.

1. Verbally, in a dynamic efficient allocation, the present value of the marginal net benefit in Period 1 ($8 - 0.4q_1 - 2$) has to equal λ. In addition, the present value of the marginal net benefit in Period 2 should also equal λ. Therefore, they must equal each other. This demonstrates the proposition shown graphically in Figure 5.2.

2. The present value of marginal user cost is represented by λ. Thus, the price in the first period ($8 - 0.4q_1$) should be equal to the sum of marginal extraction cost ($2) and marginal user cost ($1.905). Multiplying λ by $1 + r$, it becomes clear that price in the second period ($8 - 0.4q_2$) is equal to the marginal extraction cost ($2) plus the higher marginal user cost [$\lambda\ (1 + r) = (1.905)(1.10) = \$2.095$] in Period 2. These results show why the graphs in Figure 5.3 have the properties they do. They also illustrate the point that, in this case, marginal user cost rises over time.

Chapter 6

Depletable Resource Allocation

The Role of Longer Time Horizons, Substitutes, and Extraction Cost

The whole machinery of our intelligence, our general ideas and laws, fixed and external objects, principles, persons, and gods, are so many symbolic, algebraic expressions. They stand for experience; experience which we are incapable of retaining and surveying in its multitudinous immediacy. We should flounder hopelessly, like the animals, did we not keep ourselves afloat and direct our course by these intellectual devices. Theory helps us to bear our ignorance of fact.

—George Santayana, *The Sense of Beauty* (1896)

Introduction

How do societies react when finite stocks of depletable resources become scarce? Is it reasonable to expect that self-limiting feedbacks would facilitate the transition to a sustainable, steady state? Or is it more reasonable to expect that self-reinforcing feedback mechanisms would cause the system to overshoot the resource base, possibly even precipitating a societal collapse?

We begin to seek answers to these questions by studying the implications of both efficient and profit-maximizing decision making. What kinds of feedback mechanisms are implied by decisions motivated by efficiency and by profit maximization? Are they compatible with a smooth transition or are they more likely to produce overshoot and collapse?

We approach these questions in several steps, beginning by defining and discussing a simple but useful *resource taxonomy* (classification system), as well as explaining the dangers of ignoring the distinctions made by this taxonomy. We initiate the analysis by defining an efficient allocation of an exhaustible resource over time in the absence of any renewable substitute and explore the conditions any efficient allocation must satisfy. Numerical examples illustrate the implications of these conditions.

Renewable resources are integrated into the analysis by relying on the simplest possible case—the resource is assumed to be supplied at a fixed, abundant rate and can be accessed at

a constant marginal cost. Solar energy and replenishable surface water are two examples that seem roughly to fit this characterization. Integrating a renewable resource backstop into our basic depletable resource model allows us to characterize efficient extraction paths for both types of resources, assuming that they are perfect substitutes. We also explore how these efficient paths are affected by changes in the nature of the cost functions as well as by the presence or absence of externalities. Succeeding chapters will use these principles to examine the allocation of such diverse resources as energy, minerals, land, and water and to provide a basis for developing more elaborate models of renewable biological populations, such as fisheries and forests.

A Resource Taxonomy

Three separate concepts are used to classify the stock of depletable resources: (1) *current reserves*, (2) *potential reserves*, and (3) *resource endowment*. The U.S. Geological Survey (USGS) has the official responsibility for keeping records of the U.S. resource base and has developed the classification system described in Figure 6.1.

Note the two dimensions—one economic and one geological. A movement from top to bottom represents movement from cheaply extractable resources to those extracted at substantially higher costs. By contrast, a movement from left to right represents increasing geological uncertainty about the size of the resource base.

Current reserves (shaded area in Figure 6.1) are defined as known resources that can profitably be extracted at current prices. The magnitude of these current reserves can be expressed as a number.

Potential reserves, on the other hand, are most accurately defined as a function rather than a number. The amount of reserves potentially available depends upon the price people are willing to pay for those resources—the higher the price, the larger the potential reserves. Higher prices enable not only more expensive measures to recover more of the resource from conventional sources, but also measures to extract resources from previously untapped unconventional sources.

The *resource endowment* represents the natural occurrence of resources in the earth's crust. Since prices have nothing to do with the size of the resource endowment, it is a geological, rather than an economic, concept. This concept is important because it represents a physical upper limit on the availability of terrestrial resources.

The distinctions among these three concepts are significant. One common mistake in failing to respect these distinctions is using data on current reserves as if they represented the maximum potential reserves. This fundamental error can cause a huge understatement of the time until exhaustion.

A second common mistake is to assume that the entire resource endowment can be made available as potential reserves at a price people would be willing to pay. Clearly, if an infinite price were possible, the entire resource endowment could be exploited, but don't hold your breath until the arrival of infinite prices.

Other distinctions among resource categories are also useful. The first category includes all depletable, recyclable resources, such as copper. A *depletable resource* is one for which the natural replenishment feedback loop can safely be ignored. The rate of replenishment for these resources is so low that it does not offer a potential for augmenting the stock in any reasonable time frame.

A *recyclable resource* is one that, although currently being used for some particular purpose, exists in a form allowing its mass to be recovered once that original purpose is no

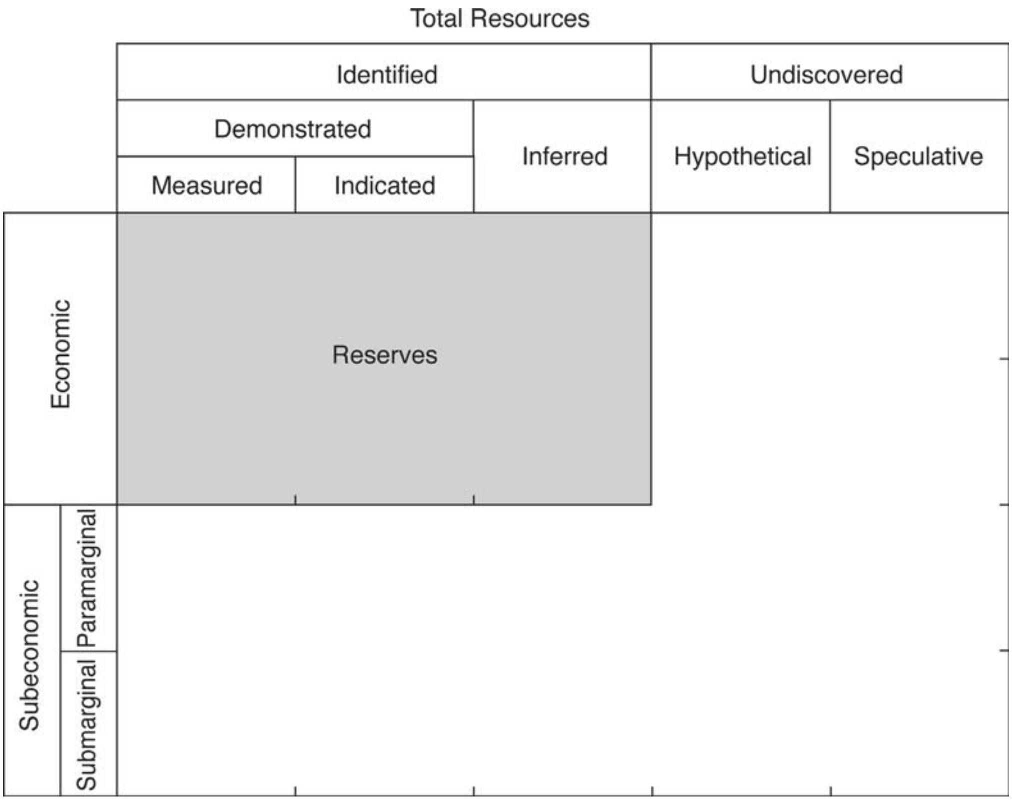

Figure 6.1 A Categorization of Resources

Terms

Identified resources: specific bodies of mineral-bearing material whose location, quality, and quantity are known from geological evidence, supported by engineering measurements.

Measured resources: material for which quantity and quality estimates are within a margin of error of less than 20 percent, from geologically well-known sample sites.

Indicated resources: material for which quantity and quality have been estimated partly from sample analyses and partly from reasonable geological projections.

Inferred resources: material in unexplored extensions of demonstrated resources based on geological projections.

Undiscovered resources: unspecified bodies of mineral-bearing material surmised to exist on the basis of broad geological knowledge and theory.

Hypothetical resources: undiscovered materials reasonably expected to exist in a known mining district under known geological conditions.

Speculative resources: undiscovered materials that may occur in either known types of deposits in favorable geological settings where no discoveries have been made, or in yet unknown types of deposits that remain to be recognized.

Source: U.S. Bureau of Mines and the U.S. Geological Survey. (1976). Principles of the Mineral Resource Classification System of the U.S. Bureau of Mines and the U.S. Geological Survey. *Geological Survey Bulletin*, 1450-A.

longer necessary or desirable. For example, copper wiring from an automobile can be recovered after the car has been shipped to the junkyard. The degree to which a recyclable resource is actually recycled is determined by economic conditions, a subject covered in Chapter 8.

The current reserves of a depletable, recyclable resource can be augmented by economic replenishment, as well as by recycling. Economic replenishment takes many forms, all sharing the characteristic that they turn previously unrecoverable resources into recoverable ones. One obvious stimulant for this replenishment is price. As price rises, producers find it profitable to explore more widely, dig more deeply, and use lower-concentration ores.

Higher prices also stimulate technological progress. Technological progress simply refers to an advancement in the state of knowledge that allows us to expand the set of feasible possibilities. Harnessing nuclear power and the advent of both horizontal drilling and hydraulic fracturing are two obvious examples. (Both are discussed in Chapter 7.)

The potential reserves of depletable, recyclable resources, however, can be exhausted. The depletion rate is affected by the demand for and the durability of the products built with the resource, and the ability to reuse the products. Except where demand is totally price-inelastic (i.e., insensitive to price), higher prices tend to reduce the quantity demanded. Durable products last longer, reducing the need for newer ones. Reusable products (e.g., rechargeable batteries or products sold at flea markets) provide a substitute for new products.

For some resources, the size of the potential reserves depends explicitly on our ability to store the resource. For example, helium is generally found commingled with natural gas in common fields. As the natural gas is extracted and stored, unless the helium is simultaneously captured and stored, it diffuses into the atmosphere. This diffusion results in such low atmospheric concentrations that extraction of helium from the air is not economical at current or even likely future prices. Thus, the useful stock of helium depends crucially on how much we decide to store.

Not all depletable resources can be recycled or reused. Depletable energy resources such as coal, oil, and gas are irreversibly transformed when they are combusted. Once turned into heat energy, the heat dissipates into the atmosphere and becomes nonrecoverable.

The endowment of depletable resources is of finite size. Current use of depletable, nonrecyclable resources precludes future use; hence, the issue of how they should be shared among generations is raised in the starkest, least forgiving form.

Depletable, recyclable resources raise this same issue, though somewhat less starkly, since recycling and reuse make the useful stock last longer, all other things being equal. It is tempting to suggest that depletable, recyclable resources could last forever with 100 percent recycling, but unfortunately the physical theoretical upper limit on recycling is less than 100 percent—an implication of a version of the entropy law defined in Chapter 2. Some of the mass is always lost during recycling or use.

Because less than 100 percent of the mass is recycled, the useful stock must eventually decline to zero. Therefore, even for recyclable, depletable resources, the cumulative useful stock is finite, and current consumption patterns still have some effect on future generations.

Renewable resources are differentiated from depletable resources primarily by the fact that natural replenishment augments the flow of renewable resources at a non-negligible rate. Solar energy, water, and biological populations are all examples of renewable resources. For this class of resources it is possible, though not inevitable, that a flow of these resources could be maintained perpetually.[1]

For some renewable resources, the continuation and volume of their flow depend crucially on humans. Soil erosion and nutrient depletion reduce the flow of food. Excessive fishing reduces the stock of fish, which in turn reduces the rate of natural increase of the fish population. What other examples can you come up with?

For other renewable resources, such as solar energy, the flow is independent of humans. The amount consumed by one generation does not reduce the amount that can be consumed by subsequent generations.

Some renewable resources can be stored; others cannot. For those that can, storage provides an additional way to manage the allocation of the resource over time. We are not left simply at the mercy of natural ebbs and flows. Food, without proper care, perishes rapidly, but under the right conditions stored food can be used to feed the hungry in times of famine. Unstored solar energy radiates off the earth's surface and dissipates into the atmosphere. While solar energy can be stored in many forms, the most common natural form of storage occurs when it is converted to biomass by photosynthesis.

Storage of renewable resources usually provides a different service than storage of depletable resources. Storing depletable resources prolongs their economic life; storing renewable resources, on the other hand, can serve as a means of smoothing out the cyclical imbalances of supply and demand. Surpluses can be stored for use during periods when deficits occur. Familiar examples include food stockpiles and the use of dams to store water to use for hydropower.

Managing renewable resources presents a different challenge from managing depletable resources, although an equally significant one. The challenge for depletable resources involves allocating dwindling stocks among generations while meeting the ultimate transition to renewable resources. In contrast, the challenge for managing renewable resources involves the maintenance of an efficient, sustainable flow. Chapters 7 through 13 deal with how the economic and political sectors have responded to these challenges for particularly significant types of resources.

Efficient Intertemporal Allocations

If we are to judge the efficiency of market allocations, we must define what is meant by efficiency in relation to the management of depletable and renewable resources. Because allocation over time is the crucial issue, dynamic efficiency becomes the core concept. The dynamic efficiency criterion assumes that society's objective is to maximize the present value of net benefits coming from the resource. For a depletable, nonrecyclable resource, this requires a balancing of the current and subsequent uses of the resource. In order to refresh our memories about how the dynamic efficiency criterion defines this balance, we shall begin with recalling and elaborating on the very simple two-period model developed in Chapter 5. We can then proceed to demonstrate how conclusions drawn from that model generalize to longer planning horizons and more complicated situations.

The Two-Period Model Revisited

In Chapter 5, we defined a situation involving the allocation, over two periods, of a finite resource that could be extracted at constant marginal cost. With a stable demand curve for the resource, an efficient allocation involved allocating more than half of the resource to the first period and less than half to the second period. How the resources were divided between the two periods was affected by the marginal cost of extraction, the marginal user cost and the discount rate.

Due to the fixed and finite nature of depletable resources, use of a unit today precludes use of that unit tomorrow. Therefore, production decisions today must take forgone future net benefits into account. Marginal user cost is the opportunity cost measure that allows intertemporal balancing to take place.

In our two-period model, the marginal cost of extraction is assumed to be constant, but the value of the marginal user cost was shown to rise over time. In fact, as was demonstrated mathematically in the appendix to Chapter 5, when the demand curve is stable over time and the marginal cost of extraction is constant, the rate of increase in the current value of the marginal user cost is equal to r, the discount rate. Thus, in Period 2, the marginal user cost would be $1 + r$ times as large as it was in Period 1.[2] Marginal user cost rises at rate r in an efficient allocation in order to preserve the balance between present versus future production.

In summary, our two-period example suggests that an efficient allocation over time of a finite resource with a constant marginal cost of extraction involves rising marginal user cost and falling quantities consumed. How can we generalize to longer time periods and different extraction circumstances?

The *N*-Period Constant-Cost Case

We begin this generalization by retaining the constant-marginal-extraction-cost assumption while extending the time horizon within which the resource is allocated. In the numerical example shown in Figures 6.2a and 6.2b, the demand curves and the marginal cost curve from the two-period case are retained. The only changes in this numerical example from the two-period case involve spreading the allocation over a larger number of years and increasing the total recoverable supply from 20 to 40. (The specific mathematics behind this and subsequent examples is presented in the appendix at the end of this chapter, but we shall guide you through the intuition that follows from that analysis in this section.)

Figure 6.2a demonstrates how the efficient quantity extracted varies over time, while Figure 6.2b shows the behavior of the marginal user cost and the marginal cost of extraction. We shall use the term "total marginal cost" to refer to the sum of the two. The marginal cost of extraction is represented by the lower line, and the marginal user cost is depicted as the vertical distance between the marginal cost of extraction and the total marginal cost. To avoid

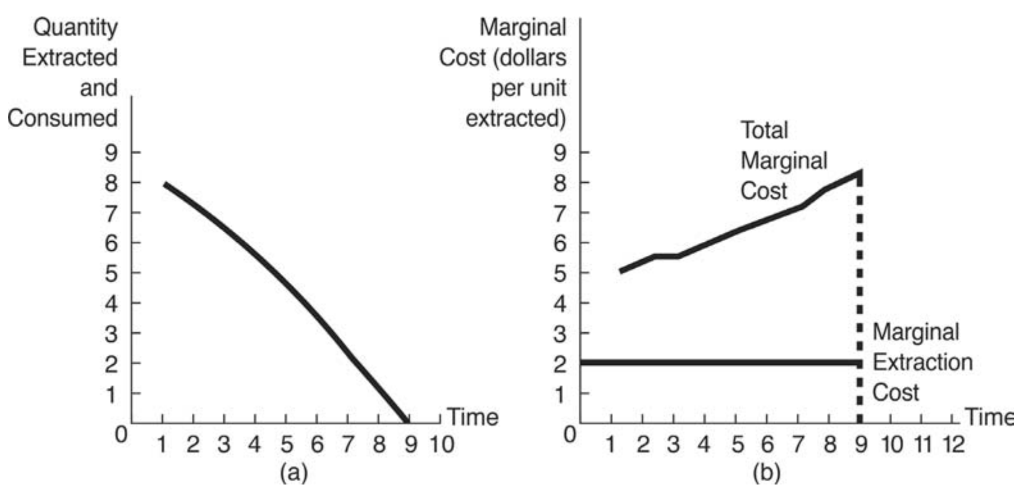

Figure 6.2 (a) Constant Marginal Extraction Cost with No Substitute Resource: Quantity Profile (b) Constant Marginal Extraction Cost with No Substitute Resource: Marginal Cost Profile

confusion, note that the horizontal axis is defined in terms of time, not the more conventional designation—quantity.

Several trends are worth noting. First of all, in this case, as in the two-period case, the efficient marginal user cost rises steadily in spite of the fact that the marginal cost of extraction remains constant. This rise in the efficient marginal user cost reflects increasing scarcity and the resulting rise in the opportunity cost of current consumption (reflecting forgone future opportunities) as the remaining stock dwindles.

In response to these rising costs over time, the extracted quantity falls over time until it finally becomes zero, which occurs precisely at the moment when the total marginal cost becomes \$8. At this point, total marginal cost is equal to the highest price anyone is willing to pay, so demand and supply simultaneously equal zero. Thus, even in this challenging case involving no increase in the cost of extraction, an efficient allocation envisions a smooth transition to the exhaustion of a resource. The resource does not "suddenly" run out (because prices have signaled the increasing scarcity), although in this case it does run out.

Transition to a Renewable Substitute

So far we have discussed the allocation of a depletable resource when no substitute is available to take its place. Suppose, however, we consider the nature of an efficient allocation when a substitute renewable resource is available at constant marginal cost. This case, for example, could describe the efficient allocation of oil or natural gas with a solar or wind substitute or the efficient allocation of exhaustible groundwater with a surface-water substitute. How could we define an efficient allocation in this circumstance?

Since this problem is very similar to the one already discussed, we can use what we have already learned as a foundation for mastering this new situation. Just as in the previous case, the depletable resource would also be exhausted in this case, but now the exhaustion will pose less of a problem, since we'll merely switch to the renewable substitute at the appropriate time.

For the purpose of our numerical example, assume the existence of a perfect substitute for the depletable resource that is infinitely available at a cost of \$6 per unit. The transition from the depletable resource to this renewable resource would ultimately transpire because the renewable resource marginal cost (\$6) is less than the maximum willingness to pay (\$8). (Can you figure out what the efficient allocation would be if the marginal cost of this substitute renewable resource was \$9, instead of \$6?)

The total marginal cost for the depletable resource in the presence of a \$6 perfect substitute would never exceed \$6, because society could always substitute the renewable resource whenever it was cheaper. Thus, while the maximum willingness to pay (\$8, the *choke price*) sets the upper limit on total marginal cost when no substitute is available, the marginal cost of extraction of the substitute (\$6 in our example) sets the upper limit in this new case as long as the perfect substitute is available at a marginal cost lower than the choke price. The efficient path for this situation is given in Figures 6.3a and 6.3b.

In this efficient allocation, the transition is once again smooth. Quantity extracted per unit of time is gradually reduced as the marginal user cost rises until the switch is made to the substitute. No abrupt change is evident once again in either marginal cost or quantity profiles.

What about the timing of the extraction of the depletable resource? When a renewable resource is available, more of the depletable resource would be extracted in the earlier periods than was the case without a renewable resource. Do you see why?

In this example, the switch is made during the sixth period, whereas in the previous example (involving no renewable substitute) the last units were exhausted at the end of the eighth

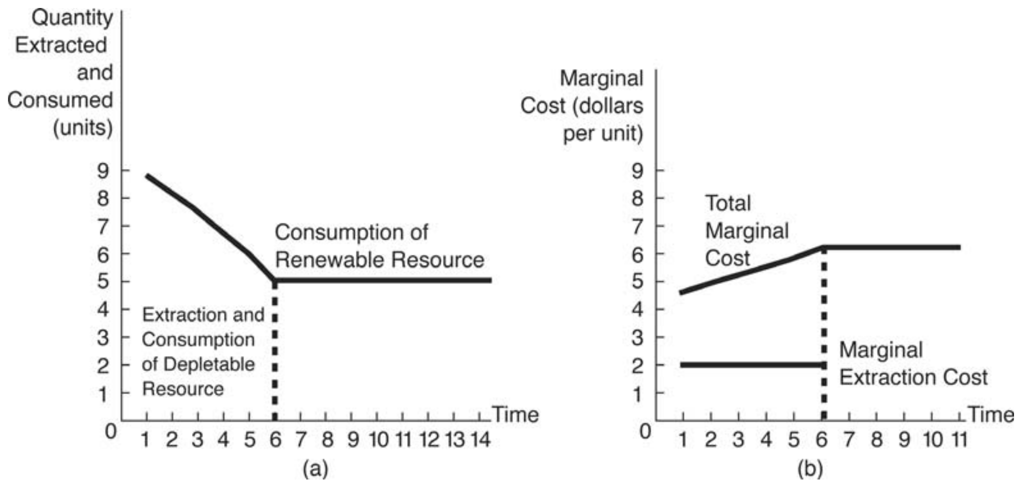

Figure 6.3 (a) Constant Marginal Extraction Cost with Substitute Resource: Quantity Profile (b) Constant Marginal Extraction Cost with Substitute Resource: Marginal Cost Profile

period. That seems consistent with common sense. When a substitute is available, the need to save some of the depletable resource for the future is certainly less pressing. The opportunity cost is lower.

At the switch point, consumption of the renewable resource begins. Prior to the switch point, only the depletable resource is consumed, while after the switch point only the renewable resource is consumed. This sequencing of consumption patterns results from the costs of the choices. Prior to the switch point, the depletable resource is cheaper. At the switch point, the total marginal cost of the depletable resource (including marginal user cost) rises to meet the marginal cost of the substitute, and the transition occurs. Due to the availability of the substitute resource, after the switch point consumption never drops below five units in any time period.

Why five? Five is the amount that maximizes the net benefit when the marginal cost equals \$6 (the price of the substitute). (Convince yourself of the validity of this statement by substituting \$6 into the willingness-to-pay function and solving for the quantity demanded.)

We shall not show the numerical example here, but it is not difficult to see how an efficient allocation would be defined when the transition is from one constant marginal-cost depletable resource to another depletable resource with a constant, but higher, marginal cost (see Figure 6.4). The total marginal cost of the first resource would rise over time until it equaled that of the second resource at the time of transition (T^*). In the period of time prior to transition, only the cheapest resource would be consumed; all of it would have been consumed by T^*.

A close examination of the total-marginal-cost path reveals two interesting characteristics worthy of our attention. First, even in this case, the transition is a smooth one; total marginal cost never jumps to the higher level. Second, the slope of the total marginal cost curve over time is flatter after transition.

The first characteristic is easy to explain. The total marginal costs of the two resources have to be equal at the time of transition. If they weren't equal, the net benefit could be increased by switching to the lower-cost resource from the more expensive resource. Total marginal

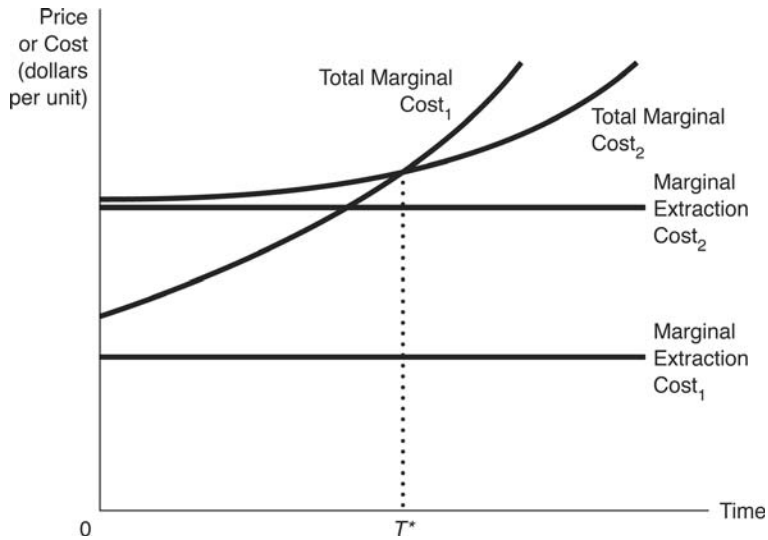

Figure 6.4 The Transition from One Constant-Cost Depletable Resource to Another

costs are not equal in the other periods. In the period before transition, the first resource is cheaper and therefore used exclusively, whereas after transition the first resource is exhausted, leaving only the second resource.

The slope of the marginal cost curve over time is flatter after transition simply because the component of total marginal cost that is growing (the marginal user cost) represents a smaller portion of the total marginal cost of the second resource than of the first. The total marginal cost of each resource is determined by the marginal extraction cost plus the marginal user cost. In both cases the marginal user cost is increasing at rate r, and the marginal cost of extraction is constant. As you can see in Figure 6.4, the marginal cost of extraction, which is constant, constitutes a much larger proportion of total marginal cost for the second resource than for the first. Hence, total marginal cost rises more slowly for the second resource, at least initially.

Increasing Marginal Extraction Cost

We have now expanded our examination of the efficient allocation of depletable resources to include longer time horizons and the availability of other depletable or renewable resources that could serve as perfect substitutes. As part of our trek toward increasing realism, we will now consider a situation in which the marginal cost of extracting the depletable resource rises with the cumulative amount extracted. This is commonly the case, for example, with minerals, where the higher-grade ores are extracted first, followed by an increasing reliance on lower-grade, higher marginal cost ones.

Analytically, this case is handled in the same manner as the previous case, except that the function describing the marginal cost of extraction is slightly more complicated[3]—it increases with the cumulative amount extracted. The dynamic efficient allocation of this resource is once again found by maximizing the present value of the net benefits, but in this case using

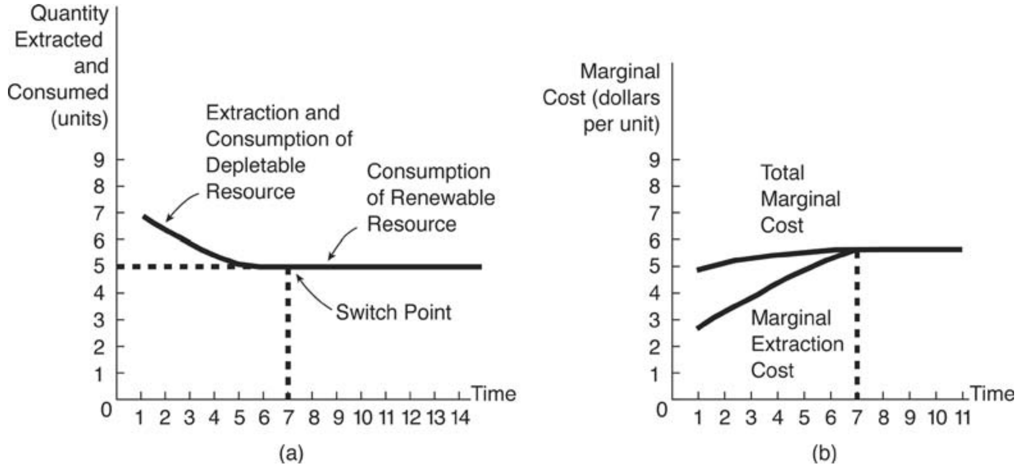

Figure 6.5 (a) Increasing Marginal Extraction Cost with Substitute Resource: Quantity Profile (b) Increasing Marginal Extraction Cost with Substitute Resource: Marginal Cost Profile

this modified cost of extraction function. The results of that maximization are portrayed in Figures 6.5a and 6.5b.

The most significant difference between this case and those that preceded it lies in the behavior of marginal user cost. In the constant marginal cost cases we noted that marginal user cost rose over time at rate *r*. When the marginal cost of extraction increases with the cumulative amount extracted, as in this case, marginal user cost *declines* over time until, at the time of transition to the renewable resource, it goes to zero. Can you figure out why?

Remember that marginal user cost is an opportunity cost reflecting forgone future marginal net benefits. In contrast to the constant marginal-cost case, in the increasing marginal-cost case every unit extracted now raises the cost of future extraction. Therefore, as the current marginal cost rises over time, the sacrifice made by future generations diminishes as an additional unit is consumed earlier; the net benefit that would be received by a future generation, if a unit of the resource were saved for them, gets smaller and smaller as the marginal extraction cost of that resource gets larger and larger. By the last period, the marginal extraction cost is so high that earlier consumption of one more unit imposes no sacrifice at all. At the switch point, the opportunity cost of current extraction (as reflected in the marginal user cost) drops to zero, and total marginal cost equals the marginal extraction cost.[4]

The increasing-cost case differs from the constant-cost case in another important way as well. In the constant-cost case, the depletable resource reserve is ultimately completely exhausted. In the increasing-cost case, however, the reserve is not exhausted; some is left in the ground because it is more expensive than the substitute.

Up to this point in our analysis, we have examined what an efficient allocation would look like in a number of circumstances. First, we examined a situation in which a finite amount of a resource is extracted at constant marginal cost. Despite the absence of increasing extraction cost, an efficient allocation involves a smooth transition to a substitute, when one is available, or to abstinence, when one is not. The complication of increasing marginal cost changes the time profile of the marginal user cost, but it does not alter the basic finding of declining consumption of depletable resources coupled with rising total marginal cost.

Can this analysis be used as a basis for judging whether current extraction profiles are efficient? As a look at the historical record reveals, the consumption patterns of most depletable resources have involved increases, not decreases, in consumption over time. Is this prima facie evidence that the resources are not being allocated efficiently?

Exploration and Technological Progress

Using the historical patterns of increasing consumption to conclude that depletable resources are not being allocated efficiently would not represent a valid conclusion. As we have noted earlier, the conclusions of any model depend on the structure of that model. The models considered to this point have not yet included a consideration of the role of population and income growth, which could cause demand to shift upward over time, or of the exploration for new resources or technological progress. These are historically significant factors in the determination of actual consumption paths.[5]

Consider how these factors might influence the efficient extraction profile. The search for new resources is expensive. As easily discovered resources are exhausted, searches are initiated in less rewarding, more costly, environments, such as the bottom of the ocean or locations deep within the earth. This suggests the *marginal cost of exploration*, which is the marginal cost of finding additional units of the resource, should be expected to rise over time, just as the marginal cost of extraction does.

As the total marginal cost for a resource rises over time, society should actively explore possible new sources of that resource. Larger increases in the marginal cost of extraction for known sources trigger larger potential increases in net benefits from finding new sources that previously would have been unprofitable to extract.

Some of this exploration would be successful: new sources of the resource would be discovered. If the marginal extraction cost of the newly discovered resources is low enough, these discoveries could lower, or at least delay, the increase in the total marginal cost of production. As a result, the new finds would tend to encourage more consumption and more extraction. Compared to a situation with no exploration possible, the model with exploration would show a smaller and slower decline in consumption, while the rise in total marginal cost would be dampened.

It is also not difficult to expand our concept of efficient resource allocations to include *technological progress*, the general term economists give to advances in the state of knowledge. In the present context, technological progress would be manifested as reductions over time in the cost of extraction. For a resource that can be extracted at constant marginal cost, a one-time breakthrough lowering the marginal cost of extraction would hasten the time of transition. Furthermore, for an increasing-cost resource, more of the total available resource would be recovered in the presence of technological progress than would be recovered without it. (Why?)

The most pervasive effects of technological progress involve continuous downward shifts in the cost of extraction over some time period. The total marginal cost of the resource could actually fall over time if the cost-reducing nature of technological progress became so potent that, in spite of increasing reliance on inferior ore, the marginal cost of extraction decreased (see Example 6.1). With a finite amount of this resource, the fall in total marginal cost would be transitory, since ultimately it would have to rise. As we shall see in the next few chapters, however, this period of transition can last quite a long time.

EXAMPLE 6.1

Historical Example of Technological Progress in the Iron Ore Industry

The term *technological progress* plays an important role in the economic analysis of mineral resources. Yet, at times, it can appear abstract, even mystical. It shouldn't! Far from being a blind faith detached from reality, technological progress refers to a host of ingenious ways in which people have reacted to impending shortages with sufficient imagination that the available supply of resources has been expanded by an order of magnitude and at reasonable cost. An interesting case from economic history illustrates how concrete a notion technological progress is.

In 1947, the president of Republic Steel, C. M. White, calculated the expected life of the Mesabi Range of northern Minnesota (the source of some 60 percent of iron ore consumed during World War II) as being in the range from 5 to 7 years. By 1955, only 8 years later, *U.S. News and World Report* concluded that worry over the scarcity of iron ore could be forgotten. The source of this remarkable transformation of a problem of scarcity into one of abundance was the discovery of a new technique of preparing iron ore, called *pelletization*.

Prior to pelletization, the standard ores from which iron was derived contained from 50 to more than 65 percent iron in crude form. A significant percentage of taconite ore containing less than 30 percent iron in crude form was available, but no one knew how to produce it at reasonable cost.

Pelletization, a process by which these ores are processed and concentrated at the mine site prior to shipment to the blast furnaces, allowed the profitable use of the taconite ores. While expanding the supply of iron ore, pelletization reduced its cost in spite of its inferior grade.

There were several sources of the cost reduction. First, substantially less energy was used; the shift in ore technology toward pelletization produced net energy savings of 17 percent in spite of the fact that the pelletization process itself required more energy. The reduction came from the discovery that the blast furnaces could be operated much more efficiently using pelletized inputs. The process also reduced labor requirements per ton by some 8.2 percent while increasing the output of the blast furnaces. A blast furnace owned by Armco Steel in Middletown, Ohio, which had a rated capacity of approximately 1500 tons of molten iron per day, was able, by 1960, to achieve production levels of 2700–2800 tons per day when fired with 90 percent pellets. Pellets nearly doubled the blast furnace productivity!

Sources: Kakela, P. J. (1978). Iron ore: Energy labor and capital changes with technology. *Science*, *202* (December 15), 1151–1157; Kakela, P. J. (1981). Iron ore: From depletion to abundance. *Science*, *212* (April 10), 132–136.

Market Allocations of Depletable Resources

In the preceding sections, we have examined in detail how the efficient allocation of substitutable, depletable, and renewable resources over time would be defined in a variety of circumstances. We must now address the question of whether actual markets can be

expected to produce an efficient allocation. Can the private market, involving millions of consumers and producers, each reacting to his or her own unique preferences, ever result in a dynamically efficient allocation? Is profit maximization compatible with dynamic efficiency?

Appropriate Property Rights Structures

The most common misconception of those who believe that even a perfect market could never achieve an efficient allocation of depletable resources is based on the idea that producers want to extract and sell the resources as fast as possible, since that is how they derive the value from the resource. This misconception makes people see markets as myopic and unconcerned about the future.

As long as the property rights governing natural resources have the characteristics of exclusivity, transferability, and enforceability (Chapter 2), the markets in which those resources are bought and sold will not necessarily lead to myopic choices for the simple reason that myopia would reduce profits. By taking marginal user cost into account, the producer maximizes profits by acting efficiently.

A resource in the ground has two potential sources of value to its owner: (1) a use value when it is sold (the only source considered by those diagnosing inevitable myopia) and (2) an asset value when it remains in the ground. As long as the price of a resource continues to rise, the resource in the ground is becoming more valuable. The owner of this resource accrues this capital gain, however, only if the resource is conserved for later sale. A producer who sells all resources in the earlier periods loses the chance to take advantage of higher prices in the future.

A profit-maximizing producer attempts to balance present and future production in order to maximize the value of the resource and, hence, profits. Since higher prices in the future provide an incentive to conserve, a producer who ignores this incentive would not be maximizing the value of the resource. Resources sold by a myopic producer would be bought by someone willing to delay extraction in order to maximize its value. As long as social and private discount rates coincide, property rights structures are well defined (no externalities), and reliable information about future prices is available, a producer who pursues maximum profits simultaneously provides the maximum present value of net benefits for society.

The implication of this analysis is that, in competitive resource markets, the price of the resource equals the total marginal cost of extracting and using the resource. Thus, Figures 6.2a through 6.5b can illustrate not only an efficient allocation but also the allocation produced by an efficient market. When used to describe an efficient market, the total marginal cost curve describes the time path that prices could be expected to follow.

Environmental Costs

Not all actual situations, however, satisfy the conditions necessary for this harmonious outcome. One of the most important situations in which property rights structures may not be well defined occurs when the extraction of a natural resource imposes an environmental cost on society that is not internalized by the producers. The aesthetic costs of strip mining, the health risks associated with uranium tailings, and the acids leached into streams from mine operations are all examples of associated environmental costs. The presence of environmental costs is both empirically and conceptually important, since it forms one of the bridges between the traditionally separate fields of environmental economics and natural resource economics.

Suppose, for example, that the extraction of a depletable resource caused some damage to the environment that was not adequately reflected in the costs faced by the extracting firms.

This would be, in the context of discussion in Chapter 2, an external cost. The cost of getting the resource out of the ground, as well as processing and shipping it, is borne by the resource owner and considered in the calculation of how much of the resource to extract. The environmental damage, however, may not be borne by the owner and, in the absence of any outside attempt to internalize that external cost, will not normally be part of the extraction decision. How would the market allocation, based on only the costs borne by the owner, differ from the efficient allocation, which is based on all costs, regardless of who ultimately bears them?

We can examine this issue by modifying the numerical example used earlier in this chapter. Assume the environmental damage can be represented by increasing the marginal cost by $1.[6] The additional dollar reflects the cost of the environmental damage caused by producing another unit of the resource. What effect do you think this external cost would have on the efficient time profile for quantities extracted?

The answers are given in Figures 6.6a and 6.6b. The result of including environmental cost in the timing of the switch point is especially interesting because it involves two different effects that work in opposite directions. On the demand side, the internalization of environmental costs results in higher prices, which tend to dampen demand. This lowers the rate of consumption of the resource, which, all other things being equal, would make it last longer.

All other things are not equal, however. The higher marginal cost also means that a smaller cumulative amount of the depletable resource would be extracted in an efficient allocation. (Can you see why?) As shown in Figures 6.6a and 6.6b, the efficient cumulative amount extracted would be 30 units instead of the 40 units extracted in the case where environmental costs were not included. This supply-side effect tends to hasten the time when a switch to the renewable resource is made, all other things being equal.

Which effect dominates—the rate-of-consumption effect or the supply effect? In our numerical example, the supply-side effect dominates and, as a result, the time of transition

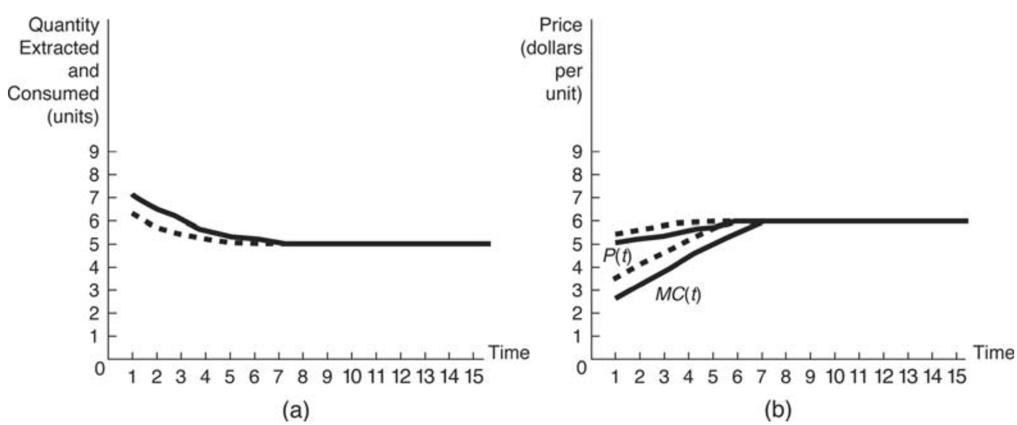

Figure 6.6 (a) Increasing Marginal Extraction Cost with Substitute Resource in the Presence of Environmental Costs: Quantity Profile (b) Increasing Marginal Extraction Cost with Substitute Resource in the Presence of Environmental Costs: Price Profile (Solid Line—without Environmental Costs; Dashed Line—with Environmental Costs)

for an efficient allocation is sooner than for the market allocation. In general, the answer depends on the shape of the marginal-extraction-cost function. With constant marginal cost, for example, there would be no supply-side effect and the market would unambiguously transition later. If the environmental costs were associated with the use of the renewable resource, rather than the depletable resource, the time of transition for the efficient allocation would have been later than the market allocation. Can you see why?

What can we learn from this analysis of the increasing cost case about the allocation of depletable resources over time when environmental side effects are <u>not</u> borne by the agent determining the extraction rate? Ignoring external costs leaves the market price of the depletable resource too low, too much of the resource would be extracted, and the rate at which it would be extracted would be too high relative to an efficient extraction profile.

Since policies that internalize these external costs can affect both the quantity extracted and price profiles, they can sometimes produce unexpected outcomes (see Example 6.2).

EXAMPLE 6.2

The Green Paradox

Common sense indicates that when pollution taxes to promote nonpolluting technology are imposed, they would lower emissions and improve welfare as long as the taxes weren't excessive. In an intriguing article, Sinn (2008) argues that in the case of global warming these demand-reducing policies could trigger (under certain conditions) price effects that could actually reduce welfare. Because this analysis suggests that polices designed to internalize an externality could actually result in lower economic welfare, this outcome was labeled "the green paradox."

The basic logic behind this finding is easily explained in terms of the depletable resource models developed in this chapter. The specific policy case examined by Sinn was a carbon tax rate that rises over time faster than the rate of interest. This carbon tax design changes the relative prices between current and future sales, increasing the relative profitability of earlier extraction. (Remember, one reason for delaying extraction was the higher prices extractors would gain in the future. With this specific tax profile the after-tax return is falling, not rising.) This policy would not only change the profit-maximizing extraction profile so that more is extracted earlier, but the present value of net benefits could fall.

Notice that this result depends on earlier, not larger, cumulative damages. In the constant MEC model, cumulative extraction (and hence, cumulative damages) is fixed so these polices would affect the timing, but not the magnitude, of the cumulative emissions. In the increasing cost MEC case, however, the cumulative emissions would actually be less; the imposition of the carbon tax would ultimately result in more of the depletable resource being left in the ground.

Is the Green Paradox a serious obstacle to climate policy? The early verdict seems to be no (van der Ploeg, 2013; Jensen et al., 2015), but the dearth of empirical evidence pointing either one way or the other leaves the door ajar.

Sources: Sinn, H.-W. (2008). Public policies against global warming: A supply side approach. *International Tax and Public Finance*, *15*, 360–394; van der Ploeg, F. (2013). Cumulative carbon emissions and the green paradox. *Annual Review of Resource Economics*, *5*, 281–300; Jensen, Svenn, Mohliny, Kristina, Pittelz, Karen, & Sterner, Thomas. (2015). An introduction to the green paradox: The unintended consequences of climate policies. *Review of Environmental Economics and Policy*, *9*, 246–265. doi:10.1093/reep/rev010

This once again demonstrates the interdependencies among the various decisions we have to make about the future. Environmental and natural resource decisions are intimately and inextricably linked.

Summary

The efficient extraction profiles for depletable and renewable resources depend on the circumstances. In the standard treatments when the resource can be extracted at a constant marginal cost, the efficient quantity of the depletable resource extracted declines over time. If no substitute is available, the quantity declines smoothly to zero. If a renewable constant-cost substitute is available, the quantity of the depletable resource extracted will decline smoothly to the quantity available from the renewable resource. In each case, all of the available depletable resource would be eventually used up and marginal user cost would rise over time, reaching a maximum when the last unit of depletable resource was extracted.

The efficient allocation of an increasing marginal-cost resource is similar in that the quantity extracted declines over time, but differs with respect to the behavior of marginal user cost and the cumulative amount extracted. Whereas marginal user cost typically rises over time when the marginal cost of extraction is constant, it declines over time when the marginal cost of extraction rises with the cumulative amount extracted. Furthermore, in the constant-cost case the cumulative amount extracted is equal to the available supply; in the increasing-cost case it depends on the relationship between the marginal extraction cost function and the cost of the substitute; some of the resource may be left in the ground unused.

Introducing technological progress and exploration activity into the model tends to delay the transition to renewable resources. Exploration expands the size of current reserves, while technological progress keeps marginal extraction cost from rising as fast as it otherwise would. If these effects are sufficiently potent, marginal cost could actually decline for some period of time, causing the quantity extracted to rise.

In the absence of environmental costs when property rights structures are properly defined, market allocations of depletable resources can be efficient. In this case profit maximization and efficiency can be compatible.

When the extraction of resources imposes an external environmental cost, however, generally market allocations will not be efficient. The market price of the depletable resource would be too low, the rate of extraction would be excessive, and too much of the resource would ultimately be extracted. (Think of this model when you read about the effects of fracking in Chapter 7.)

In an efficient market allocation, the transition from depletable to renewable resources is smooth and exhibits no overshoot-and-collapse characteristics. Whether the actual market allocations of these various types of resources are efficient remains to be seen. To the extent markets negotiate an efficient transition, a laissez-faire policy would represent an appropriate response by the government. On the other hand, when the market is not capable of yielding an efficient allocation, then some form of government intervention may be necessary. In the next few chapters, we shall examine these questions for a number of different types of depletable and renewable resources.

Discussion Question

1. One current practice is to calculate the years remaining for a depletable resource by taking the prevailing estimates of current reserves and dividing it by current annual consumption. How useful is that calculation? Why?

Self-Test Exercises

1. To anticipate subsequent chapters where more complicated renewable resource models are introduced, consider a slight modification of the two-period depletable resource model. Suppose a biological resource is renewable in the sense that any of it left unextracted after Period 1 will grow at rate k. Compared to the case where the total amount of a constant-MEC resource is fixed, how would the efficient allocation of this resource over the two periods differ? (*Hint*: It can be shown that $MNB_1/MNB_2 = (1 + k)/(1 + r)$, where *MNB* stands for marginal net benefit.)
2. Consider an increasing marginal-cost depletable resource with no effective substitute. (a) Describe, in general terms, how the marginal user cost for this resource in the earlier time periods would depend on whether the demand curve for that resource was stable or shifting outward over time. (b) How would the allocation of that resource over time be affected?
3. Many states are now imposing severance taxes on resources being extracted within their borders. In order to understand the effect of these taxes on the allocation of the mineral over time, assume a stable demand curve. (a) How would the competitive allocation of an increasing marginal-cost depletable resource be affected by the imposition of a per-unit tax (e.g., $4 per ton) if there exists a constant-marginal-cost substitute? (b) Comparing the allocation without a tax to one with a tax, in general terms, what are the differences in cumulative amounts extracted and the price paths?
4. For the increasing marginal-extraction-cost model of the allocation of a depletable resource, how would the ultimate cumulative amount taken out of the ground be affected by (a) an increase in the discount rate, (b) the extraction by a monopolistic, rather than a competitive, industry, and (c) a per-unit subsidy paid by the government for each unit of the abundant substitute used?
5. Suppose you wanted to hasten the transition from a depletable fossil fuel to solar energy. Compare the effects of a per-unit tax on the depletable resource to an equivalent per-unit subsidy on solar energy. Would they produce the same switch point? Why or why not?
6. Suppose a tax on the extraction of a depletable resource is enacted and it will first take effect 10 years in the future. This resource is assumed to have a renewable, constant MEC substitute that will remain untaxed.

   a. For a depletable resource characterized by a constant MEC how would, if at all, this pending law affect the extraction profile over time in terms of both the timing of the extraction and the cumulative amount extracted. Why?
   b. If the depletable resource is characterized by an increasing MEC, how would your answer in (a) change, if at all. Why?

Notes

1 Even renewable resources are ultimately finite because their renewability depends on energy from the sun and the sun is expected to serve as an energy source for only the next 5 or 6 billion years. Because the finiteness of renewable resources is sufficiently far into the future, the distinction between depletable and renewable resources remains useful as a practical matter.

2 The condition that marginal user cost rises at rate r turns out to be true only when the marginal cost of extraction is constant. Later in this chapter we show how the marginal user cost is affected when marginal extraction cost is not constant. Remember in Chapter 1 how we noted that the outcome of any model depends upon the assumptions that undergird it? This is one example of that point.

3 The new marginal cost of extraction is $MCt = \$2 + 0.1Q_t$ where Q_t is cumulative extraction to date.
4 Total marginal cost cannot be greater than the marginal cost of the substitute. Yet, in the increasing marginal extraction cost case, at the time of transition the marginal extraction cost must also equal the marginal cost of the substitute. If that weren't true, it would imply that some of the resource that was available at a marginal cost lower than the substitute would remain unused. This would clearly be inefficient, since net benefits could be increased by simply using it instead of the more expensive substitute. Hence, at the switch point, in the rising marginal-cost case, the marginal extraction cost has to equal total marginal cost, implying a zero marginal user cost.
5 To derive how a rising demand curve over time due to either rising income or population growth would affect the extraction profile, complete self-test exercise (2) at the end of this chapter.
6 Including environmental damage, the marginal cost function would be raised to $\$3 + 0.1Q$ instead of $\$2 + 0.1Q$.

Further Reading

Andre, F., & Cerda, E. (2006). On the dynamics of recycling and natural resources. *Environmental & Resource Economics*, *33*(2), 199–221. This article provides a formal examination of how the recyclability of depletable resources affects extraction profiles and sustainability.

Banzhaf, H. Spencer. (July 2016). The environmental turn in natural resource economics: John Krutilla and "conservation reconsidered." Resources for the Future Discussion paper DP 16-27. Available from: www.rff.org/files/document/file/RFF-DP-16-27.pdf. A discussion of how a classic paper in the early twentieth century used economics to unify the traditionally separate views of conservationists and preservationists.

Conrad, J. M., & Clark, C. W. (1987). *Natural Resource Economics: Notes and Problems*. Cambridge: Cambridge University Press. Reviews techniques of dynamic optimization and shows how they can be applied to the management of various resource systems.

Fischer, C., & Laxminarayan, R. (2005). Sequential development and exploitation of an exhaustible resource: Do monopoly rights promote conservation? *Journal of Environmental Economics and Management*, *49*(3), 500–515. Examines the conditions under which a monopolist would extract a depletable resource more quickly or more slowly than a competitive industry.

Strand, J. (2010). Optimal fossil-fuel taxation with backstop technologies and tenure risk. *Energy Economics*, *32*(2), 418–422. This article examines the time paths for optimal taxes and extraction profiles for a depletable resource that creates a negative stock externality (think climate change), involves increasing marginal extraction cost, and is subject to competition from an unlimited backstop resource causing no externality.

Additional references and historically significant references are available on this book's Companion Website: www.routledge.com/cw/Tietenberg

Appendix

Extensions of the Constant Extraction Cost Depletable Resource Model: Longer Time Horizons and the Role of an Abundant Substitute

In the appendix to Chapter 5, we derived a simple model to describe the efficient allocation of a constant-marginal-cost depletable resource over time and presented the numerical solution for a two-period version of that model. In this appendix, the mathematical derivations for the extension to that basic model will be documented, and the resulting numerical solutions for these more complicated cases will be explained.

The *N*-Period, Constant-Cost, No-Substitute Case

The first extension involves calculating the efficient allocation of the depletable resource over time when the number of time periods for extraction is unlimited. This is a more difficult calculation because how long the resource will last is no longer predetermined; the time of exhaustion must be derived as well as the extraction path prior to exhaustion of the resource.

The equations describing the allocation that maximizes the present value of net benefits are

$$\frac{a - bq_t - c}{(1+r)^{t-1}} - \lambda = 0,\ t = 1, \ldots, T \tag{1}$$

$$\sum_{t=1}^{n} q_t = \overline{Q} \tag{2}$$

The parameter values assumed for the numerical example presented in the text are

a = \$8, b = 0.4, c = \$2, $\overline{Q}$ = 40, and r = 0.10

The allocation that satisfies these conditions is

$q_1 = 8.004$	$q_4 = 5.689$	$q_7 = 2.607$	$T = 9$
$q_2 = 7.305$	$q_5 = 4.758$	$q_8 = 1.368$	$\lambda = 2.7983$
$q_3 = 6.535$	$q_6 = 3.733$	$q_9 = 0.000$	

The optimality of this allocation can be verified by substituting these values into the above equations. (Due to rounding, these add to 39.999, rather than 40.000.)

Practically speaking, solving these equations to find the optimal solution is not a trivial matter, but neither is it very difficult. One method of finding the solution for those without the requisite mathematics involves developing a computer algorithm (computation procedure) that converges on the correct answer. One such algorithm for this example can be constructed as follows: (1) assume a value for λ; (2) using Equation set (1) solve for all q's based upon this λ; (3) if the sum of the calculated q's exceeds $\overline{Q}$, adjust λ upward or if the sum of the calculated q's is less than $\overline{Q}$, adjust λ downward (the adjustment should use information gained in previous steps to ensure that the new trial will be closer to the solution value); (4) repeat steps (2) and (3) using the new λ; (5) when the sum of the q's is sufficiently close to $\overline{Q}$ stop the calculations. As an exercise, those interested in computer programming might construct a program to reproduce these results.

Constant Marginal Cost with an Abundant Renewable Substitute

The next extension assumes the existence of an abundant, renewable, perfect substitute, available in unlimited quantities at a cost of \$6 per unit. To derive the dynamically efficient allocation of both the depletable resource and its substitute, let q_t be the amount of a constant-marginal-cost depletable resource extracted in year t and q_{st} the amount used of another constant-marginal-cost resource that is perfectly substitutable for the depletable resource. The marginal cost of the substitute is assumed to be \$$d$.

With this change, the total benefit and cost formulas become

$$\text{Total benefit } = \sum_{t=1}^{T} a(q_t + q_{st}) - \frac{b}{2}(q_t + q_{st})^2 \tag{3}$$

$$\text{Total cost} = \sum_{t=1}^{T} (cq_t + dq_t) \tag{4}$$

The objective function is thus

$$PVNB = \sum_{t=1}^{T} \frac{a(q_t + q_{st}) - \frac{b}{2}(q_t^2 + q_{st}^2 + 2q_t q_{st}) - cq_t - dq_{st}}{(1+r)^{t-1}} \tag{5}$$

subject to the constraint on the total availability of the depletable resource

$$\overline{Q} - \sum_{t=1}^{T} q_t \geq 0 \tag{6}$$

Necessary and sufficient conditions for an allocation maximizing this function are expressed in Equations (7), (8), and (9):

$$\frac{a - b(q_t + q_{st}) - c}{(1+r)^{t-1}} - \lambda \leq 0,\ t = 1, \ldots, T \tag{7}$$

Any member of Equation set (7) will hold as an equality when $q_t > 0$ and will be negative when

$$a - b(q_t + q_{st}) - d \leq 0,\ t = 1, \ldots, T \tag{8}$$

Any member of Equation set (8) will hold as an equality when $q_{st} > 0$ and will be negative when $q_{st} = 0$

$$\overline{Q} - \sum_{t=1}^{T} q_t \geq 0 \tag{9}$$

For the numerical example used in the text, the following parameter values were assumed: $a = \$8$, $b = 0.4$, $c = \$2$, $d = \$6$, $Q = 40$, and $r = 0.10$. It can be readily verified that the optimal conditions are satisfied by

$$q_1 = 8.798 \quad q_3 = 7.495 \quad q_5 = 5.919$$
$$q_2 = 8.177 \quad q_4 = 6.744$$
$$q_{s6} = 2.137 \quad q_{st} = \begin{cases} 5.000 \text{ for } t > 6 \\ 0 \text{ for } t < 6 \end{cases}$$
$$q_6 = 2.863 \quad \lambda = 2.481$$

The depletable resource is used up before the end of the sixth period and the switch is made to the substitute resource at that time. From Equation set (8), in competitive markets the switch occurs precisely at the moment when the resource price rises to meet the marginal cost of the substitute.

The switch point in this example is earlier than in the previous example (the sixth period rather than the ninth period). Since all characteristics of the problem except for the availability of the substitute are the same in the two numerical examples, the difference can be attributed to the availability of the renewable substitute.

Chapter 14

Economics of Pollution Control

An Overview

Democracy is not a matter of sentiment, but of foresight. Any system that doesn't take the long run into account will burn itself out in the short run.

—Charles Yost, *The Age of Triumph and Frustration* (1964)

Introduction

In Chapter 2 we introduced a schematic describing the relationship between the natural and economic systems. One side depicted the flow of mass and energy to the economic system, while the other depicted the flow of waste products back to the environment. In the last few chapters we dealt extensively with different types of natural resources and maintaining efficient and sustainable levels for both stocks and flows of those resources. Now we turn to examining how a balance can be achieved in the reverse flow of waste products back to the environment. Because the waste flows are inexorably intertwined with the flow of mass and energy into the economy, establishing a balance for waste flows will have feedback effects on the input flows as well.

Two questions must be addressed: (1) what is the appropriate level of flow of pollution? and (2) how should the responsibility for achieving this flow level be allocated among the various sources of the pollutant when reductions are needed?

In this chapter we lay the foundation for understanding the policy approach to controlling the flow of these waste products by developing a general framework for analyzing pollution control. This framework allows us to define efficient and cost-effective allocations for a variety of pollutant types, to compare these allocations to market allocations, and to demonstrate how efficiency and cost-effectiveness can be used to formulate desirable policy responses. This overview is then followed by a series of chapters that apply these principles by examining the policy approaches that have been adopted in the United States and in the rest of the world to establish control over waste flows.

A Pollutant Taxonomy

The amount of waste products emitted determines the load upon the environment. The damage done by this load depends on the capacity of the environment to assimilate the waste products (see Figure 14.1). We call this ability of the environment to absorb pollutants its *absorptive capacity*. If the emissions load exceeds the absorptive capacity, then the pollutant accumulates in the environment.

Pollutants for which the environment has little or no absorptive capacity are called *stock pollutants*. Stock pollutants accumulate over time as emissions enter the environment. Examples of stock pollutants include nonbiodegradable bottles tossed by the roadside; heavy metals, such as lead, that accumulate in the soils near the emissions source; and persistent synthetic chemicals, such as dioxin and PCBs (polychlorinated biphenyls).

Pollutants for which the environment has some absorptive capacity are called *fund pollutants*. For these pollutants, as long as the emissions rate does not exceed the absorptive capacity of the environment, the pollutants do not accumulate. Examples of fund pollutants are easy to find. Many organic pollutants injected into an oxygen-rich stream will be transformed by the resident bacteria into less-harmful inorganic matter. Carbon dioxide is absorbed by plant life and the oceans.

The point is *not* that the mass is destroyed; the law of conservation of mass suggests this cannot be the case. Rather, when fund pollutants are injected into the air or water, they may be transformed into substances that are not considered harmful to people or to the ecological system, or they may be so diluted or dispersed that the resulting concentrations are not harmful.

Pollutants can also be classified by their zone of influence, defined both horizontally and vertically. The horizontal dimension deals with the spatial domain over which damage from an emitted pollutant is experienced. The damage caused by *local* pollutants is experienced near the source of emission, while the damage from *regional* pollutants is experienced at greater distances from the source of emission. The limiting case is a *global* pollutant, where the damage affects the entire planet. The categories are not mutually exclusive; it is possible for a pollutant to be in more than one category. Sulfur oxides and nitrogen oxides, for example, are both local and regional pollutants.

The vertical zone of influence describes whether the damage is caused mainly by ground-level concentrations of an air pollutant or by concentrations in the upper atmosphere. For some pollutants, such as lead or particulates, the damage is determined mainly by concentrations

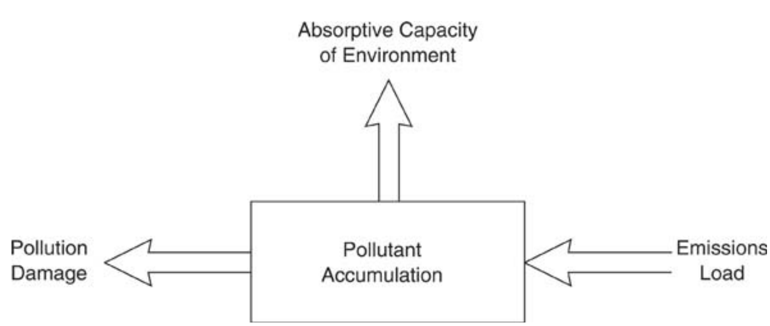

Figure 14.1 Relationship between Emissions and Pollution Damage

of the pollutant near the earth's surface. For others, such as ozone-depleting substances or greenhouse gases (described in Chapter 17), the damage is related more to their concentrations in the upper atmosphere. This taxonomy will prove useful in designing policy responses to these various types of pollution problems. Each type of pollutant requires a unique policy response. The failure to recognize these distinctions leads to counterproductive policy.

Defining the Efficient Allocation of Pollution

Pollutants are the residuals of production and consumption. These residuals must eventually be recycled or returned to the environment in one form or another. Since their presence in the environment may depreciate the service flows received, an efficient allocation of resources must take this cost into account. What is meant by the efficient allocation of pollution depends on the nature of the pollutant.

Stock Pollutants

The efficient allocation of a stock pollutant must take into account the fact that the pollutant accumulates in the environment over time and that the damage caused by its presence increases and persists as the pollutant accumulates. By their very nature, stock pollutants create an interdependency between the present and the future, since the damage imposed in the future depends on current actions.

The damage caused by pollution can take many forms. At high enough exposures to certain pollutants, human health can be adversely impacted, possibly even leading to death. Other living organisms, such as trees or fish, can be harmed as well. Damage can even occur to inanimate objects, as when acid rain causes sculptures to deteriorate or when particulates cause structures to discolor.

It is not hard to establish what is meant by an efficient allocation in these circumstances using the intuition we gained from the discussion of depletable resource models. Suppose, for example, that we consider the allocation of a commodity that we refer to as X. Suppose further that the production of X involves the generation of a proportional amount of a stock pollutant. The amount of this pollution can be reduced, but that takes resources away from the production of X. The damage caused by the presence of this pollutant in the environment is further assumed to be proportional to the size of the accumulated stock. As long as the stock of pollutants remains in the environment, the damage persists.

The dynamic efficient allocation, by definition, is the one that maximizes the present value of the net benefit. In this case the net benefit at any point in time, t, is equal to the benefit received from the consumption of X minus the cost of the damage caused by the presence of the stock pollutant in the environment.

This damage is a cost that society must bear, and in terms of its effect on the efficient allocation, this cost is not unlike that associated with extracting minerals or fuels. While for minerals the extraction cost rises with the cumulative amount of the depletable resource extracted, the damage cost associated with a stock pollutant rises with the cumulative amount deposited in the environment. The accretion of the stock pollutant is proportional to the production of X, which creates the same kind of linkage between the production of X and this pollution cost as exists between the extraction cost and the production of a mineral. They both rise over time with the cumulative amount produced. The one major difference is that the extraction cost is borne only at the time of extraction, while damage persists as long as the stock pollutant remains in the environment.

We can exploit this similarity to infer the efficient allocation of a stock pollutant. As discussed in Chapter 6, when extraction cost rises, the efficient quantity of a depletable resource extracted and consumed declines over time.

Exactly the same pattern would emerge for a commodity that is produced jointly with a stock pollutant. The efficient quantity of *X* (and therefore, the addition to the accumulation of this pollutant in the environment) would decline over time as the marginal cost of the damage rises. The price of *X* would rise over time, reflecting the rising social cost of production. To cope with the increasing marginal damage, the amount of resources committed to controlling the pollutant would increase over time. Ultimately, a steady state would be reached where additions to the amount of the pollutant in the environment would cease and the size of the pollutant stock would stabilize. At this point, all further emission of the pollutant created by the production of *X* would be controlled (perhaps through recycling). The price of *X* and the quantity consumed would remain constant. The damage caused by the stock pollutant would persist.

As was the case with rising extraction cost, technological progress could modify this efficient allocation. Specifically, technological progress could reduce the amount of pollutant generated per unit of *X* produced; it could create ways to recycle the stock pollutant rather than injecting it into the environment; or it could develop ways of rendering the pollutant less harmful. All of these responses would lower the marginal damage cost associated with a given level of production of *X*. Therefore, more of *X* could be produced with technological progress than without it.

Stock pollutants are, in a sense, the other side of the intergenerational equity coin from depletable resources. With depletable resources, it is possible for current generations to create a burden for future generations by using up resources, thereby diminishing the remaining endowment. Stock pollutants can create a burden for future generations by passing on damages that persist well after the benefits received from incurring the damages have been forgotten. Though neither of these situations automatically violates the weak sustainability criterion, they don't automatically satisfy it either.

Fund Pollutants

To the extent that the emission of fund pollutants exceeds the assimilative capacity of the environment, they accumulate and share some of the characteristics of stock pollutants. When the emissions rate is low enough, however, the discharges can be assimilated by the environment, with the result that the link between present emissions and future damage may be broken.

When this happens, current emissions cause current damage, and future emissions cause future damage, but the level of future damage is independent of current emissions. This independence of allocations among time periods allows us to explore the efficient allocation of fund pollutants using the concept of static, rather than dynamic, efficiency. Because the static concept is simpler, this affords us the opportunity to incorporate more dimensions of the problem without unnecessarily complicating the analysis.

The normal starting point for the analysis would be to maximize the net benefit from the waste flows. However, pollution is more easily understood if we deal with a mathematically equivalent formulation involving the minimization of two rather different types of costs: damage costs and control or avoidance costs.

To examine the efficient allocation graphically, we need to know something about how control costs vary with the degree of control and how the damages vary with the amount of pollution emitted. Though our knowledge in these areas is far from complete, economists normally agree on the shapes of these relationships.

Generally, the marginal damage caused by a unit of pollution increases with the amount emitted. When small amounts of the pollutant are emitted, the incremental damage is quite small. However, when large amounts are emitted, the marginal unit can cause significantly more damage. It is not hard to understand why. Small amounts of pollution are easily diluted in the environment, and the body can tolerate small quantities of substances. However, as the amount in the atmosphere increases, dilution is less effective and the body is less tolerant.

Marginal control costs commonly increase with the amount controlled. For example, suppose a source of pollution tries to cut down on its particulate emissions by purchasing an electrostatic precipitator that captures 80 percent of the particulates as they flow past in the stack. If the source wants further control, it can purchase another precipitator and place it in the stack above the first one. This second precipitator captures 80 percent of the remaining 20 percent, or 16 percent of the uncontrolled emissions. Thus, the first precipitator would achieve an 80 percent reduction from uncontrolled emissions, while the second precipitator, which costs the same as the first, would achieve only a further 16 percent reduction. Obviously each unit of emissions reduction by the second precipitator costs more than by the first.

In Figure 14.2 we use these two pieces of information on the shapes of the relevant curves to derive the efficient allocation. A movement from right to left refers to greater control and less pollution emitted. The efficient allocation is represented by Q^*, the point at which the damage caused by the marginal unit of pollution is exactly equal to the marginal cost of avoiding it.[1]

Greater degrees of control (points to the left of Q^*) are inefficient because the further increase in avoidance costs would exceed the reduction in damages. Hence, total costs would rise. Similarly, levels of control lower than Q^* would result in a lower cost of control but the increase in damage costs would be even larger, yielding an increase in total cost. Increasing or decreasing the amount controlled causes an increase in total costs. Hence, Q^* must be efficient.

The diagram suggests that under the conditions presented, the optimal level of pollution is not zero. If you find this disturbing, remember that we confront this principle every day. Take the damage caused by automobile accidents, for example. Obviously, a considerable

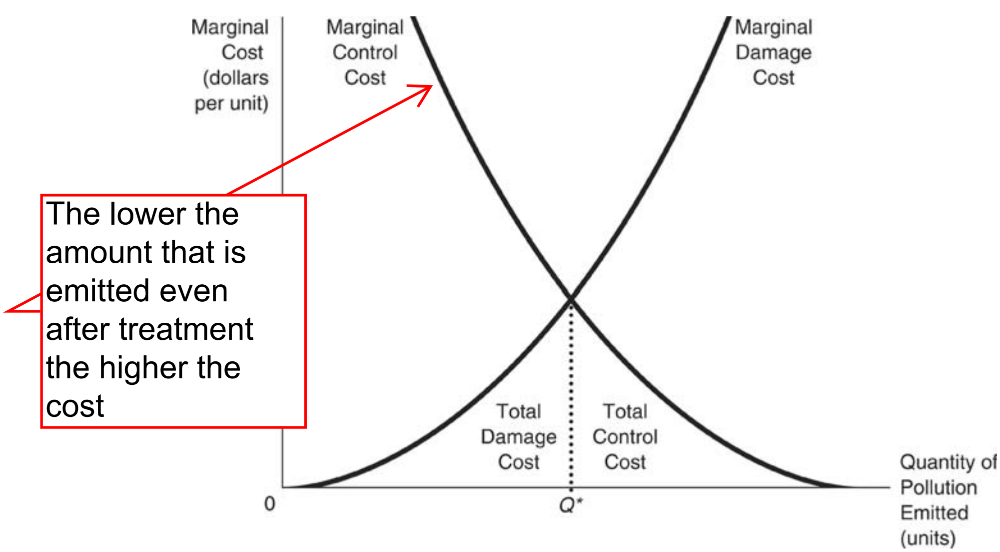

Figure 14.2 Efficient Allocation of a Fund Pollutant

amount of damage is caused by automobile accidents, yet we do not reduce that damage to zero because the cost of doing so would be too high.

The point is *not* that we do not know how to stop automobile accidents. All we would have to do is eliminate automobiles! Rather, the point is that since we value the benefits of automobiles, we take steps to reduce accidents (such as using speed limits) only to the extent that the costs of accident reduction are commensurate with the damage reduction achieved. The efficient level of automobile accidents is not zero.

The second point is that in some circumstances the optimal level of pollution *may* be zero, or close to it. This situation occurs when the damage caused by even the first unit of pollution is so severe that it is higher than the marginal cost of controlling it. This would be reflected in Figure 14.2 as a leftward shift of the damage cost curve of sufficient magnitude that its intersection with the vertical axis would lie above the point where the marginal cost curve intersects the vertical axis. This circumstance seems to characterize the treatment of highly dangerous radioactive pollutants such as plutonium.

Additional insights are easily derived from our characterization of the efficient allocation. For example, it should be clear from Figure 14.2 that the optimal level of pollution generally is not the same for all parts of the country. Areas that have higher population levels or are particularly sensitive to pollution would have a marginal damage cost curve that intersected the marginal control cost curve close to the vertical axis. Efficiency would imply lower levels of pollution for those areas. Areas that have lower population levels or are less sensitive should have higher efficient levels of pollution.

Examples of ecological sensitivity are not hard to find. For instance, some areas are less sensitive to acid rain than others because the local geological strata neutralize moderate amounts of the acid. Thus, the marginal damage caused by a unit of acid rain is lower in those fortunate regions than in other, less tolerant regions. It can also be argued that pollutants affecting visibility are more damaging in national parks and other areas where visibility is an important part of the aesthetic experience than in other more industrial areas.

Market Allocation of Pollution

Since air and water are treated in our legal system as common-pool resources, at this point in the book it should surprise no one that the market misallocates them. Our previously derived conclusion that free-access resources are overexploited certainly also applies here. Air and water resources have been overexploited as waste repositories. However, this conclusion only scratches the surface; much more can be learned about market allocations of pollution.

When firms create products, rarely does the process of converting raw material into outputs use 100 percent of the mass. The typical firm has several alternatives to control the amount of the residual. It can use inputs more completely so that less is left over. It can also produce less output, so that smaller amounts of the residual are generated. Recycling the residual is sometimes a viable option, as is removing the most damaging components of the waste stream and disposing of the rest.

Pollutant damages are commonly externalities to the firms that produce them.[2] When pollutants are injected into water bodies or the atmosphere, they cause damages to those firms and consumers (as well as to flora and fauna) downstream or downwind of the source, not to the source itself. These costs are typically *not* borne by the emitting source and hence not considered by it, although they certainly are borne by society at large.[3] As with other services that are systematically undervalued, the disposal of wastes into the air or water becomes inefficiently attractive. In this case the firm minimizes its costs when it chooses not

to abate anything, since the only costs it bears are the control costs. What is cheapest for the firm is not cheapest for society.

In the case of stock pollutants, the problem is particularly severe. Uncontrolled markets would lead to an excessive production of the product that generates the pollution, too few resources committed to pollution control, and an inefficiently large amount of the stock pollutant in the environment. Thus, the burden on future generations caused by the presence of this pollutant would be inefficiently large.

The inefficiencies associated with pollution control and the previously discussed inefficiencies associated with the extraction or production of minerals, energy, and food exhibit some rather important differences. For private property resources, the market forces provide automatic signals of impending scarcity. These forces may be understated (as when the vulnerability of imports is ignored), but they operate in the correct direction. Even when some resources are treated as open-access (fisheries), the possibility for a private property alternative (fish farming) is enhanced. When private property and open-access resources sell in the same market, the private property owner tends to ameliorate the excesses of those who exploit open-access properties. Efficient firms are rewarded with higher profits.

With pollution, no comparable automatic amelioration mechanism is evident.[4] Because this cost is borne partially by innocent victims rather than producers, it does not find its way into product prices. Firms that attempt unilaterally to control their pollution are placed at a competitive disadvantage; due to the added expense, their costs of production are higher than those of their less conscientious competitors. Not only does the unimpeded market fail to generate the efficient level of pollution control, but also it penalizes those firms that might attempt to control an efficient amount. Hence, the case for some sort of government intervention is particularly strong for pollution control.

Efficient Policy Responses

Our use of the efficiency criterion has helped demonstrate why markets fail to produce an efficient level of pollution control as well as trace out the effects of this less-than-optimal degree of control on the markets for related commodities. It can also be used to define efficient policy responses.

In Figure 14.2 we demonstrated that, for a market as a whole, efficiency is achieved when the marginal cost of control is equal to the marginal damage caused by the pollution. This same principle applies to each emitter. Each emitter should control its pollution until the marginal cost of controlling the last unit is equal to the marginal damage it causes. One way to achieve this outcome would be to impose a legal limit on the amount of pollution allowed by each emitter. If the limit were chosen precisely at the level of emission where marginal control cost equaled the marginal damage, efficiency would have been achieved for that emitter.

An alternative approach would be to internalize the marginal damage caused by each unit of emissions by means of a tax or charge on each unit of emissions. Either this per-unit charge could increase with the level of pollution (following the marginal damage curve for each succeeding unit of emission) or the tax rate could be constant as long as the rate were equal to the marginal social damage at the point where the marginal social damage and marginal control costs cross (see Figure 14.2). Since the emitter is paying the marginal social damage when confronted by these fees, pollution costs would be internalized. The efficient choice would also be the cost-minimizing choice for the emitter.[5]

While the efficient levels of these policy instruments can be easily defined in principle, they are very difficult to implement in practice. To implement either of these policy instruments,

we must know the level of emissions at which the two marginal cost curves cross for every emitter. That is a tall order, one that imposes an unrealistically high information burden on control authorities. Control authorities typically have very poor information on control costs and little reliable information on marginal damage functions.

How can environmental authorities allocate pollution control responsibility in a reasonable manner when the information burdens are apparently so unrealistically large? One approach, the choice of several countries including the United States, is to select specific legal levels of pollution based on some other criterion, such as providing adequate margins of safety for human or ecological health. Once these thresholds have been established by whatever means, only half of the problem has been resolved. The other half deals with deciding how to allocate the responsibility for meeting predetermined pollution levels among the large numbers of emitters.

This is precisely where the cost-effectiveness criterion comes in. Once the objective is stated in terms of meeting the predetermined pollution level at minimum cost, it is possible to derive the conditions that any cost-effective allocation of the responsibility must satisfy. These conditions can then be used as a basis for choosing among various kinds of policy instruments that impose more reasonable information burdens on control authorities.

Cost-Effective Policies for Uniformly Mixed Fund Pollutants

Defining a Cost-Effective Allocation

We begin our analysis with uniformly mixed fund pollutants, which analytically are the easiest to deal with. The damage caused by these pollutants depends simply on the amount entering the atmosphere. Thus, the policy can focus simply on controlling the total amount of emissions in a manner that minimizes the cost of control. What can we say about the cost-effective allocation of control responsibility for uniformly mixed fund pollutants?

Consider a simple example. Assume that two emissions sources are currently emitting 15 units each for a total 30 units. Assume further that the control authority determines that the environment can assimilate 15 units in total, so that a reduction of 15 units is necessary. How should this 15-unit reduction be allocated between the two sources in order to minimize the total cost of the reduction?

We can demonstrate the answer with the aid of Figure 14.3, which is drawn by measuring the marginal cost of control for the first source from the left-hand axis (MC_1) and the marginal cost of control for the second source from the right-hand axis (MC_2). Note that a total 15-unit reduction is achieved for every point on this graph; each point represents some different combination of reduction by the two sources that sums to 15. Drawn in this manner, the diagram represents all possible allocations of the 15-unit reduction between the two sources. The left-hand axis, for example, represents an allocation of the entire reduction to the second source, while the right-hand axis represents a situation in which the first source bears the entire responsibility. All points in between represent different degrees of shared responsibility. What allocation minimizes the cost of control?

In the cost-effective allocation, the first source cleans up ten units, while the second source cleans up five units. The total variable cost of control for this particular assignment of the responsibility for the reduction is represented by area *A* plus area *B*. Area *A* is the cost of control for the first source; area *B* is the cost of control for the second. Any other allocation would result in a higher total control cost. (Convince yourself that this is true.)

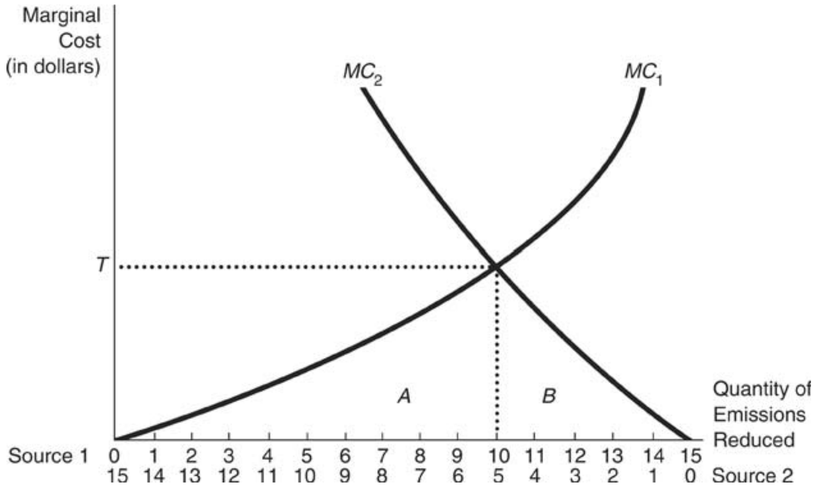

Figure 14.3 Cost-Effective Allocation of a Uniformly Mixed Fund Pollutant

Figure 14.3 also demonstrates the cost-effectiveness equimarginal principle introduced in Chapter 3. *The cost of achieving a given reduction in emissions will be minimized if and only if the marginal costs of control are equalized for all emitters.*[6] This is demonstrated by the fact that the marginal cost curves cross at the cost-effective allocation.

Cost-Effective Pollution Control Policies

This proposition can be used as a basis for choosing among the various policy instruments that the control authority might use to achieve this allocation. Sources have a large menu of options for controlling the amount of pollution they inject into the environment. The cheapest method of control will differ widely not only among industries but also among plants in the same industry. The selection of the cheapest method requires detailed information on the possible control techniques and their associated costs.

Generally, plant managers are able to acquire this information for their plants when it is in their interest to do so. However, the government authorities responsible for meeting pollution targets are not likely to have this information. Since the degree to which these plants would be regulated depends on cost information, it is unrealistic to expect these plant managers to transfer unbiased information to the government. Plant managers would have a strong incentive to overstate control costs in hopes of reducing their ultimate control burden.

This situation poses a difficult dilemma for control authorities. The cost of incorrectly assigning the control responsibility among various polluters is likely to be large. Yet the control authorities do not have sufficient information at their disposal to make a correct allocation. Those who have the information—the plant managers—are not inclined to share it. Can the cost-effective allocation be found? The answer depends on the approach taken by the control authority.

Emissions Standards. We start our investigation of this question by supposing that the control authority pursues a traditional legal approach by imposing a separate emissions limit on each source. In the economics literature this approach is referred to as the

"command-and-control" approach. An *emissions standard* is a legal limit on the amount of the pollutant an individual source is allowed to emit. In our example it is clear that the two standards should add up to the allowable 15 units, but it is not clear how, in the absence of information on control costs, these 15 units are to be allocated between the two sources.

The easiest method of resolving this dilemma—and the one chosen in the earliest days of pollution control—would be simply to allocate each source an equal reduction. As is clear from Figure 14.3, this strategy would not be cost-effective. While the first source would have lower costs compared to the cost-effective allocation, this cost reduction would be substantially smaller than the cost increase faced by the second source. Compared to a cost-effective allocation, total costs would increase if both sources were forced to clean up the same amount.

When emissions standards are the policy of choice, there is no reason to believe that the authority will assign the responsibility for emissions reduction in a cost-minimizing way. This is probably not surprising. Who would have believed otherwise?

Surprisingly enough, however, some policy instruments do allow the authority to allocate the emissions reduction in a cost-effective manner even when it has no information on the magnitude of control costs. These policy approaches rely on economic incentives to produce the desired outcome. The two most common approaches are known as emissions charges and emissions trading.

Emissions Charges. An *emissions charge* is a fee, collected by the government, levied on each unit of pollutant emitted into the air or water. The total payment any source would make to the government could be found by multiplying the fee times the amount of pollution emitted. Emissions charges reduce pollution because paying the fees costs the firm money. To save money, the source seeks ways to reduce its pollution.

How much pollution control would the firm choose? A profit-maximizing firm would control, rather than emit, pollution whenever it proved cheaper to do so. We can illustrate the firm's decision with Figure 14.4. The level of uncontrolled emission is 15 units and the emissions charge is T. Thus, if the firm were to decide against controlling any emissions, it would have to pay T times 15, represented by area $0TBC$.

Is this the best the firm can do? Obviously not, since it can control some pollution at a lower cost than paying the emissions charge. It would pay the firm to reduce emissions until the marginal cost of reduction is equal to the emissions charge. After that point it is cheaper

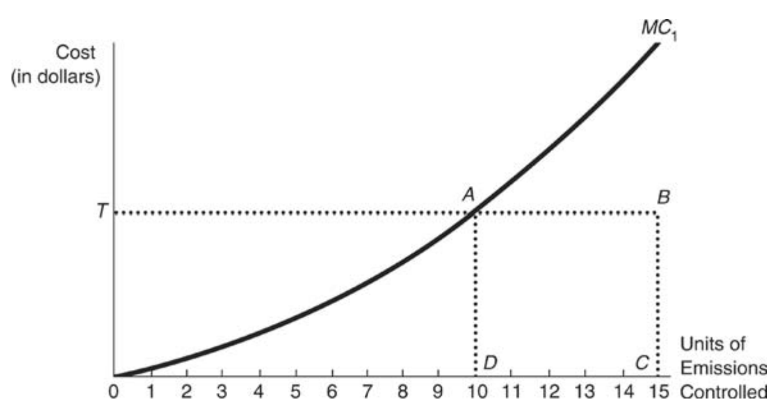

Figure 14.4 Cost-Minimizing Control of Pollution with an Emissions Charge

for the firm to pay the tax since the marginal cost curve rises above the tax. The firm would minimize its cost by choosing to clean up ten units of pollution and to emit five units. At this allocation the firm would pay control costs equal to area $0AD$ and total emissions charge payments equal to area $ABCD$ for a total cost of $0ABC$. This is clearly less than $0TBC$, the amount the firm would pay if it chose not to clean up any pollution.

Let's carry this one step further. Suppose that we levied the same emissions charge on both sources discussed in Figure 14.3. Each source would then control its emissions until its marginal control cost equaled the emissions charge. (Faced with an emissions charge T, the second source would clean up five units.) Since they both face the same emissions charge, they will *independently* choose levels of control consistent with equal marginal control costs. This is precisely the condition that yields a cost-minimizing allocation.

This is a remarkable finding. We have shown that as long as the control authority imposes the same emissions charge on all sources, the resulting incentives are *automatically* compatible with minimizing the costs of achieving that level of control. This is true in spite of the fact that the control authority may not have sufficient knowledge of control costs.

However, we have not yet dealt with the issue of how the appropriate level of the emissions charge is determined. Each level of a charge will result in *some* level of emissions reduction. Furthermore, as long as each firm minimizes its own costs, the responsibility for meeting that reduction will be allocated in a manner that minimizes control costs for all firms. How high should the charge be set to ensure that the resulting emissions reduction is the *desired* level of emissions reduction?

Without having the requisite information on control costs, the control authority cannot establish the correct tax rate on the first try. It is possible, however, to develop an iterative, trial-and-error process to find the appropriate charge rate. This process is initiated by choosing an arbitrary charge rate and observing the amount of reduction that occurs when that charge is imposed. If the observed reduction is larger than desired, it means the charge should be lowered; if the reduction is smaller, the charge should be raised. The new reduction that results from the adjusted charge can then be observed and compared with the desired reduction. Further adjustments in the charge can be made as needed. This process can be repeated until the actual and desired reductions are equal. At that point the correct emissions charge would have been found.

The charge system not only causes cost-minimizing sources to choose a cost-effective allocation of the control responsibility, it also stimulates the development of newer, cheaper means of controlling emissions, as well as promoting technological progress. This is illustrated in Figure 14.5.

The reason for this is rather straightforward. Control authorities base the emissions standards on specific technologies. As new technologies are discovered by the control authority, the standards are tightened. These stricter standards force firms to bear higher costs. Therefore, with emissions standards, firms have an incentive to hide technological changes from the control authority.

With an emissions charge system, the firm saves money by adopting cheaper new technologies. As long as the firm can reduce its pollution at a marginal cost lower than T, it pays to adopt the new technology. In Figure 14.5 MC^0 represents the MC before the new technology is adopted and MC^1 is the new lower marginal cost with the adoption of the new technology. The firm saves A and B by adopting the new technology and voluntarily increases its emissions reduction from Q^0 to Q^1.

With an emissions charge, the minimum cost allocation of meeting a predetermined emissions reduction can be found by a control authority even when it has insufficient information on control costs. An emissions charge also stimulates technological advances in

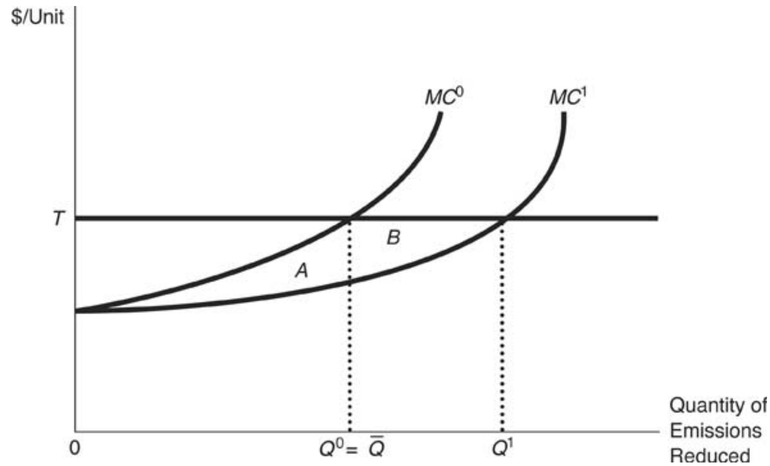

Figure 14.5 Cost Savings from Technological Change: Charges versus Standards

emissions reduction. Unfortunately, the process for finding the appropriate rate takes some experimenting. During the trial-and-error period of finding the appropriate rate, sources would be faced with a volatile emissions charge. Emissions charges that kept changing in the search for the right level would make planning for the future difficult for the firms subject to that charge. Investments that would make sense under a high emissions charge might not make sense when it falls. From either a policymaker's or business manager's perspective, this process leaves much to be desired.

Cap-and-Trade. Is it possible for the control authority to find the cost-minimizing allocation without going through a trial-and-error process? It is possible if cap-and-trade (a form of emissions trading) is the chosen policy. Under this system, all sources face a collective limit on their emissions (the cap) and they are allocated (or sold) allowances to emit. Each allowance authorizes a specific amount of emissions (commonly 1 ton). The control authority issues exactly the total number of allowances needed to produce the desired emissions level. These can be distributed among the firms either by auctioning them off to the highest bidder or by granting them directly to firms free of charge (an allocation referred to as "gifting"). However they are acquired, the allowances are freely transferable; they can be bought and sold. Firms emitting more than their holdings would buy additional allowances from firms who are emitting less than authorized. Any emissions by a source in excess of those allowed by its allowance holdings at the end of the year would cause the source to face severe monetary sanctions.

Why this system automatically leads to a cost-effective allocation can be seen in Figure 14.6. This figure treats the same set of circumstances as in Figure 14.3. Consider first the gifting alternative. Suppose that the first source was allocated seven allowances (each corresponds to one emission unit). Because it has 15 units of uncontrolled emissions, this would mean it must control eight units. Similarly, suppose that the second source was granted the remaining eight allowances. It would have to clean up seven units. Notice that both firms have an incentive to trade. The marginal cost of control for the second source (C) is

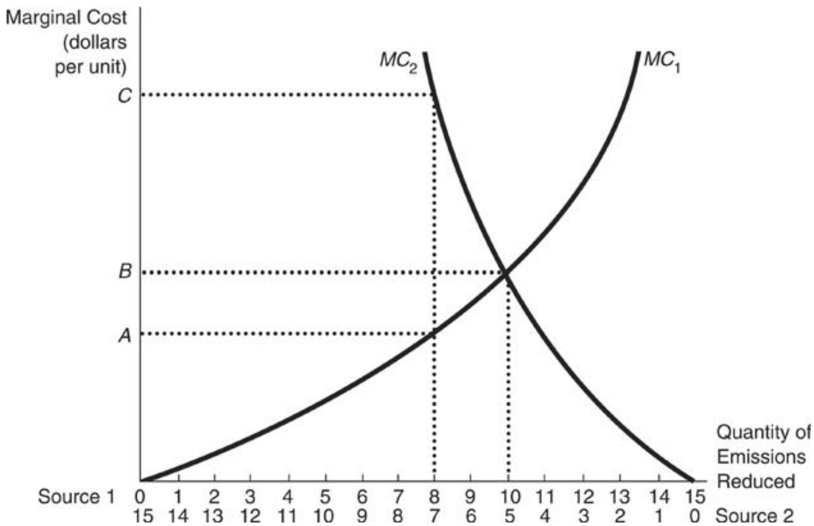

Figure 14.6 Cost-Effectiveness and Emissions Trading

substantially higher than that for the first (*A*). The second source could lower its cost if it could buy an allowance from the first source at a price lower than *C*. Meanwhile, the first source would be better off if it could sell an allowance for a price higher than *A*. Because *C* is greater than *A*, grounds for trade certainly exist.

A transfer of allowances would take place until the first source had only five allowances left (and controlled ten units), while the second source had ten allowances (and controlled five units). At this point, the allowance price would equal *B*, because that is the marginal value of that allowance to both sources, and neither source would have any incentive to trade further. The allowance market would be in equilibrium.

Notice that the market equilibrium for an emission-allowance system is the cost-effective allocation! Simply by issuing the appropriate number of allowances (15) and letting the market do the rest, the control authority can achieve a cost-effective allocation without having even the slightest knowledge about control costs. This system allows the government to meet its policy objective, while allowing greater flexibility in how that objective is met.

How would this equilibrium change if the allowances were auctioned off? Interestingly, it wouldn't; both allocation methods lead to the same result. With an auction, the allowance price that clears demand and supply is *B*, and we have already demonstrated that *B* supports a cost-effective equilibrium.

The incentives created by this system ensure that sources use this flexibility to achieve the objective at the lowest possible cost. As we shall see in the next two chapters, this remarkable property has been responsible for the prominence of this type of approach in current attempts to reform the regulatory process.

How far can the reforms go? Can developing countries use the experience of the industrialized countries to move directly into using these market-based instruments to control pollution?

As Debate 14.1 points out, that may be easier said than done.

DEBATE 14.1

Should Developing Countries Rely on Market-Based Instruments to Control Pollution?

Since the case for using market-based instruments seems so strong in principle, some observers have suggested that developing countries should capitalize on the experience of the industrialized countries to move directly to market-based instruments (such as emissions charges or cap-and-trade) to control pollution. The desirability of this strategy is seen as flowing from the level of poverty in developing countries; abating pollution in the least expensive manner would seem especially important to poorer nations. Furthermore, since developing countries are frequently also starved for revenue, revenue-generating instruments (such as emissions charges or auctioned allowances) would seem especially useful. Proponents also point out that a number of developing countries already use market-based instruments.

Another school of thought suggests that the differences in infrastructure between the developing and industrialized countries make the transfer of lessons from one context to another fraught with peril. To illustrate their more general point, they note that the effectiveness of market-based instruments presumes an effective monitoring and enforcement system, something that is frequently not present in developing countries. In its absence, the superiority of market-based instruments is much less obvious.

Some middle ground is clearly emerging. Those who are skeptical do not argue that market-based instruments should never be used in developing countries, but rather that they may not be as universally appropriate as the most enthusiastic proponents seem to suggest. They see themselves as telling a cautionary tale. And proponents are certainly beginning to see the crucial importance of infrastructure. Recognizing that some developing countries may be much better suited (by virtue of their infrastructure) to implement market-based systems than others, proponents are beginning to see capacity building as a logical prior step for those countries that need it.

For market-based instruments, as well as for other aspects of life, if it looks too good to be true, it probably is.

Sources: World Bank. (2000). *Greening Industry*: *New Roles for Communities, Markets and Governments*. Washington, D.C.: World Bank and Oxford University Press; Russell, C. S., & Vaughan, W. J. (2003). The choice of pollution control policy instruments in developing countries: Arguments, evidence and suggestions. In H. Folmer & T. Tietenberg (Eds.), *The International Yearbook of Environmental and Resource Economics 2003/2004*. Cheltenham, U.K.: Edward Elgar, 331–371.

Other Policy Dimensions

Two main pollution control policy instruments rely on economic incentives—charges and cap-and-trade. Both of these allow the control authority to distribute the responsibility for

control in a cost-effective manner. The major difference between them we have discussed so far is that the appropriate charge can be determined only by an iterative trial-and-error process over time, whereas for the cap-and-trade approach the allowance price can be determined immediately by the market. Can other differences be identified? As it turns out, yes.

The Revenue Effect

One of the differentiating characteristics of these instruments is their ability to raise revenue. Environmental taxes and auctioned allowances raise revenue, but cap-and-trade programs that gift the allowances to users free of charge do not. Does this difference matter?

It does, for at least two reasons.[7] First, a number of authors (Parry & Bento, 2000; Bovenberg & Goulder, 1996; Goulder, 1997; Parry, 1995) have noted that the revenue from environmental taxes or auctioned transferable allowances could be substituted for the revenue from distortionary taxes, thereby reducing those taxes and their associated distortions. When this substitution is made, the calculations indicate that it allows an increase in the present value of net benefits from the application of this instrument, an effect that has been called the "double dividend." This effect creates a preference for instruments that can raise revenue as long as both the implementation of a revenue-raising instrument and the use of this revenue to reduce distortionary taxes are politically feasible.

The second important consideration is that the revenue from taxes or auctions could be used to reduce the burden on low-income households. The empirical evidence suggests that gifting allowances produces a regressive distribution of the control burden. (A regressive distribution is one that places a higher relative cost burden on low-income households or individuals as a percentage of their income.) That same evidence has also demonstrated that when the revenue from auctions or taxes is targeted appropriately, the regressiveness of the policy can be eliminated.

A final consequence of raising revenues involves their political feasibility. It seems quite clear that, until 2008, using a free-distribution ("gifting") approach for the initial allocation of allowances was a necessary ingredient to build the necessary political support for cap-and-trade legislation to be implemented (Raymond, 2003). Existing users frequently have the power to block implementation, while potential future users do not. This made it politically expedient to allocate a substantial part of the economic rent from these resources to existing users as the price of securing their support, sometimes in creative ways (see Example 14.1).

While this strategy reduces the adjustment costs to existing users, generally it raises them for new users. Interestingly in the climate change case, the empirical evidence suggests that only a small fraction of the total revenue would be needed to assure that the profits of carbon suppliers would be unchanged by a switch to a revenue raising approach (Bovenberg & Goulder, 2002). Gifting all allowances therefore may not be inevitable in principle, even if political feasibility considerations affect the design.

While the earliest programs gifted the allowances to large emitters, later programs have tended to rely more on auctions. On January 1, 2009, the historic tendency to "gift" allowances changed with the implementation of the Regional Greenhouse Gas Initiative (RGGI) in nine Northeastern states, from Maryland to Maine. This cap-and-trade program covers CO_2 emissions from large fossil fuel-fired electricity-generating plants.

A number of RGGI states have chosen to auction nearly 100 percent of these allowances, using a sealed-bid system, with the revenue returned to the states. Most states have chosen to use the revenue to promote energy efficiency (see Example 14.2), although two states—New York and New Hampshire—chose to siphon off some of the money for budget relief. New Jersey also pursued this latter option and subsequently dropped out of RGGI.

EXAMPLE 14.1

The Swedish Nitrogen Oxide Charge

One of the dilemmas facing those who wish to use charges to control pollution is that the amounts of revenue extracted from those subject to the tax can be considerable and that additional expense can produce a lot of political resistance to the policy. This resistance can be lowered if the revenue is rebated to those who pay it. However if all firms know they are getting their money back, the economic incentive to limit emissions is lost. Is it possible to design a system of rebates that will promote political feasibility without undermining abatement incentives?

The Swedish nitrogen oxide charge was designed specifically to resolve this dilemma. It was first imposed in 1992 on large energy sources. Some 120 heating plants and industrial facilities with about 180 boilers were subject to the tax.

It was intended from the beginning to have a significant incentive effect, not to raise revenue. Although the charge rate is high by international standards (thereby producing an effective economic incentive), the revenue from this tax is not retained by the government, but rather is rebated to the emitting sources (thereby lowering resistance to the policy by the regulated sources).

It is the form of this rebate that makes this an interesting scheme. While the tax is collected on the basis of *emissions*, it is rebated on the basis of *energy production*. In effect, this system rewards plants that emit little nitrogen oxide per unit of energy and penalizes plants that emit more per unit of energy. Designed in this way it provides incentives to reduce emissions per unit of energy produced.

As expected, emissions per unit of energy produced fell rather dramatically. Over the period from 1992 to 2007, the plants were able to reduce the amount of emissions per unit of input energy by 67 percent. According to one study (OECD, 2010) there were three main explanations for this:

- Cumulative energy output produced by the plants increased by 74 percent over the period. The expansion in output mostly took place in plants that were relatively emission-efficient.
- Regulated plants invested in NOx mitigation and were therefore able to produce more energy output with fewer emissions.
- Innovations in mitigation technology made it possible to reach even lower emission intensity levels for the same output level.

Note, however, that rebating the revenue means that this tax cannot produce a double dividend.

Sources: Organisation for Economic Co-operation and Development. (2013). The Swedish tax on nitrogen oxide emissions: Lessons in environmental policy reform. OECD Environment Policy Paper No. 2; OECD. (2010). *Innovation Impacts of the Swedish NOx Charge*. Paris: OECD. Available at www.oecd.org/greengrowth/consumption-innovation/43211635.pdf.

Using the revenue from auctions to promote investment in energy efficiency reduces the cost of meeting the carbon targets. Costs are reduced not only because less energy is used (and hence less carbon emitted), but also because the lower demand for energy lowers the price not only of the allowances, but of electricity too. (Can you see why?)

It would be hard to overemphasize what a departure from the previous norm this venture into auctioning represents. Allowing emitters to pollute up to the emissions standard without paying for the right to pollute (the traditional approach) implies that emitters have an implicit property right to pollute already; they don't have to buy it.

A cap-and-trade program with allowance auctions implies, in contrast, that the atmosphere is held in trust for the community. Institutions that use the atmosphere for emissions must therefore pay to acquire that scarce right. Notice that this understanding of who actually holds the property right to the atmosphere completely changes the lens through which this regulation is viewed.

EXAMPLE 14.2

RGGI Revenue: The Maine Example

The revenue received by Maine from the quarterly RGGI auctions is received by Efficiency Maine (EM), a specially created, quasi-independent organization. The enabling statute requires EM to spend most of the RGGI funds on energy efficiency measures such as more efficient lighting, motors, heating, and air conditioning, as well as on building weatherization.

For large customers such as paper mills, the money is allocated in response to applications from the large customers for specific projects. These are evaluated on the basis of the amount of kilowatt-hours reduced (for electricity) or tons of CO_2 reduced (for fossil fuels) *per EM dollar expended*. (Notice how focusing on public dollars, as opposed to the sum of public and private dollars, provides an incentive for cost sharing on the part of companies—putting more of their own money and less public money into the project raises the ratio of the savings per EM dollar and, hence, increases the likelihood of success of their proposed funding request.)

To be funded, all proposals must also pass a benefit-cost test to assure the resources are being used efficiently. EM's 2016 annual report notes that the FY2016 benefit-cost ratios for these large customers were 3.26 for the electric programs and 3.16 for the thermal programs. The comparable numbers for all EM programs were respectively 2.63 and 1.89.

The investments in energy efficiency incentivized by these funds have been very cost-effective. The data demonstrate that at the margin saving energy is cheaper than buying it. In other words investing in energy savings actually lowers energy costs. Lower energy costs have made participating Maine firms more cost competitive and have saved jobs and bolstered the local economy, while reducing emissions of one of the gases that contributes to climate change.

Sources: The Regional Greenhouse Gas Initiative auction results website: www.rggi.org/market/co2_auctions/results#state_proceeds (accessed June 19, 2017); Acadia Center. (2016). Regional greenhouse gas initiative status report: Part I—measuring success. Available at: http://acadiacenter.org/document/measuring-rggi-success/ (accessed June 19, 2017); Efficiency Maine (EM). (2016). *FY2016 Annual Report*. Available at: www.efficiencymaine.com/docs/FY2016-Annual-Report.pdf (accessed on June 19, 2017).

Responses to Changes in the Regulatory Environment

One major additional difference between charges and allowances concerns the manner in which these two systems react to changes in external circumstances in the absence of further decisions by the control authority. This is an important consideration, because bureaucratic procedures are notoriously sluggish and changes in policies are usually rendered slowly.[8] We consider three such circumstances: growth in the number of sources, inflation, and technological progress.

If the number of sources were to increase in a cap-and-trade program, the demand for allowances would shift to the right. Given a fixed supply of allowances, the price would rise, as would the control costs, but the amount of emissions would remain the same. If charges were being used, in the absence of additional action by the control authority the charge level would remain the same. This implies that the amount the existing sources would control would be unchanged by the growth. Therefore, the arrival of new sources would cause a deterioration of air or water quality in the region due to the added emissions by the new sources. The costs of abatement would rise, since the costs of control paid by the new sources must be considered, but by a lesser amount than with cap-and-trade, because of the lower amount of pollution being controlled. If the economy is growing, the allowance system ensures that emissions will not rise.

Inflation in the cost of control would automatically result in higher allowance prices in a cap-and-trade program, but with a charge system it would result in lower control. Essentially, the real charge (the nominal charge adjusted for inflation) declines with inflation if the nominal charge remains the same.

We should not, however, conclude that over time, charges always result in less control than allowances. Suppose, for example, technological progress in designing pollution control equipment were to cause the marginal cost of abatement to fall. With cap-and-trade this would result in lower prices and lower abatement costs, but the same aggregate degree of control. With a charge system, the amount controlled would actually increase (see Figure 14.5) and, therefore, would result in more control than a cap-and-trade program that, prior to the fall in costs, controlled the same amount.

Instrument Choice under Uncertainty

Another major difference between allowances and charges involves the cost of being wrong. Suppose that we have very imprecise information on damages caused and avoidance costs incurred by various levels of pollution and yet we have to choose either a charge level or an allowance level and live with it. What can be said about the relative merits of allowances versus charges in the face of this uncertainty?

The answer depends on the circumstances. Allowances offer a greater amount of certainty about the quantity of emissions, while charges confer more certainty about the marginal cost of control. Allowance markets allow an aggregate emissions standard to be met with certainty, but they offer less certainty about marginal costs. When the objective is to minimize total costs (the sum of damage cost and control costs), allowances would be preferred when the costs of being wrong are more sensitive to changes in the quantity of emissions than to changes in the marginal cost of control. Charges would be preferred when control costs were more important. What circumstances give rise to a preference for one or the other?

When the marginal damage curve is steeply sloped and the marginal cost curve is rather flat, certainty about emissions is more important than certainty over control costs. Smaller deviations of actual emissions from expected emissions can cause a rather large deviation in damage costs, whereas control costs would be relatively insensitive to the degree of control.

Allowances would prevent large fluctuations in these damage costs and therefore would yield a lower cost of being wrong than charges.

Suppose, however, that the marginal control cost curve was steeply sloped, but the marginal damage curve was flat. Small changes in the degree of control would have a large effect on abatement costs but would not affect damages very much. In this case it makes sense to rely on charges to give more precise control over control costs, accepting the less dire consequences from possible fluctuations in damage costs.

Theory is not strong enough to dictate a choice. Empirical studies are necessary to establish a preference for particular situations.

One interesting current application of these insights involves the control of the gases that intensify climate change. As we will see in the next chapters, the shape of the marginal cost curve matters for the choice of policy instrument. For our most complex issue, greenhouse gases, growing scientific evidence suggests that climatic responses to temperature increases may well be highly nonlinear, characterized by thresholds or abrupt changes. This understanding of the science leads to a greater sensitivity of damages to the level of emissions reduction, shifting the preference toward cap-and-trade (Keohane, 2009). These cases suggest that a preference either for allowances or for charges in the face of uncertainty is not universal; it depends on the circumstances.

Summary

In this chapter we developed the conceptual framework needed to evaluate current approaches to pollution control policy. We have explored many different types of pollutants, and found that context matters. Different policy approaches are appropriate for different circumstances.

Stock pollutants pose the most serious intertemporal problems. The efficient production of a commodity that generates a stock pollutant could be expected to decline over time. Theoretically, a point would be reached when all of the pollutant would be recycled. After this point, the amount of the pollutant in the environment would not increase. The amount already accumulated, however, would continue to cause damage perpetually unless some natural process could reduce the amount of the pollutant over time.

The efficient amount of a fund pollutant was defined as the amount that minimizes the sum of damage and control costs. Using this definition, we were able to derive two propositions of interest: (1) the efficient level of pollution would vary from region to region; and (2) the efficient level of pollution would not generally be zero, although in some particular circumstances it might.

Since pollution is a classic externality, markets will generally produce more than the efficient amount of both fund pollutants and stock pollutants. For both types of pollutants, this will imply higher-than-efficient damages and lower-than-efficient control costs. For stock pollutants, an excessive amount of pollution would accumulate in the environment, imposing a detrimental externality on future generations as well as on current generations.

The market would not provide any automatic ameliorating response to the accumulation of pollution as it would in the case of natural resource scarcity. Firms attempting to unilaterally control their pollution could be placed at a competitive disadvantage. Hence, the case for some sort of government intervention is particularly strong for pollution control.

While policy instruments could, in principle, be defined to achieve an efficient level of pollution for every emitter, it is very difficult in practice because the amount of information required by the control authorities is unrealistically high.

Cost-effectiveness analysis provides a way out of this dilemma. In the case of uniformly mixed fund pollutants, uniform emissions charges or an allowance system focused purely on emissions could be used to attain the cost-effective allocation, even when the control authority has no information whatsoever on either control costs or damage costs. Uniform emissions standards would not, except by coincidence, be cost-effective. In addition, either emissions trading or charges would provide more incentives for technological progress in pollution control than would emissions standards.

The fact that auctioned allowances or taxes can raise revenue is also an important characteristic. If the revenue from pollution charges or auctioned allowances can be used to reduce revenue from other, more distortionary taxes (such as labor or income taxes), greater welfare gains can be achieved from revenue-raising instruments than instruments that raise no revenue. On the other hand, historically at least, transferring some or all of that revenue back to the sources either by gifting the allowances or including some sort of tax rebate has been an important aspect of securing the political support for implementing the system. Revenue use for this purpose, of course, cannot be used to reduce distortionary taxes or lower the regressive nature of the program.

The allowance approach and the charge approach respond differently to growth in the number of sources, to inflation, to technological change, and to uncertainty. As we shall see in the next few chapters, some countries have chosen to rely on emissions charges, while others have chosen to rely on cap-and-trade.

Discussion Question

1. In his book *What Price Incentives?*, Steven Kelman suggests that from an ethical point of view, the use of economic incentives (such as emissions charges or emissions trading) in environmental policy is undesirable. He argues that transforming our mental image of the environment from a sanctified preserve to a marketable commodity has detrimental effects not only on our use of the environment, but also on our attitude toward it. His point is that applying economic incentives to environmental policy weakens and cheapens our traditional values toward the environment.

    a. Consider the effects of economic incentive systems on prices paid by the poor, on employment, and on the speed of compliance with pollution-control laws—as well as the Kelman arguments. Are economic incentive systems more or less ethically justifiable than the traditional regulatory approach?
    b. Kelman seems to feel that because emissions allowances automatically prevent environmental degradation, they are more ethically desirable than emissions charges. Do you agree? Why or why not?

Self-Test Exercises

1. Two firms can control emissions at the following marginal costs: $MC_1 = \$200q_1$, $MC_2 = \$100q_2$, where q_1 and q_2 are, respectively, the amount of emissions reduced by the first and second firms. Assume that with no control at all, each firm would be emitting 20 units of emissions or a total of 40 units for both firms.

    Compute the cost-effective allocation of control responsibility if a total reduction of 21 units of emissions is necessary.

2. Assume that the control authority wanted to reach its objective in 1 by using an emissions charge system.
   a. What per-unit charge should be imposed?
   b. How much revenue would the control authority collect?
3. In a region that must reduce emissions, three polluters currently emit 30 units of emissions. The three firms have the following marginal abatement cost functions that describe how marginal costs vary with the amount of emissions each firm reduces.

Firm Emissions Reduction	*Firm 1 Marginal Cost*	*Firm 2 Marginal Cost*	*Firm 3 Marginal Cost*
1	$1.00	$1.00	$2.00
2	$1.50	$2.00	$3.00
3	$2.00	$3.00	$4.00
4	$2.50	$4.00	$5.00
5	$3.00	$5.00	$6.00
6	$3.50	$6.00	$7.00
7	$4.00	$7.00	$8.00
8	$4.50	$8.00	$9.00
9	$5.00	$9.00	$10.00
10	$5.50	$10.00	$11.00

   Suppose this region needs to reduce emissions by 14 units and plans to do it using a form of cap-and-trade that auctions allowances off to the highest bidder.

   a. How many allowances will the control authority auction off? Why?
   b. Assuming no market power, how many of the allowances would each firm be expected to buy? Why?
   c. Assuming that demand equals supply, what price would be paid for those allowances? Why?
   d. If the control authority decided to use an emissions tax rather than cap-and-trade, what tax rate would achieve the 14-unit reduction cost-effectively? Why?

Notes

1 At this point, we can see why this formulation is equivalent to the net benefit formulation. Since the benefit is damage reduction, another way of stating this proposition is that marginal benefit must equal marginal cost. That is, of course, the familiar proposition derived by maximizing net benefits.
2 Note that pollution damage is not inevitably an externality. For any automobile rigged to send all exhaust gases into its interior, those exhaust gases would not be an externality to the occupants.
3 Actually the source certainly considers some of the costs, if only to avoid adverse public relations. The point, however, is that this consideration is likely to be incomplete; the source is unlikely to internalize all of the damage cost.
4 Affected parties do have an incentive to negotiate among themselves, a topic covered in Chapter 2. As pointed out there, however, that approach works well only in cases where the number of affected parties is small.
5 Another policy choice is to remove the people from the polluted area. The government has used this strategy for heavily contaminated toxic-waste sites, such as Times Beach, Missouri, and Love Canal, New York, as we shall see in Chapter 19.

Answers to Self-Test Exercises

Chapter 1

1. A shortage would promote higher prices, thereby lowering demand until it equaled the new smaller supply. Since this acts to reduce rather than intensify the shortage, it is a negative feedback loop.

   If consumers anticipate these higher prices, however, thereby buying and hoarding extra amounts before the prices rise, this is an example of a positive feedback loop because it intensifies the shortage.

Chapter 2

1. a. This is a public good, so add the 100 demand curves vertically. This yields $P = 1{,}000 - 100q$. This demand curve would intersect the marginal-cost curve when $P = 500$, which occurs when $q = 5$ miles.

   b. The economic surplus is represented by a right triangle, where the height of the triangle is \$500 (\$1,000, the point where the demand curve crosses the vertical axis, minus \$500, the marginal cost) and the base is 5 miles. The area of a right triangle is 1/2 × base × height = 1/2 × \$500 × 5 = \$1,250.

2. a. Set $MC = P$, so $80 - 1q = 1q$. Solving for q finds that $q = 40$ and $P = 40$.

   b. Consumer surplus = \$800. Producer surplus = \$800. Consumer surplus plus producer surplus = \$1,600 = economic surplus.

   c. The marginal revenue curve has twice the slope of the demand curve, so $MR = 80 - 2q$. Setting $MR = MC$, yields $q = 80/3$ and $P = 160/3$. Using Figure 2.8, producer surplus is the area under the price line (FE) and over the marginal-cost line (DH). This can be computed as the sum of a rectangle (formed by FED and a horizontal line drawn from D to the vertical axis) and a triangle (formed by DH and the point created by the intersection of the horizontal line drawn from D with the vertical axis).
   The area of any rectangle is base × height. The base = 80/3 and the

$$\text{Height} = P - MC = \frac{160}{3} - \frac{80}{3} = \frac{80}{3}.$$

   Therefore, the area of the rectangle is 6400/9. The area of the right triangle is

$$\frac{1}{2} \times \frac{80}{3} \times \frac{80}{3} = \frac{3{,}200}{9}.$$

$$\text{Producer surplus} = \frac{3{,}200}{9} + \frac{6{,}400}{9}$$

$$= \frac{\$9{,}600}{9}.$$

$$\text{Consumer surplus} = \frac{1}{2} \times \frac{80}{3} \times \frac{80}{3}$$

$$= \frac{\$3{,}200}{9}.$$

1. $\dfrac{\$9{,}600}{9} > \$800$
2. $\dfrac{\$3{,}200}{9} < \$800$

3. $\dfrac{\$12{,}800}{9} < \$1{,}600$

3. The policy would not be consistent with efficiency. As the firm considers measures to reduce the magnitude of any spill, it would compare the marginal costs of those measures with the expected marginal reduction in its liability from reducing the magnitude of the spill. Yet the expected marginal reduction in liability from a smaller spill would be zero. Firms would pay $\$X$ regardless of the size of the spill. Since the amount paid cannot be reduced by controlling the size of the spill, the incentive to take precautions that reduce the size of the spill will be inefficiently low.
4. If "better" means efficient, this common belief is not necessarily true. Damage awards are efficient when they equal the damage caused. Ensuring that the award reflects the actual damage will appropriately internalize the external cost. Larger damage awards are more efficient only to the extent that they more closely approximate the actual damage. Whenever they promote an excessive level of precaution that cannot be justified by the damages, awards that exceed actual cost are inefficient. Bigger is not always better.

5. a. Descriptive. It is possible to estimate this linkage empirically.
   b. Normative. A descriptive analysis could estimate the impacts of expenditures on endangered species, but moving from that analysis to a conclusion that expenditures would be wasted requires injecting values into the analysis.
   c. Normative. A descriptive analysis could compare the effects of privatized and nonprivatized fisheries, but moving from these results to a conclusion that the fisheries must be privatized to survive normally requires an injection of values. If the data revealed that all privatized fisheries survived and none of the others did, the move to "must" would have a very strong descriptive underpinning.
   d. Descriptive. This linkage could be estimated empirically directly from the data.
   e. Normative. This statement could be descriptive if it was stated as "birth control programs actually contribute to a rise in population" since this is an empirical relationship that could be investigated.

      However, as stated, it allows a much wider scope of aspects to enter the debate and weighing the importance of those aspects will normally require value judgments.

6. a. A pod of whales is a common-pool resource to whale hunters. It is characterized by nonexclusivity and divisibility.
   b. A pod of whales is a public good to whale watchers since it is characterized by both nondivisibility and nonexclusivity.
   c. The benefits from reductions of greenhouse gas emissions are public goods because they are both nondivisible and nonexclusive.
   d. For residents, a town water supply is a common-pool resource because it is both divisible and nonexclusive to town residents. It is not a common-pool resource for nonresidents since they can be excluded.
   e. Bottled water is neither; it is both divisible and exclusive. In fact it is a private good.

Chapter 3

1. With risk neutrality, the policy should be pursued because the expected net benefits (0.85 × \$4,000,000 + 0.10 × \$1,000,000 + 0.05 × –\$10,000,000 = \$3,000,000) are positive. Related Discussion Question: Looking at these numbers, do you think risk neutrality is how you would actually think about this situation? Or would you be more risk averse and weigh the third outcome more heavily than its expected likelihood?
2. a. Cost-effectiveness in this case (according to the second equimarginal principle) requires that that target be met (10 fish removed) and the marginal costs of each method be equal. We know that $q_1 + q_2 + q_3 = 10$ and that $MC_1 = MC_2 = MC_3$. The key is to reduce this to one equation with one unknown. Since $MC_1 = MC_2$ we know that $\$10q_1$ will equal $\$5q_2$, or $q_1 = .5q_2$. Similarly, $MC_2 = MC_3$, so $\$5q_2 = \$2.5q_3$ or $q_3 = 2q_2$. Substituting these values into the first equation yields $.5q_2 + 1q_2 + 2q_2 = 10$. So $q_2 = 10/3.5 = 2.86$ (to two decimal places.) That means $q_1 = 1.43$ and $q_3 = 5.72$. (The fact that this adds to 10.01 rather than 10.00 is due to rounding.)
   b. All three of these methods have a marginal cost that increases with the amount removed. Thus the cost of removing the first fish for each is cheaper than removing the second fish with that method, and so on. Consider the marginal cost of removing the last fish if all fish are removed by method three. In that case the marginal cost would be \$2.5 × 10 or \$25. Notice that the cost-effective allocation, the cost of removing the last fish when the marginal costs are equal (using q_1 for the calculation) is \$10 × \$1.43 = \$14.30. In the case of increasing marginal costs using a combination is much cheaper.
   c. In this case you would only use method three because the marginal cost of removing each fish would be \$2.5. This is lower than the *MC* for method 1 (\$10) and lower than the *MC* for method 2 (\$5). Note that the marginal costs only have to be equal for the methods that are actually used. The marginal costs for unused methods will be higher.
3. Since the benefit cost test requires that the present value of benefits be greater than the present value of the costs, we can find the maximum allowable current cost by calculating the present value of the benefits. This can be calculated as $\$500,000,000,000/(1 + r)^{50}$ where r is either 0.10 or 0.02. Whereas with a 10 percent discount rate the present value is approximately \$4.3 billion, with a 2 percent discount rate it is approximately \$185.8 billion. Clearly the size of the discount rate matters a lot in determining efficient current expenditures to resolve a long-range problem.

Chapter 4

1. In order to maximize net benefits, Coast Guard oil-spill prevention enforcement activity should be increased until the marginal benefit of the last unit equals the marginal cost of providing that unit. Efficiency requires that the level of the activity be chosen so as to equate marginal benefit with marginal cost. When marginal benefits exceed marginal cost (as in this example), the activity should be expanded.

2. a. According to the figures given, the per-life cost of the standard for unvented space heaters lies well under the implied value of life estimates given in the chapter, while per-life cost implied by the proposed standard for formaldehyde lies well over those estimates. In benefit-cost terms, the allocation of resources to fixing unvented space heaters should be increased, while the formaldehyde standard should be relaxed somewhat to bring the costs back into line with the benefits.
   b. Efficiency requires that the marginal benefit of a life saved in government programs (as determined by the implied value of a human life in that context) should be equal to the marginal cost of saving that life. Marginal costs should be equal only if the marginal benefits are equal and, as we saw in the chapter, risk valuations (and hence the implied value of human life) depend on the risk context, so it is unlikely they are equal across all government programs.

3. a. The total willingness to pay for this risk reduction is $200 million ($50 per person × 4 million exposed people.) The expected number of lives saved would be 40 (1/100,000 risk of premature death × 4,000,000 exposed population). The implied value of a statistical life would be $5,000,000 ($200,000,000 total willingness to pay/40 lives saved).
   b. The program is expected to save 160 lives ((6/100,000 – 2/100,000) × 4,000,000). According to the value of a statistical life in (a), the program will have more benefits than costs as long as it costs no more than $800,000,000 ($5,000,000 value per life × 160 lives saved).

Chapter 5

1. a. Ten units would be allocated to each period.
   b. $P = \$8 - 0.4q = \$8 - \$4 = \$4$
   c. User cost $= P - MC = \$4 - \$2 = \$2$

2. Because in this example the static allocations to the two periods (those that ignore the effects on the other period) are feasible within the 20 units available, the marginal user cost would be zero. With a marginal cost of $4.00, the net benefits in each period would independently be maximized by allocating 10 units to each period. In this example no intertemporal scarcity is present, so price would equal a $4.00 marginal cost.
3. Refer to Figure 5.2. In the second version of the model, the lower marginal extraction cost in the second period would raise the marginal net benefit curve in that period (since marginal net benefit is the difference between the unchanged demand curve and the lower *MC* curve). This would be reflected in Figure 5.2 as a parallel leftward shift out of the curve labeled "Present Value of Marginal Net Benefits in Period 2." This shift would immediately have two consequences: it would move the intersection to the left (implying relatively more would be extracted in the second period), and the intersection would take

place at a higher vertical distance from the horizontal axis (implying that the marginal user cost would have risen).

4. a. The higher discount rate would lower the present value of the net benefit function in the second period. This would be reflected as a rotation of that function downward to the right. The new function would necessarily cross the $PVMNB_1$ function at a point further to the right and lower than before the discount rate change. The fact that the intersection is further to the right implies that more is being allocated to period 1 and less to period 2. The fact that the intersection is lower implies that the present value of the marginal user cost has declined.
   b. Since a higher discount rate lowers the present value of allocations made to the second period, allocating relatively more of the resources to the first period will increase the present value derived from them. The present value of the marginal user cost is lower since the marginal opportunity cost of using the resources earlier has gone down.

5. a. Increasing the second period demand is reflected in the two-period model by a shift (not a rotation) in the $PVMNB_2$ curve upward and to the left. After the shift, this new function will necessarily intersect the $PVMNB_1$ curve closer to the left-hand axis and higher up on the Y-axis. This implies an increase in the relative amount allocated to the second period (thereby reducing the amount allocated to the first period) and a higher present value of the marginal user cost.
   b. When demand is increasing in the future (hence making the marginal resources relatively more valuable), it makes sense to save more for the future. This is accomplished by a rise in the marginal user cost, which results in higher prices. The higher prices provide the incentive to save more for the future. More is consumed in the second period despite the higher prices because the demand curve has shifted out.

Chapter 6

1. From the hint, $MNB_1/MNB_2 = (1 + k)/(1 + r)$. Notice that when $k = 0$, this reduces to $MNB_2 = MNB_1(1 + r)$, the case we have already considered. When $k = r$, then $MNB_1 = MNB_2$; the effect of stock growth exactly offsets the effect of discounting, and both periods extract the same amount. If $r > k$, then $MNB_2 > MNB_1$. If r 6<k, then $MNB_2 < MNB_1$.

2. a. With a demand curve shifting out over time, the marginal net benefits from a given future allocation increase over time. This raises the marginal user cost (since it is the opportunity cost of using the resource now) and, hence, the total marginal cost. Thus, the initial user cost would be higher.
   b. Less of the resource would be consumed in the present; more would be saved for the future.

3. a. This turns out to have the same effect as the environmental cost pictured in Figures 6.6a and 6.6b. The tax serves to raise the total marginal cost and, hence, the price. This tends to lower the amount consumed in all periods compared to a competitive allocation.

   b. The tax also serves to reduce the cumulative amount extracted because it raises the marginal cost of each unit extracted. Some resources that would have been extracted without the tax would not be extracted with the tax; their after-tax cost to the producer exceeds the cost of the substitute. The price would be higher with the tax

in all periods prior to the without-tax switch point. After that time the price would be equal to the price of the substitute with or without the tax.

4. The cumulative amount ultimately taken out of the ground is determined by the point at which the marginal extraction cost equals the maximum price consumers will pay for the depletable resource. In this model the maximum price is the price of the substitute. Neither the monopoly nor the discount rate affects either the marginal extraction cost or the price of the substitute, so they will have no effect on the cumulative amount ultimately extracted. The subsidy, however, has the effect of lowering the net price (price minus subsidy) of the substitute. The intersection of marginal extraction cost and the net price will, therefore, occur when a smaller cumulative amount has been extracted than would be the case in the absence of the subsidy.
5. They would not produce the same switch point. The switch would be faster under the subsidy. While they would result in the same cumulative amount of the depletable resource being extracted, the speed with which it would be extracted would be faster with the subsidy. By raising the after-tax price the tax would reduce demand (and hence the speed with which the depletable resource would be used up), while the subsidy would, by lowering the marginal user cost, increase demand (and hence increase the rate at which the depletable resource was extracted).

6. a. The impending tax would lower the after-tax, per-unit revenue to the extractors of the depletable resource once it was enacted. In anticipation of this change suppliers would have an incentive to shift extraction to periods before the tax is imposed; relatively more would be extracted earlier. Because the cumulative amount extracted is determined by the price of the substitute, which is not changed by the act, the total amount extracted would remain unchanged. Only the timing would be affected.
   b. In this case the pending tax would have the same incentive to shift extraction earlier as in (a), but the cumulative amount extracted of the depletable resource would go down. The rising marginal cost, including the tax, would hit the marginal cost of the abundant resource at a lower level of cumulative extraction.

Chapter 7

1. During a recession, the demand curve shifts inward, causing downward pressure on prices. If price is supported, then the quantity supplied must be reduced. Since the burden of holding the price up falls on the cartel, while the competitive fringe can keep on producing, the demand reduction causes production to fall most heavily in OPEC nations. This causes the cartel market share to fall. To protect their individual market shares, members start cutting prices. In growing markets, cartel market shares can be protected without cutting prices.

2. a.

$$\text{Producer surplus} = \frac{\$3{,}200}{9},\ P = MC = \frac{\$80}{3}.$$

$$\text{Consumer surplus} = \frac{\$9{,}600}{9},\ q = \frac{80}{3}.$$

   b. This is the mirror image of the monopoly allocation. The net benefits are identical in the two allocations, but they are distributed among producers and consumers rather differently. With this form of price control, the consumer surplus is larger and the

of origin. Making them transferable across national boundaries allows them to be sold to the highest bidder. This transferability provides incentives for the credit supplier to create the credits and to sell them to the highest bidders. The buyers who acquire the credits are likely to have the most to gain from their acquisition. Efficiency is enhanced because both the buyer and the seller gain from the transaction and so net benefits are increased.

b. Conservation banking allows landowners to fulfill their conservation obligations on one site by acquiring the requisite conservation entitlements from a much larger project on another site. If these entitlements were not transferable from one site to another, the original owner would have to fulfill the obligation on her own land. Because that approach would likely be smaller in scale, typically would involve less-suitable, fragmented habitat, and would be more expensive, transferability allows the obligation to be met with lower cost and at a better (less fragmented) scale, while making more appropriate habitat available to the endangered species.

2. With a rise in demand in the recreational fishery its members are likely to want to increase their catch shares. In the absence of inter-sector transferability this is likely to occur only if an administrative process changes the historical catch shares to allocate more to the recreational fishery and less to the commercial fishery. Any such change is likely to be opposed by the commercial fishery members since each share transferred represents a monetary loss for them. With inter-sector transferability, however, the recreational fishery members would have to buy the additional shares. They would do this by offering a higher price for catch shares, resulting in a shift in some shares from the commercial to a recreational fishery. Transferability reduces conflict because the transactions are voluntary and in this case the sellers gain, not lose.

Chapter 14

1. In a cost-effective allocation of emissions reduction, the marginal control costs should be equal. So $\$200q_1 = \$100q_2$. Furthermore, the total reduction is 21 units, so $q_1 + q_2 = 21$. Solving the first of these equations for q_1 yields $q_1 - 0.5q_2$. Substituting this into the second yields $0.5q_2 + q_2 = 21$. Solving this for q_2 results in $q_2 = 14$ and $q_1 = 7$.

2. a. From the text we know $T = MC_1 = MC_2$. From Problem 1(a) we know $MC_1 = MC_2 = \$1{,}400$. Therefore, $T = \$1{,}400$.

   b. Revenue = $T(20 - q_1) + T(20 - q_2) = \$1{,}400(13) \times \$1{,}400(6) = \$26{,}600$.

3. a. The control authority would auction off 16 allowances (30, which is the current level of emissions, minus 14, which is the required reduction).

   b. The market-clearing price would be \$4. Since demand would equal supply and marginal abatement costs would be equal for all firms, a \$4 marginal abatement cost produces the required 14 units of reduction.

   c. With a \$4 price, Firm 1 would reduce emissions by 7 units so it would need to buy 3 allowances. Firm 2 would reduce emissions by 4 units and hence would need to buy 6 allowances, and firm 3 would reduce 3 units of emissions and therefore it would need to buy 7 allowances. Note that this produces the required 14 units of reduction and accounts for the 16 allowances that were made available by the control authority.

   d. We know that the cost-effective allocation is achieved when the $MC_1 = MC_2 = MC_3 = \$4$. This allocation will be achieved with an emissions charge if the firms set their MCs equal to \$4. Hence the required tax rate is \$4.